PRACTICAL ELECTRICITY.

PRACTICAL ELECTRICITY:

A

LABORATORY AND LECTURE COURSE,

For First Year Students of Electrical Engineering,

BASED ON THE

International Definitions of the Electrical Units.

COMPLETELY RE-WRITTEN

BY

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VOLUME I.

CURRENT, PRESSURE, RESISTANCE, ENERGY, POWER AND CELLS.

WITH 247 ILLUSTRATIONS.

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PREFACE.

EXACTLY ten years have elapsed since the preface to the first edition of this book was written—a decade which has seen a vast development in the applications of electricity to industrial purposes, and the springing up in all parts of the kingdom of Technical Schools and Colleges where much attention is devoted to the study of electrotechnics. Hence, to-day it is far more easy for a student to connect his experimental apparatus with the electric light mains and use a comparatively large current at a pressure of 100 volts, than it was in 1886 to obtain a small current at a much lower pressure from the battery which he had to set up for the purpose. This possibility of carrying out the experiments on a larger scale has led to considerable simplification in certain cases; for example, in experimentally determining the heat equivalent of electric energy, it is no longer necessary to distract the beginner's attention with a variety of corrections for the loss of heat, &c.

After many issues of the book had appeared in its original form, it seemed desirable to bring it up to date; and since the practice, not unfrequently resorted to by writers, of inserting a number of new patches in an antiquated ground work, would be out of place in a book which had been written to aid electrotechnical teaching and not for purposes of profit, a proposition was made to entirely rewrite it. This the publishers accepted; and, guided by the success which the book had achieved, they generously, and I anticipate wisely, modified the arrangements so as to justify my devoting a large amount

of time to the preparation of what in reality is an entirely new book, although called by its old title "PRACTICAL ELECTRICITY."

The reception of the first edition took me by surprise. I anticipated that the book would be regarded as "faddy," and that the critics, while admitting that perhaps it would do well enough for my own classes, would not recommend its use for students in general. It did not occur to me that the world was ready for using such a text-book and prepared to adopt the methods of teaching advocated in its pages. To-day, however, the following reasons suggested in the original preface for even elementary students of electricity spending much time in the laboratory would be advanced by many teachers:—

"One of the great difficulties experienced by people in mastering the *quantitative* science of electricity, arises from the fact that we do not number an electrical sense among our other senses, and hence we have no intuitive perception of electrical phenomena. During childhood we did not have years of unconscious experimenting with electrical forces as we had with the forces connected with the sensations of heaviness and lightness, loudness and softness, heat and cold. Beyond a shock or two taken perhaps from some medical galvanic apparatus, or from a Leyden jar, our senses have never been affected by electrical action, and hence we ought to begin the study of electricity as a child begins its early education. Quite an infant has distinct ideas about hot and cold, although it may not be able to put its ideas into words, and yet many a student of electricity of mature years has but the haziest notions of the exact meaning of high and low potential, the electrical analogues of hot and cold. That it is desirable that students should learn physics, as they learn to ride the bicycle, by experimenting themselves, is now generally admitted, and this is especially true in the case of electricity, since it is by experimenting, and *only* by experimenting, that a student

can obtain such a real grasp of electricity that its laws become, so to say, a part of his nature."

"Hence, in the courses of electricity which I arranged at the City and Guilds of London Technical College, Finsbury, and at their Central Technical College, Exhibition Road, for every hour that a student spends at lecture, he spends several in the laboratory."

When Dr. Hopkinson this year, 1896, in his Inaugural Address as President of the Institution of Electrical Engineers, advocated commencing the study of electricity with the electric current, more than one teacher testified to the value of the method by claiming it as his own, apparently forgetful that when this order of treating the subject was introduced by the author in 1879 there was no precedent for such an innovation. Indeed, when even seven years later there appeared the first edition of "PRACTICAL ELECTRICITY" it was thought advisable to introduce the method by inserting the following explanatory paragraphs :—

"Readers who have been accustomed only to the ordinary books, commencing with certain chapters on statical electricity, continuing with one or more on magnetism, and ending with some on current electricity, will be surprised at the arrangement of the subjects in this book, and will probably be astonished at what they will condemn, at the first reading, as a total want of order. But so far from the various subjects having been thrown together hap-hazard, the order in which they have been arranged has been a matter of the most careful consideration, and has been arrived at by following what appears to me to be the *natural* as distinguished from the *scholastic* method of studying electricity. I have endeavoured to treat the subject *analytically* rather than *synthetically*, because that race of successful experimental philosophers —children—adopt this method.

"For example, it is not by studying geometrical optics, much less physical optics, that an infant gradually learns to appreciate the distance of objects; and later on it is

not by studying a treatise on struts, nor by listening to a course of lectures on structures, that the child finds out that the table has legs, hard legs, round legs. Feeling, looking, trying, in fact a simple course of experimental investigation, gives a child its knowledge; and this, therefore, I venture to think, is the method we should adopt when commencing the study of electricity."

"The subject of current is treated first, because in almost all the industries in which electricity is practically made use of it is the electric current that is employed; also, because currents can be compared with one another, and the unit of current (the ampere) defined, without any knowledge of potential difference or resistance. Potential difference is next considered, and resistance the last of the three, because the very idea of resistance implies a previous acquaintance with the ideas of current and potential difference, since the resistance of a conductor is the name given to the ratio of the potential difference (measured *electrostatically*) between its terminals to the current passing through it. And it is Ohm's experimental proof that this ratio was constant for a given conductor under given conditions, together with the numberless experimental verifications that this conclusion has received, that has led to resistance gradually coming to be considered as a fixed definite property of a given conductor like its weight or length."

The international, or Board of Trade, unit of P.D.—the volt—cannot, however, be defined until the definition of the unit of resistance—the ohm—has been fixed, because for legal purposes the units of current and resistance have been taken as the primary ones, and Ohm's law has been employed to fix the third or derived unit—viz. that of P.D. Hence, the actual sequence adopted in the present volume is (1) current and the ampere; (2) the *relative* measurement of P.Ds. with a *zero electrostatic voltmeter*; (3) Ohm's law; (4) resistance, and the ohm; (5) the volt, and current-voltmeters. Electric

energy and power, with their units—the joule and the watt—are next treated; and, lastly, the conception of the E.M.F. in a circuit, and the necessity for the E.M.F. of a good cell being constant, are derived from the laws of energy.

It should be obvious that any method of trying to experimentally prove Ohm's law with a *current*-voltmeter, such as may be found in certain text-books, begs the question. If a voltmeter be used it must be of the *electrostatic* type, and to simplify the definition of one P.D. being twice another this electrostatic voltmeter should be a *zero* instrument, which, without the need of any independent electrification, would be suitable for measuring P.Ds. no larger than those commonly employed in laboratories for sending currents. Such an instrument I have long felt the need of, and now—thanks to the ingenuity of Mr. Mather—it is available for use, and will be found described for the first time in pages 163–166 of the present volume.

It will be observed that the apparatus required for each experiment is mounted complete on a board. This is to enable it to be easily carried backwards and forwards between the laboratory and the lecture-room without disarranging it. At first sight it might appear that the student, finding each set of apparatus joined up quite complete, with current laid on all ready for carrying out the experiment, would be deprived of all incentive to exercise his own ingenuity in overcoming experimental difficulties, and, therefore, would fail to acquire habits of self-reliance. For first year students, however, I have found it a good plan to have each set of apparatus complete in position; firstly, because it is only with some such arrangement that fifty or more students can commence work almost simultaneously, and in the course of two or three hours have *all* performed some *quantitative* experiment; secondly, because when the apparatus is so arranged that even beginners can perform several experiments successfully, they acquire faith in the

possibility of success, and are less discouraged with the difficulties they subsequently meet with when selecting and arranging the apparatus for conducting some investigation.

The practical side of electricity has grown so rapidly that the original single volume has expanded into two. The present Volume I. of the rewritten book is intended to assist students in acquiring experimentally an *exact* working knowledge of current, difference of potentials, resistance, energy, and power, with their electric transmission, cells and their cost of working. This subject of the *cost* of converting chemical energy into electric energy is not, as far as I am aware, to be found in any text-book. Hence, in view of the "booms" in primary batteries, which appear to be periodic, the question of *cost* has been entered into in considerable detail.

The past four years have seen the legalisation in several countries of an international system of electrical units, so that, while the units of length, volume, mass, and money vary from country to country, there is now but one ampere, one ohm, and one volt throughout the whole world; a fact of which electrical engineers may feel justly proud. Some thirty pages at the end of the book are, therefore, devoted to "A Short History of the Absolute Unit of Resistance, and of the Electrical Standards of the Board of Trade."

In spite of the fact that the present Volume I. contains some 140 pages more than the original book, the subjects of secondary cells, electric quantity, coulombmeters, capacity, &c., have had to be left for a second volume. This has arisen not merely from primary cells, including dry cells and the Clark's standard cell, having been treated somewhat fully, but from the subjects of electric energy and power, the various meters used for measuring these quantities, the efficiency of electric transmission, the ratio of the power received to the maximum power receivable in various cases of transmission, &c., having been entered into at length in

consequence of the commercial importance that the electric transmission of energy now possesses. And it may be mentioned that generally where problems of maxima or minima have been considered, attention has been directed to the kind of change that is produced in the value of the quantity under consideration, when the value of the variable is altered from that required to make the quantity a maximum or a minimum.

In fact, the aim has been to treat a few subjects fairly thoroughly in a simple manner, and not to prepare a list of short instructions for carrying out a large number of experiments, nor to write a treatise, mainly of value as an electrical dictionary, which should give a little information about everything that can be comprised under the head of electricity, whether it be electric eels, the history of the invention of the telegraph, the aurora, or the earliest forms of frictional machines.

In the letterpress, *small* capitals have been used to represent instruments, parts of apparatus, &c., while *large* capitals systematically stand for electric quantities other than resistances, these being throughout designated by *small* letters in *italics*. Thus A, A, *a* stand respectively for an ammeter, the current in amperes flowing through it, and its resistance in ohms.

In the preface written in 1886 it was mentioned that, with the exception of two or three blocks that had been lent, the 180 figures had been specially drawn for the book, and were not time-honoured representations of historical apparatus. Of these 180 figures only 64, however, have been employed in the present volume, partly because the fresh matter required many new figures to illustrate it, and partly because several of the blocks specially executed for the original book have lost their freshness from the appreciative use of them by other writers. Hence, 183 of the 247 figures contained in the present volume will not be found in the former book, and 163 of these fresh illustrations have been specially drawn for this new edition.

A large number of new examples have been added, and any that have been reproduced from the original book have been reworked, either to check the accuracy of the results, or because the so-called legal units referred to have been replaced by those that have now been adopted internationally.

My thanks are due to my past and present assistants —Dr. Sumpner, Mr. Haycraft, and Mr. Severs—for much assistance in the preparation of this book; to my daughter for compiling a very comprehensive and judiciously arranged index; and to Messrs. Spiers, Twyman, and other students for carefully examining the proofs. In conclusion, I desire to express to-day even more warmly than in October, 1886, my indebtedness to Mr. Mather for the very earnest, thoughtful, and painstaking way in which for many years he has assisted me in developing the course of instruction for students of electrical technology, of which the present volume represents part of the elementary portion.

W. E. AYRTON.

October, 1896.

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PRACTICAL ELECTRICITY.

CHAPTER I.

THE ELECTRIC CURRENT AND ITS MEASUREMENT.

1. What is meant by an Electric Current, and by its Direction of Flow —2. Properties of an Electric Current—3. Measuring the Strength of a Current—4. Conductors and Insulators—5. The Strength of an Electric Current: by which of its Properties shall it be Directly Measured?—6. Definition of the Unit Current; Ampere —7. Definition of the Direction of the Current—8. Objection to the Usual Mode of Constructing Voltameters—9. Description of Practical Forms of Sulphuric Acid Voltameters—10. Relative Advantages of Voltameters and Galvanometers—11. Meaning of the Relative and the Absolute Calibration of a Galvanometer—12. Experiment for Calibrating a Galvanometer Relatively or Absolutely—13. Graphically Recording the Results of an Experiment—14. Practical Value of Drawing Curves to Graphically Record the Results of Experiments—15. To Construct a Galvanometer Scale from which the Relative Strengths of Currents can be at once Ascertained.

1. What is meant by an Electric Current, and by its Direction of Flow.—In the various industries in which electricity is employed, as in the telegraph, telephone, electric lighting, electrotyping, electroplating, torpedo exploding, electric traction, the electric transmission of power, and in the working of machinery by the aid of electromotors, it is the so-called "*electric current*" that is made use of. Hence a knowledge of the laws of this electric current, a clear conception of its so-called properties, combined with a practical acquaintance with the modes of measuring it, must be of especial importance for a right understanding of the working of the apparatus employed in the above-mentioned

industries. Indeed, such knowledge is absolutely necessary if the user of electrical apparatus is desirous of employing it to the best advantage, of being able to correct faults when they occur, as well as of effecting improvements in the instruments themselves.

It is customary to speak of an electric current as if it had an independent existence apart from the "*conductor*" through which it is said to be flowing, just as a current of water is correctly spoken of as something quite distinct from the pipe through which it flows. But in reality we are sure neither of the direction of flow of an electric current, nor whether there is any motion of anything at all. And the student must not assume that the conventional expression, "The current flows from the copper pole of a galvanic battery to the zinc pole through the external circuit," implies any knowledge of the real direction of flow any more than the railway expressions, "up train" and "down train," mean that either train is necessarily going to a higher level than the other. In the case of a stream of water flowing along a river-bed we are quite certain that there is water in motion, and everyone is agreed as to which way the water is flowing; a cork or a piece of wood thrown on the water indicates by its motion the direction in which the water is moving.

Nor, again, must an electric current be supposed to be like waves of sound travelling along, since in this latter case, although there is no actual travelling along of matter, still the direction of motion of the wave of sound is perfectly definite. Indeed, a wire along which an electric current is flowing is more like a wire at each end of which a musical instrument is being played, so that the sound is travelling in both directions along the wire at the same time. In short, the statement that an electric current is flowing along a wire is only a short way of expressing the fact that the wire and the space around the wire are in a different state from that in which they are when no electric current is said to be flowing. So

that when a body and the space around the body possess certain properties that they do not usually possess, an electric current is said to be flowing through that body.

2. Properties of an Electric Current.—These properties are :—

(1) A suspended magnet put in nearly any position near a body through which an electric current is said to be flowing will be deflected, showing that a force is exerted on the magnet (Fig. 1). This force is mutual, so

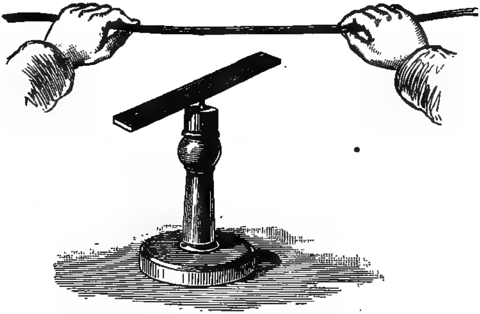


Fig. 1.

that if a magnet be brought near any substance traversed by an electrical current, this substance will generally be acted upon by a force tending to move it (Fig. 2). Also any piece of soft iron put near a conductor carrying a current will become magnetised (Fig. 3). The action in all these cases is just as if the body conveying the current had become magnetic. This is further shown by the fact that any two wires through each of which a current of electricity is passing, act upon each other with a magnetic force in nearly every position in which the wires may be placed relatively to one another (Fig. 4).

(2) If the circuit through which an electric current

is said to be flowing be partly solid and partly liquid,

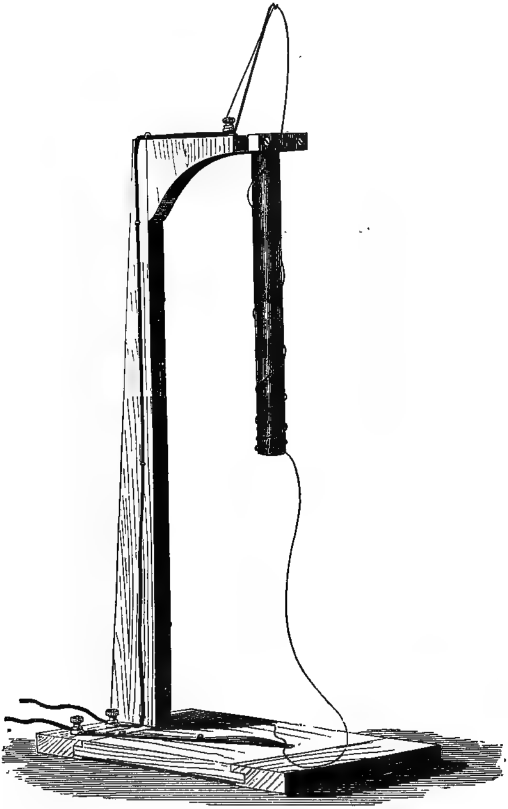


Fig. 2.—A Piece of Tinsel Coiling itself round a Magnet when a Current Flows through the Tinsel. Suggested by Prof. Jamieson.

then the liquid will generally be decomposed into two parts, one part going to one side of the liquid in the

direction in which the current may be said to be flowing, and the other part going to the other side of the liquid in the opposite direction to the flow of the current (Fig. 5).

(3) The body conveying the current becomes more or less heated (Fig. 6).

In popular language the current is said:

(1) *To deflect the magnet, and magnetise the iron.*

(2) *To decompose the liquid.*

(3) *To heat the body through which it is flowing.*

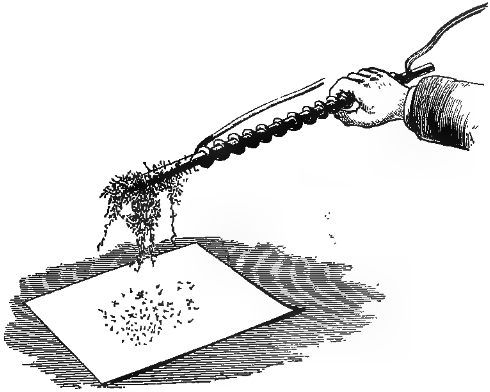


Fig. 3.—Iron Rod Picking up Nails when a Current Flows through a Wire Coiled round it. Wire may be Bare if Rod be Varnished.

But as we have no evidence of the current apart from the conductor through which it is said to flow, it is more accurate to speak of a current being said to flow through a conductor in which these effects are found to be produced, than to say that the current produces these effects. The latter expression, however, for brevity's sake, is generally adopted; and, indeed, the heat generated in a wire conveying a current has so many analogies with the heat produced in a pipe by the

friction of a stream of water passing through it, that we can frequently assist ourselves by thinking of an electric current as a stream of matter passing through the wire as water would pass through a pipe filled with sponge,

Fig. 4.—Two Coils Standing on Narrow Bases Falling Down when a Current Flows through them in Opposite Directions.

or loosely packed with sand. But the analogy, like many other analogies, must not be pressed too far, especially as there is this very great difference between a current of water flowing in a pipe and a current of electricity in a wire, viz. that in the former case no

effects are produced external to the pipe, whereas in the latter the whole space surrounding the wire is affected. For example, an electric current is flowing through a thick flexible conductor, a compass needle brought within

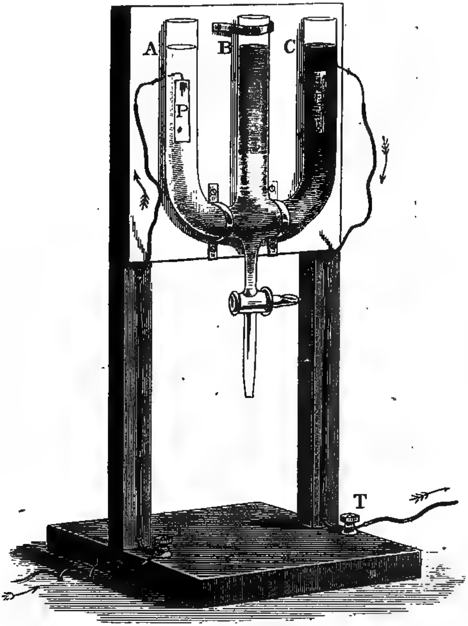


Fig. 5.—Tube A B C contains Solution of Common Salt with Drop of Hydrochloric Acid, and is Coloured Red with Litmus. When a Current Flows Chlorine is liberated in limb A, which Bleaches Liquid, while Caustic Soda is Formed in limb C, making Liquid Dark Blue.

two or three inches of it is deflected. But suppose not merely is there a current of electricity flowing, but also a stream of water passing through the interior of the conductor, the conductor being in reality a pipe. The

water stream, however, is a perfectly steady one, therefore it makes no sound; the water has been previously brought to the temperature of the pipe, therefore the presence of the water inside the pipe cannot be detected by the pipe feeling hotter or colder to the touch; and,

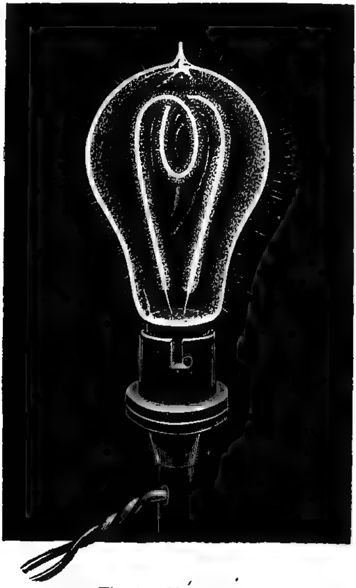


Fig. 6.—Glow Lamp.

consequently, it would be extremely difficult to detect this stream of water by any test made outside the pipe.*

The Magnetic, Chemical, and Heating effects of a current are practically utilised in a number of electrical instruments; for instance:

* A compo gas pipe answers very well for this experiment.

Magnetic Property.—Needle telegraph,* the Morse instrument, telephones, electric bells, arc lamps, dynamo machines, electromotors, and, in fact, all instruments using electromagnets.

Chemical Property.—Electroplating, electrotyping, the extraction of aluminium and other metals from their ores, the production of sodium and chlorine from salt, the manufacture of pure copper, the cleansing of the mercury used in separating gold from sand, &c.

Heating Property.—Electric welding, electric cooking apparatus, electric lamps, contrivances for lighting gas or oil lamps electrically, fuses for torpedoes, &c.

The heating effect of the current is, as we shall see, the effect which always occurs when a current flows; that is to say, it is impossible for a current to flow through a body without some heat being produced; and not only is heat produced by the ordinary currents flowing through telegraph wires, which are sometimes not much more than three-thousandths of the strength of the current flowing through an incandescent lamp, but even the currents used with the Bell telephone worked without a battery produce a definite amount of heat in a given telephone circuit, even though such telephone currents are very weak compared with the currents used in telegraphy. The actual measurement of the heat produced by such very weak currents, however, would be extremely difficult, if not impossible, to carry out with existing apparatus.

3. Measuring the Strength of a Current.—As, then, the production of heat always accompanies the passage of a current, it might seem that the amount of heat produced in a given time ought to be taken as a measure of the strength of the current. But, in addition

* It is desirable to show in operation to students as many as possible of the instruments enumerated under the three heads, Magnetic Property, Chemical Property, and Heating Property, but at this early stage it is only necessary to describe the instruments in so far as their operation illustrates the respective property of the current.

to the difficulty of measuring the small amount of heat produced by weak currents, the only way we have of measuring the amount of heat given to a body is an indirect one, and consists in measuring its rise of temperature by means of a thermometer. But as a thermometer measures merely rise of temperature, and not the amount of heat, and, since by changing the mass or the nature of the material warmed, or by increasing or diminishing the facility that the body may have for cooling, the rise of temperature of a body through which a current is passing can be varied without changing the strength of the current, it follows that various precautions have to be adopted, and further experiments have generally to be made to enable us to deduce from the observed rise of temperature the real amount of heat that was given to the body.

In order to ascertain which of the properties of a current can be best employed for measuring its strength, an experiment may be made with the following apparatus :—

A, B, C, D, E (Fig. 7) are instruments so arranged that the *same* electric current will be sent through them all by the "*battery*," *b b*, on joining the wires P and Q. A is a coil of cotton- or silk-covered wire, with a magnet *m* suspended so as to turn freely inside the coil, the whole arrangement forming what is called a "*galvanoscope.*" B is an "*electromagnet*" consisting of a coil of cotton- or silk-covered wire wound in opposite directions round the ends of a piece of iron of horse-shoe form. C is a "*sulphuric acid voltameter*" consisting of two platinum plates dipping into moderately dilute sulphuric acid in a vessel V, closed by an air-tight stopper *s*, through which passes a glass tube *t*, open at both ends, and with its lower end nearly touching the bottom of V. This tube is graduated in fractions of a cubic centimetre or cubic inch. D consists of two thin copper plates *p*, *p*, dipping into a solution of copper sulphate (the blue vitriol of commerce), and is called a "*copper voltameter.*" E is a coil of bare

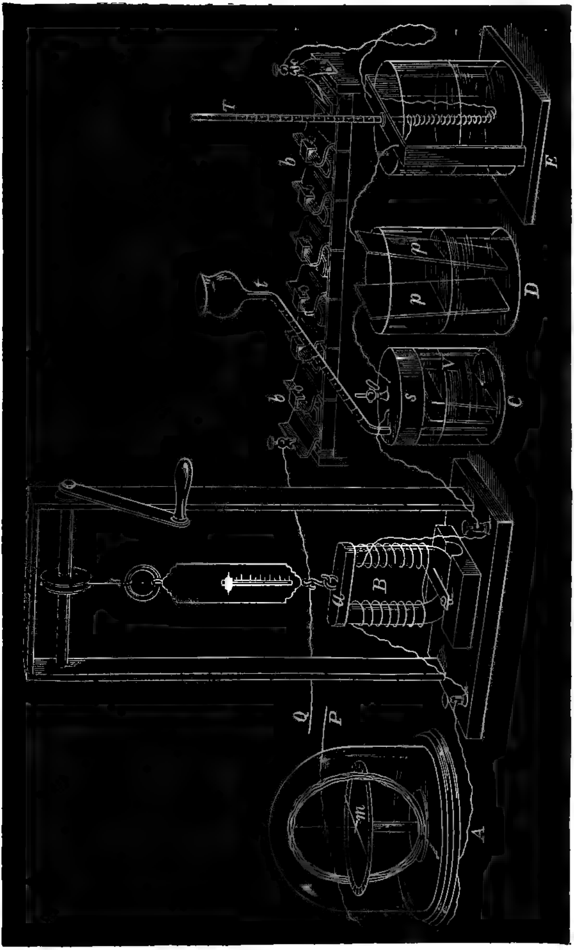


Fig. 7.

wire immersed in paraffin oil, the temperature of which can be measured by the thermometer T, the arrangement being called a "*calorimeter.*"

Connect the two wires P and Q, and allow the current to pass for a convenient time through these five pieces of apparatus, then it will be found that :—

(1) The liquid has risen a distance d_1 in the tube t of the voltameter C, indicating that the passing of the current through the liquid from one of the platinum plates to the other has caused c_1 cubic inches of gas to be generated.

(2) One of the plates in the copper voltameter has increased in weight by W_1 grains.

(3) The mercury in the thermometer T of the calorimeter E has risen through D_1°.

(4) The magnetic "*needle*" m of the galvanoscope A has all the time been kept deflected from its original position through a number of degrees N_1°.

(5) If at any time during the passage of the current the "*armature*" a was placed carefully on the ends of the horse-shoe electromagnet B it required a pull of w_1 lbs., as measured by the spring balance, to pull it off, when the handle at the top of the apparatus was slowly turned.

Next increase the strength of the current passing through the apparatus C, D, E, A, B, by increasing the number of cells forming the battery b b or in any other way, such as will be described later on, and repeat the experiment for the same time as before, then each of the effects previously observed with these instruments will be increased, and instead of the results c_1, W_1, D_1°, N_1°, w_1, we shall obtain c_2, W_2, D_2°, N_2°, w_2. But it will be found that the new values do not all bear the same ratio to the corresponding old ones. For example, if c_2 is twice c_1, then N_2° may be more or less than twice N_1°; but will generally be less than twice, while D_2° and w_2 may be found to be much greater than twice D_1° and w_1 respectively. On the other hand, if the strength of the second current be so chosen as to make D_2° exactly twice D_1°,

then generally it will be found that w_2 is rather more than twice w_1, while c_2 and W_2 are less than twice c_1 and W_1 respectively.

If, then, we *arbitrarily* define the strength of the current as being *directly* proportional to the gas evolved in the sulphuric acid voltameter, we must conclude that if c_2 is exactly double c_1 we have doubled the current strength; but, on the other hand, if we prefer to say that strength of current is directly proportional to the angular deflection of the needle m in the galvanoscope A, then we must conclude that, as N_2° is less than twice N_1°, we have not quite doubled the strength of the current; whereas if we prefer to say that current strength shall be regarded as proportional to the force required to detach the armature a of the electromagnet B, or, instead, proportional to the rise of temperature of the liquid in the calorimeter E in a given time, then we must conclude that the strength of the current has been more than doubled. Which of these is right and which wrong? As long as no one of the effects varies we may be safe in concluding that the strength of the current is constant, but if the different effects to which we have been referring vary from one time to another, then which of them shall we take to represent by the magnitude of its variations the change that has taken place in the current strength?

In the case of measuring the velocity of a stream of water, or the number of gallons of water per minute discharged by a river, no two experimenters could differ. One of them, by the employment of better-constructed measuring instruments, or it may be from having greater experience in making such measurements, might get answers slightly different from, and more accurate than, those obtained by the other experimenter. But they could not have such totally different conceptions of what should be meant by the velocity of the water in a particular part of the channel, or of the total discharge in gallons per minute, that the results obtained by one

observer were, apart from all mere errors of experiments, twice as great as those obtained by the other. And this is because they would be dealing with the actual flow of a material substance—water.

The flow of an electric current, however, being merely a *conventional* method of expressing the fact that a conductor has acquired certain properties that it does not usually possess, there is no question of right or wrong, but only one of convenience, in selecting whichever we choose of the so-called properties of the current as the one we *arbitrarily* decide to employ as the measure of the current strength.

4. Conductors and Insulators.—The various pieces of apparatus in Fig. 7 are joined by bits of copper wire, but as long as there is even one break in the continuity, as at P Q, no current can be sent by the battery *b b* through the circuit, because the air separating the wire P from the wire Q "*insulates*" or is an "*insulator*." If P be pressed against Q, but with even a thin piece of paper, or silk, or indiarubber, &c., between, still no current will flow, because all these substances are more or less good *insulators*. If, however, the ends of the wires P and Q be rubbed *clean* with *emery paper*, or be scraped clean with the back of a knife or a file, and then pressed together, the current will flow, since there is good "*conductivity*" or little "*resistance*" between the clean surfaces of metals pressed together.

5. The Strength of an Electric Current: by which of its Properties shall it be Directly Measured?—To assist us in deciding whether the amount of the magnetic action, or of the chemical action, or the amount of heat produced in a given time, shall be *arbitrarily* taken as that magnitude to which the current strength shall be defined as being *directly* proportional, we observe that of the five pieces of apparatus A, B, C, D, E employed in the previous experiment, C and D are the only two which give results that steadily increase in the *same proportion* when the current is increased. Consequently

while, on the one hand, our estimate of the relative strength of two currents would be quite different according as we selected the angular deflection of the

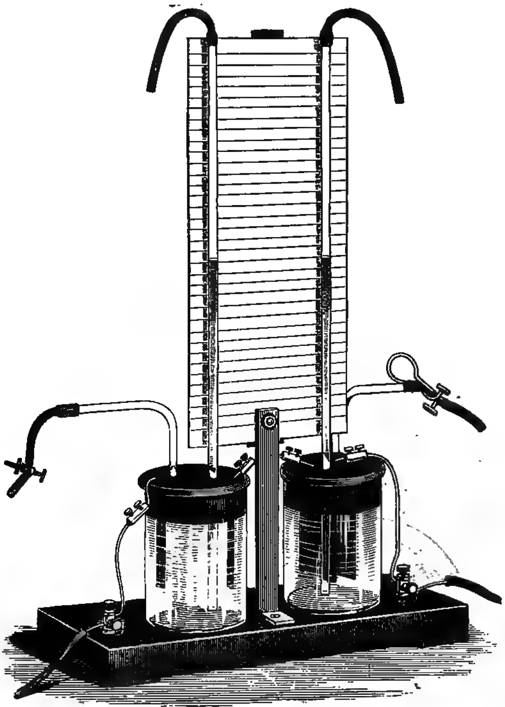


Fig. 8.—Two Sulphuric Acid Voltameters having Platinum Plates of Different Sizes and at Different Distances Apart.

magnet *m* (Fig. 7), or the force of detachment of the armature *a* to be the direct measure of the current strength; on the other hand, we should arrive at practically the same estimate whether we chose to say that

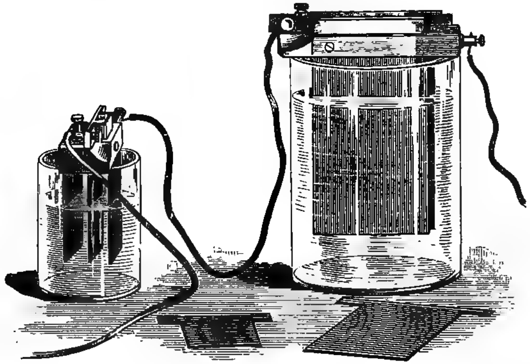


Fig. 9.—Two Copper Voltameters having Plates of Different Sizes and at Different Distances Apart.

the current was directly proportional to the rate of production of gas in the sulphuric acid voltameter C, or to the rate of deposition of copper in the copper voltameter D.

But in addition to this *agreement* between the relative amounts of *different* chemical actions produced

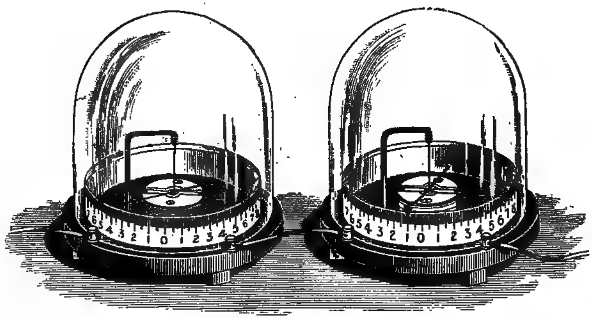


Fig. 10.—Galvanoscope to the Left Wound with Many Turns of Fine Wire; Galvanoscope to the Right with a Few Turns of Thick Wire. For details of Construction of Galvanoscope *see* page 125.

by two currents there is another equally important fact, viz. that *the rate at which a particular chemical effect is produced by a current is practically independent of the size and shape of the apparatus.* Thus, suppose we have *two* sulphuric acid voltameters, the platinum plates being of totally different shapes and sizes (Fig. 8);

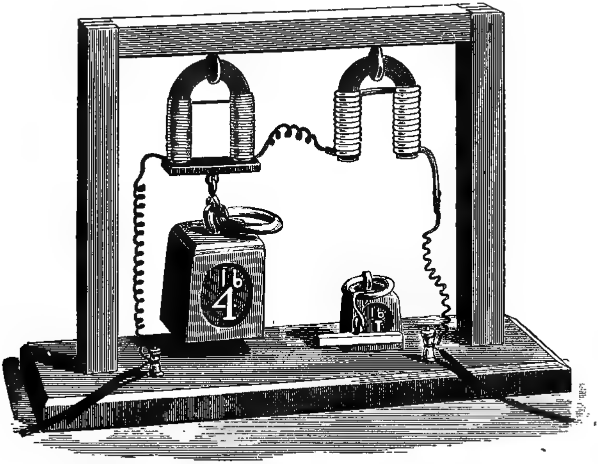


Fig. 11.—Electromagnet to the Left Wound with Many Turns of Fine Wire; Electromagnet to the Right with a Few Turns of Thick Wire.

two copper voltameters also of different shapes and sizes (Fig. 9), the copper plates, for example, being much larger, and, either much nearer together, or much farther apart in the one than in the other; also *two* galvanoscopes (Fig. 10), which may look very much like one another, but the bobbin of the instrument to the right is wound with a few turns of thick wire, while that of the other galvanoscope to the left is wound with many turns of fine wire; *two* electromagnets (Fig. 11), which differ from one another in the same sort of way as

do the galvanoscopes, and *two* calorimeters (Fig. 12), the two instruments in each case being selected so as to be distinctly different in size and form. Then,

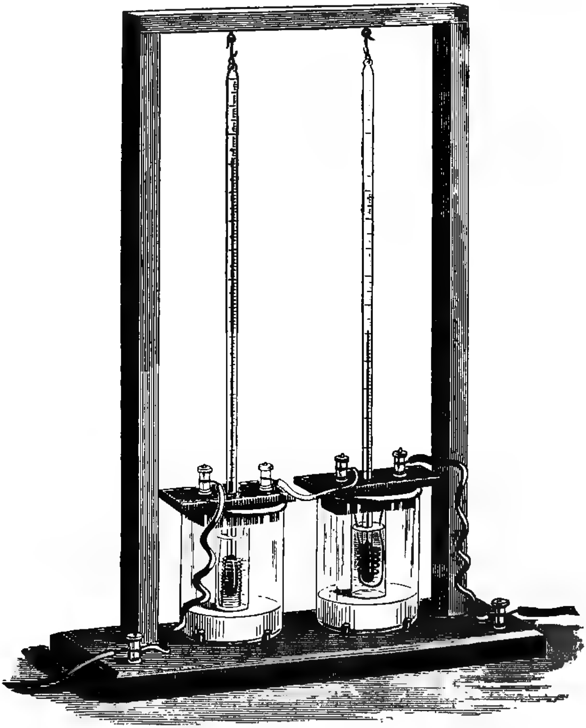


Fig. 12.—Thermometer to the Left Surrounded with Several Turns of Fine Wire; Thermometer to the Right with a Few Turns of Thick Wire.

if an experiment be made with each pair of apparatus, a certain current being sent through both sulphuric acid voltameters for a certain time, and a

current, which may or may not be, of the same strength as the former, through both the copper voltameters, &c., the following results will be observed: In the two sulphuric acid voltameters quantities of gas equal in volume at the same pressure and temperature, and, therefore, possessing the same mass, will be developed in the same time, in spite of the platinum plates being of a very different size and at a very different distance apart in the two voltameters.* Similarly, in spite of the difference in size and form in the two copper voltameters, the increase in weight of the plate of the one will be practically the same as the increase in weight of the corresponding plate of the other, unless the current be so strong that the deposited copper tumbles to the bottom instead of forming an adherent deposit. But in the case of the two galvanoscopes, the two electromagnets, and the two calorimeters, although the current passing through the two apparatus in any one pair is the same, the effects depend on the shape, on the size, and on very many details in the arrangement, &c. Hence, to specify the strength of a current by the magnitude of the deflection of the needle of a galvanoscope, it would be necessary to state the exact mode of constructing each part of the galvanoscope in great detail, as well as the exact position of the instrument relatively to neighbouring magnetic pieces of iron. Whereas, to specify the strength of a current by the amount of gas produced in a given time in a sulphuric acid voltameter, or by the amount of copper deposited in a given time on one of the plates of a copper voltameter, neither the shape nor size of the plates, nor

* Equality of pressure may be obtained by using for the voltameters two vessels of the same size as well as two tubes of the same bore, and filling the vessels with the same quantity of dilute sulphuric acid of the same specific gravity. In that case, if the level of the liquid in the two tubes be the same to start with, the liquids will be found to rise at exactly the same rate in them on the same current being sent through the two voltameters.

the distance between them, need be taken into account within wide limits.

In both the voltameters it is chemical decomposition that takes place—in the former, this decomposition being the splitting up of the liquid into gases; in the latter, the splitting up of the copper sulphate, and the deposit of copper on one of the copper plates, together with *a loss of an equal weight of the metal* of the other copper plate to give back to the solution the amount of copper taken out of it. In A and B (Fig. 7) the effects produced are both magnetic, but we have found that N_2° does not bear to N_1° the same ratio that w_2 bears to w_1; whereas in the case of the voltameters we always find that c_2 bears to c_1 almost exactly the same ratio that W_2 bears to W_1. Consequently, as far as we have seen at present, the amount of chemical action produced in a given time by a current appears to be a more direct measure of its strength than the magnitude of the magnetic effect produced.

Let us examine this point still farther. In Fig. 7 all the apparatus is joined up "*in series*"—that is to say, the current passing through any one instrument passes through every other. But in Fig. 13 C_2 and C_3 are two sulphuric acid voltameters "*in parallel,*" and not *in series*, with one another. For the current which comes along the wire W_1 and passes through sulphuric acid voltameter C_1 divides into two portions, one of which passes through C_2 and the other through C_3, the two portions then recombine and flow away together by the wire W_2. Also from the construction of the apparatus it will be seen that the rise of liquid in the tube T_1 measures the production of gas in the voltameter C_1, while the rise of liquid in the tube T_2 measures the *sum* of the quantities of gas produced in the voltameters C_2 and C_3 together. Now, experiment shows that, if the precautions referred to in the note on page 19 for using the apparatus in Fig. 8 be taken, the liquid rises at exactly the same rate in the tube T_1 that it does in the tube T_2. Consequently

the rate of production of gas in C_1 is equal to the sum of the rates of production in C_2 and C_3 together.

Further, whether C_1, C_2, and C_3 be all sulphuric-acid voltameters, or all copper voltameters, or all silver volta-

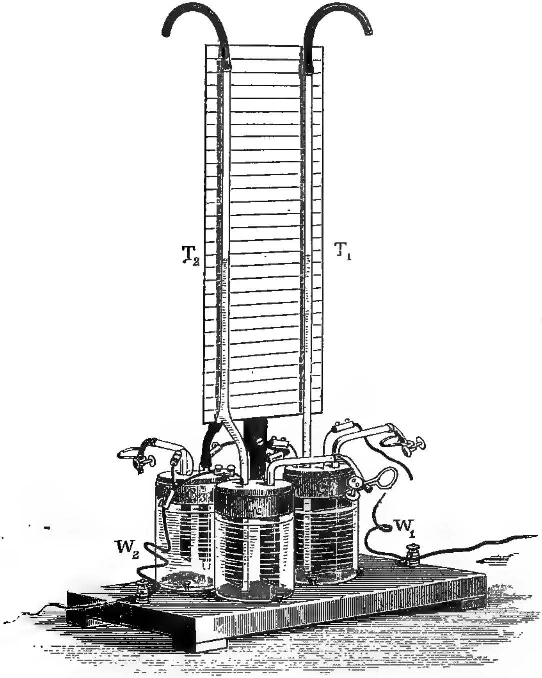


Fig. 13.—Voltameters C_2 and C_3 in Parallel with One Another, but in Series with Voltameter C_1.

meters, or, indeed, all voltameters of the same character, it will be found that, no matter what be the shapes or sizes of the different voltameters, and no matter what be the areas of the platinum, copper, or silver plates immersed in the

respective liquids, or the distances apart of the plates, the amount of chemical action produced in a given time in c_1 is almost exactly equal to the sum of the amounts of chemical action produced in c_2 and c_3 together. The plates, in any one of the voltameters, c_1, may be large or small, near together or far apart—may be, in fact, moved about while the chemical action is going on. The current may be strong and the chemical action take place rapidly, or it may be weak and the action proceed slowly, and it may be varied while the action is progressing; but the same general result still remains true. Measure the amount of chemical action that has taken place in c_2 and in c_3, add the two together, and it will be found to be practically equal to the action that has taken place in c_1 in the same time.

Now, when a river divides in consequence of the existence of an island in mid-stream, we know that the number of gallons of water flowing per minute on each side of the island must together equal the total number of gallons per minute flowing in the main stream, simply because the water which does not go past one side of the island must go past the other; and similarly, if we are to look upon a current of electricity in the same way as a current of water, we must expect that, when it divides into two parts, the sum of these parts must always be equal to the whole, whether the current which divides is a large one or a small one. The experiment just described shows that, if we say that a current is directly proportional to the rate at which chemical action is produced in a voltameter, this statement will always be true, whatever be the current in the main circuit; but it will *not* generally be true if we take any of the other effects occurring in the instruments indicated in Fig. 7 (page 11) as a direct measure of a current. Thus, in Fig. 13, if c_1, c_2, c_3 represent galvanoscopes, such as A in Fig. 7, the deflection of the first will not generally be equal to the sum of the deflections of the other two; and even if this were the case for one current in

the main circuit, it would not be the case for any other. Nor will any simple relation be found to connect the deflection of the first instrument with those of the other two, *unless elaborate precautions be taken in the construction of the apparatus.*

6. Definition of the Unit Current; Ampere.—We may, therefore, define the *strength of a current as being proportional to the amount of chemical decomposition it can produce in a given time;* and *an unvarying current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of* 0·001118 *of a gramme per second, is taken as a current of one ampere.*

The metal deposited by the current does not adhere well to the plate of a voltameter or "*electrolytic cell,*" if the action proceeds too rapidly; also errors will arise in the estimation of a current by the *electrolytic* method, unless certain precautions be carefully attended to. Thus, when measuring a current of about one *ampere*

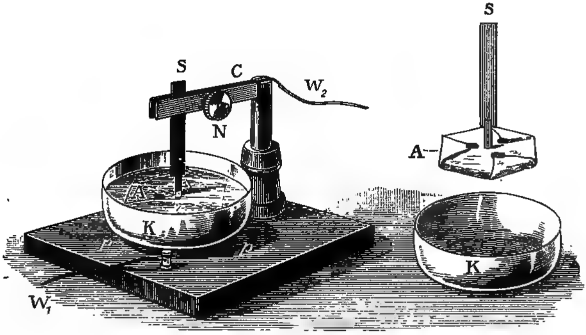


Fig. 14.—Silver Voltameter for Measuring Currents of about One Ampere.

with a silver voltameter, it is advisable to adopt the following arrangement:—The "*cathode,*" sometimes spelt "*kathode,*" or plate on which the silver is deposited, should take the form of a light bowl K (Fig.

14), not less than 10 centimetres * in diameter, and from 4 to 5 centimetres in depth, and made of platinum, so that it may be easily cleaned with nitric acid. The "*anode*," or plate from which the silver is electrically removed, should be a disc of pure silver, A, of about 30 square centimetres in area, and from 2 to 3 millimetres thick.

Riveted to the *anode* is a strip of pure silver which is braized to a brass strip S, and by means of the metal clamp C and nut N the anode is held so that its edge is equidistant all round from the rim of the *cathode*, and its upper surface just below the level of the liquid; this may conveniently consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the salt to 85 parts of distilled water.

Electric contact is made between the wire W_1 and the bowl by means of three metal pins *p*, on which the latter rests; and the wire W_2 is electrically joined to the anode disc by the strip S being held fast in the metal clamp C, to which the wire W_2 is attached.

In addition to the surface of the anode plate being turned into silver nitrate by the passage of the current, there is a tendency for small bits of silver to become detached and to fall into the bowl, thus making its weight too great. To prevent this, the anode should be wrapped round with pure filter paper, secured at the back with sealing-wax.

When making an observation, the current should be allowed to pass for about half an hour, and be maintained as constant as possible.

The preceding is based on the Reports issued in 1891, 1892, and 1894 by the Committee † appointed to advise

* One metre is 39·370 inches, therefore 10 centimetres correspond with a little less than 4 inches. One square metre is 1,550 square inches, therefore 30 square centimetres is a little less than $4\frac{3}{4}$ square inches.

† This Committee consisted of Sir Courtenay Boyle, Mr. Hopwood and Major Cardew representing the Board of Trade; Mr. Preece and the late Mr. Graves representing the Postal Telegraph Department;

the Board of Trade on Electrical Standards, and the following, extracted from the "Order in Council" made by Her Majesty on the 23rd of August, 1894, is their description of the

"METHOD OF MAKING A MEASUREMENT.

"The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry, it is weighed carefully.

"It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support, to which a binding screw is attached. This copper support must be insulated.

"The anode is then immersed in the solution, so as to be well covered by it and supported in that position; the connections to the rest of the circuit are made.

"Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed. Care must be taken that the clock is keeping correct time during this interval.

"The solution is now removed from the bowl, and the deposit is washed with distilled water, and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol, and dried in a hot-air bath at a temperature of about 160°C. After cooling in a desiccator it is weighed again. The gain in weight is the silver deposited.

"To find the current in amperes, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed, and by ·001118.

"The result will be the time-average of the current, if during the interval the current has varied.

"In determining by this method the constant of an instrument, the current should be kept as nearly constant as possible, and the readings of the instrument taken at frequent observed intervals of time. These observations give a curve from which the reading corresponding to the mean current (time-average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading."

Fig. 15 shows a desiccator, such as is referred to in the extract from the Board of Trade Reports. The

Lord Kelvin and Lord Rayleigh the Royal Society; Prof. Carey Foster and Mr. Glazebrook the British Association; and Dr. J. Hopkinson and the Author the Institution of Electrical Engineers.

platinum bowl K (Fig. 14) is placed on the triangle T (Fig. 15), resting on the rim of a vessel V, containing strong sulphuric acid, calcium chloride, or other water-absorbing substance, and the whole is covered over with a glass bell-jar G.

To obtain a uniform adherent deposit of silver, it is desirable, as already stated, that the cathode should possess

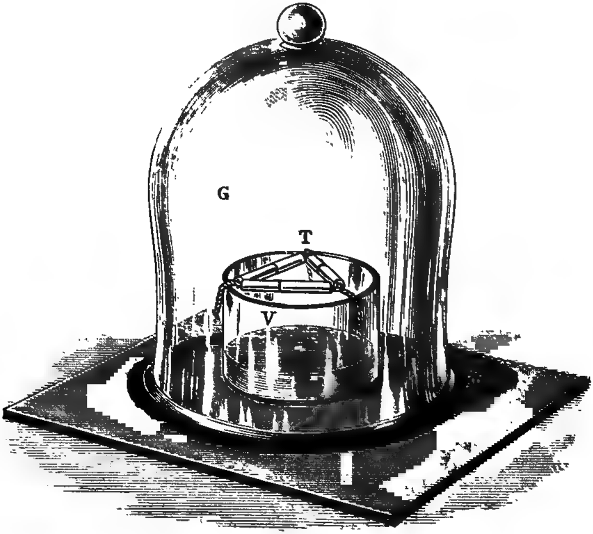


Fig. 15.—Desiccator Used with the Silver Voltameter.

about 30 square centimetres of surface for every ampere passing. Hence, if a large current of several hundred amperes had to be measured by means of a silver voltameter, the apparatus would necessarily be large and costly. In the voltametric measurements of large currents, therefore, it is usual to replace the platinum bowl and the silver disc by copper plates, and the solution of silver nitrate by one of copper sulphate.

If a number of voltameters containing, for example, solutions of silver nitrate, copper sulphate, zinc sulphate, &c., respectively, be placed in series, and a current be sent through them for a certain time, the weights of the metals deposited on the cathodes of the respective voltameters, or the weights of the other constituents of the respective salts set free at the anodes (and acting on the material of the anodes, or on the liquid in the neighbourhood of the anodes), are *approximately* proportional to the chemical equivalents. Thus, since the atomic weights of silver, copper, and zinc are respectively 107·94, 63·44, and 65·38, and, since silver is monatomic while copper and zinc are diatomic, it follows that, as an ampere is the current that deposits 0·001118 gramme of silver per second, the weights of copper and zinc that will be deposited per second per ampere are respectively *about*

$$\frac{1}{2} \times \frac{63 \cdot 44}{107 \cdot 94} \times 0 \cdot 001118 \text{ or } 0 \cdot 0003286 \text{ gramme,}$$

and

$$\frac{1}{2} \times \frac{65 \cdot 38}{107 \cdot 94} \times 0 \cdot 001118 \text{ or } 0 \cdot 0003386 \text{ gramme.}$$

The first quantitative experiments on "*electrolysis*," the name given to electric decomposition, were carried out by Faraday in 1833, and although he found that the proportion of the weights of different substances liberated by a given current flowing for a certain time differed sometimes by as much as 2 per cent. from the ratio of their chemical equivalents, he attributed this to inaccuracy in his experiments. He, therefore, concluded that the "*electrochemical equivalents*" of substances were directly proportional to their chemical equivalents. We now know, however, that although this law is very nearly true it is not absolutely so.

Among the many investigations that have been conducted for comparing the rates of deposit of copper and

silver the most complete is probably that carried out by Prof. T. Gray. He finds that the amount of copper deposited per second per ampere varies somewhat with the size of the cathode and the temperature of the copper sulphate bath. If, however, the anode and cathode plates have each an area of about 50 square centimetres per ampere passing, and if the solution in the bath be formed by dissolving pure copper sulphate in distilled water until the density becomes 1·18, and afterwards adding about 1 per cent. of sulphuric acid, the weight of copper deposited per second per ampere is very approximately 0·0003286 gramme, and is but little affected by temperature.*

It will be observed that the weight of silver deposited per second per ampere in a silver voltameter is nearly four times as great as the weight of copper deposited in a copper voltameter. This reason would alone render the silver voltameter much to be preferred for the measurement of small currents.

Although the deposition of copper is now frequently employed for the measurement of large currents it must

* The following table gives a summary of the results obtained by Prof. T. Gray (Phil. Mag., Vol. XXV., page 179) :—

TABLE I.

ELECTROCHEMICAL EQUIVALENTS OF COPPER IN GRAMMES PER SECOND PER AMPERE.

Area of Cathode in Square Centimetres per Ampere of Current.	Temperature, 2°C.	Temperature, 12°C.	Temperature, 23°C.	Temperature, 28°C.	Temperature, 35°C.
50	·0003288	·0003287	·0003286	·0003286	·0003282
100	·0003288	·0003284	·0003283	·0003281	·0003274
150	·0003287	·0003281	·0003280	·0003278	·0003267
200	·0003285	·0003279	·0003275	·0003268	·0003252
250	·0003283	·0003278	·0003275	·0003268	·0003252
300	·0003282	·0003278	·0003272	·0003262	·0003245

not be forgotten that the definition of the ampere in the Order in Council is based on the amount of silver, and *not* on the amount of copper, deposited per second. Hence, while experience has convinced us that the current which deposits 0·001118 gramme of silver per second in a silver voltameter, constructed and used as above described, will also deposit very approximately 0·0003286 gramme of copper per second in a copper voltameter, when constructed as described by Prof. Gray, it is to be remembered that, should more accurate experiments show us that this number 0·0003286 ought to be slightly changed, such a change will only modify the result arrived at by Prof. Gray, and will not affect the value of the ampere as now defined by law.

A current of one ampere, when passed through a solution of dilute sulphuric acid, decomposes about 0·00009326 gramme of the liquid per second. The acid in the voltameter may be conveniently diluted with water until the specific gravity of the mixture is about 1·1, which corresponds with a mixture of about 15 per cent. by weight of pure sulphuric acid at 15°C.

The volume of mixed gas (oxygen and hydrogen) that is produced per second by the decomposition, corresponding with a current of one ampere, is about 0·1733 cubic centimetre, when the temperature at which the gas is evolved is 0°Centigrade, and the atmospheric pressure that of 76 centimetres of mercury. When the temperature is C°Centigrade, and the height of the barometer h centimetres, the volume of gas evolved by one ampere in one second is about—

$$\frac{0{\cdot}1733 \times 76 \times (273 + C^\circ)}{h \times 273} \text{ cubic centimetres.}$$

Example 1.—How many amperes would deposit 5 grammes of copper in half an hour, the current being supposed constant?

0·0003286 gramme is deposited in 1 second by 1 ampere.

$\therefore$ 5 grammes are deposited in 1 second by

$$\frac{5}{0{\cdot}0003286} \text{ amperes.}$$

Hence 5 grammes are deposited in 30 × 60 seconds by

$$\frac{5}{0{\cdot}0003286 \times 30 \times 60} \text{ amperes.}$$

Answer.—About 8·453 amperes.

Example 2.—How many grammes of copper would be deposited by a steady current of 40 amperes acting for 5 hours?

1 ampere acting for 1 second deposits 0·0003286 gramme, therefore 40 amperes acting for 60 × 60 × 5 seconds deposit 0·0003286 × 40 × 60 × 60 × 5 grammes.

Answer.—About 236·6 grammes.

Example 3.—How many amperes would deposit 9 grammes of copper in $2\frac{1}{2}$ hours, the current being constant? *Answer.*—About 3·043 amperes.

Example 4.—How many grammes of copper would be deposited by a steady current of 1·5 ampere acting for 16 seconds? *Answer.*—About 0·007886 gramme.

Example 5.—How many grammes of sulphuric acid would be decomposed by a steady current of 12 amperes acting for one hour? *Answer.*—About 4·029 grammes.

Example 6.—A current is passed through two voltameters in succession, the plates in one being of silver, and in the other of copper. After the current has ceased a deposit of 2·03 grammes of silver is found in the former voltameter. How much copper has been deposited in the latter?

Answer.—0·597 gramme.

Example 7.—If the mixed gas produced in a sulphuric acid voltameter be at 20°C., and the barometer stand at 77·5 centimetres, what volume of gas would

be produced in half a minute by a steady current of 18 amperes?

1 ampere in 1 second produces about

$$\frac{0{\cdot}1733 \times 76 \times (273 + 20)}{77{\cdot}5 \times 273} \text{ cubic centimetres of gas,}$$

therefore 18 amperes in 30 seconds produce about

$$\frac{0{\cdot}1733 \times 76 \times 293 \times 18 \times 30}{77{\cdot}5 \times 273} \text{ cubic centimetres of gas.}$$

Answer.—About 98·5 cubic centimetres of gas.

Example 8.—If the temperature of the mixed gas in a sulphuric acid voltameter be 19°·5 C., and the height of the barometer 75 centimetres, what current would produce 50 cubic centimetres of mixed gas in one minute?

Answer.—About 4·43 amperes.

Example 9.—A silver voltameter and a copper voltameter are arranged like C_2, C_3, in Fig. 13, so that the main current divides between them. A steady current of 3 amperes is kept flowing in the main circuit for one hour, and it is then found that the deposit of copper in the copper voltameter is 0·4 gramme. What is the deposit of silver in the other voltameter?

Answer.—About 10·71 grammes.

7. Definition of the Direction of the Current.—The next thing to define is the direction of the current, which, as already explained, can only be done in a conventional way. In the case of a sulphuric acid voltameter, we have hitherto only spoken of the total quantity of gas given off at both platinum plates, but if these gases be collected in separate tubes, as can very conveniently be done in the Hoffmann's voltameter (Fig. 16), then it is found that at one of the plates P oxygen gas is given off, and at the other P hydrogen, and the current is said to travel through the liquid towards the plate at which the hydrogen is given off, or *the current flows through the liquid with the hydrogen.* Hence, in the

Hoffmann's voltameter, shown in Fig. 16, the current would be said to flow through the liquid, in the short horizontal tube, from right to left.

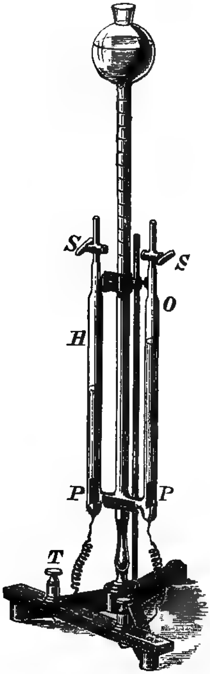


Fig. 16.—Hoffmann's Sulphuric Acid Voltameter.

The gases are evolved exactly in the proportions in which they have to be combined together to form water—viz. two (or more accurately 2·00245 at 15°C.) volumes of hydrogen and one of oxygen.* So that the electrolytic action effected by sending a current from one platinum plate to another in dilute sulphuric acid, is exactly the same as if the water had simply been decomposed. That sulphuric acid must be added to distilled water in order that an electric current may flow through it and produce oxygen and hydrogen, may easily be shown experimentally; but we are not sure of the exact action of the sulphuric acid. It may be that the sulphuric acid has to be added merely to make the non-conducting distilled water more conducting, in order that it may become possible to send a strong current through the mixture by means of an ordinary battery; or it may be that it is the sulphuric acid that is decomposed by the current, and that the water is decomposed by a secondary chemical action.

* That the gases are hydrogen and oxygen can be proved by the fact that on turning the stop-cocks S, S, the one gas H when lighted will burn with a pale blue flame, and the other O will ignite a glowing piece of wood.

In the latter case the action would be represented in chemical symbols as follows:—

Electrical decomposition $H_2SO_4 = H_2 + SO_4$.
Subsequent chemical action $H_2O + SO_4 = H_2SO_4 + O$.

Whichever may be the true explanation, the effect of the *electrolysis* of dilute sulphuric acid is that about two volumes of hydrogen come off at one platinum plate and one volume of oxygen at the other, and the weights of the hydrogen and oxygen gas liberated per second per ampere are about 0·00001043, and 0·00008285 gramme respectively. (*See* note, page 62.)

If an acid, a silver, a copper, and a zinc voltameter be all joined together, so that the same current passes through them, then it will be found that the hydrogen in the first, the silver in the second, the copper in the third, and the zinc in the fourth, all travel in the same direction; so that if through the liquid in an acid voltameter the current be said to go in the direction in which the hydrogen travels, then through the liquids in a silver, a copper, and in a zinc voltameter, it must be said to go in the direction in which the silver, the copper, and the zinc travel. Or generally *the current in a voltameter* may be said *to travel with the metal from the anode towards the cathode*, hydrogen behaving in this respect, and, as is well known, in other respects, like a metal.

The components into which an "*electrolyte*" is decomposed by the passage of a current are called "*ions*," and the *ion* which travels *with* the current is called the "*electropositive ion*," while the one which travels against the current is called the "*electronegative ion*."

With the definition given above of the direction of a current we find that if a compass needle, *n s* (Fig. 17), be pivoted so as to turn in a plane at right angles to the plane of the paper, and a current flow along any wire, A B C D, which is in the plane of the paper, then *the*

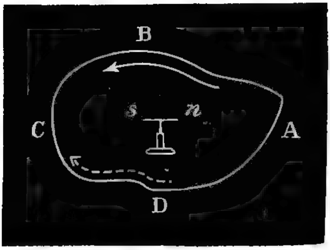


Fig. 17.

north-seeking end * *of the compass needle will come towards the observer if the current-flow round the wire in the direction indicated by the continuous arrow—that is, counterclockwise; whereas the south-seeking end of the needle will come towards the observer if the current flow in the direction of the dotted arrow—that is, clockwise.*

Similarly, if A B (Fig. 18) be any bit of wire in the plane of the paper, the north-seeking end of the needle (n, say) will come towards the observer if the current flow along this bit of wire, A B, in such a direction that A B may be regarded as forming part of a counterclockwise circuit round the needle.

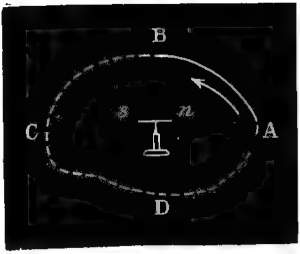


Fig. 18.

Therefore, in the upper three of the illustrations of Fig. 19, the end n

* The "*north-seeking*" end of a magnet is the one that points towards the geographical north. The simple expression "*north*" end is confusing, since in England it refers generally to the end of a magnet that points to the north, while in France it refers to the end that points to the south, the French using that definition because that end is attracted by the earth's magnetism situated in the southern hemisphere, and the *unlike* ends *attract one another*. Calling the ends of magnets "*red*" and "*blue*" is equally confusing, as some people use one of these two colours, and others the other colour, to stand for the same end. As, however, the north-seeking end of a magnet is usually marked by instrument-makers with a scratch or a cut, it would probably be best to call the *north-seeking* and "*south-seeking*" ends of a magnet the "*marked end*" and "*unmarked end*" respectively.

will come towards the observer, while in the lower three it will be the end *s* that will come out towards the observer.

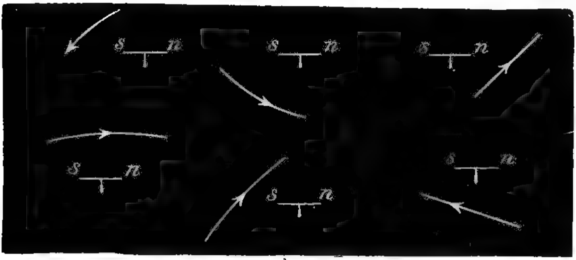


Fig. 19.

Or, again, if a wire conveying a current be coiled round a piece of iron shown end-on to an observer, then *the end of the iron nearest him will act as the north-seeking end of a magnet when the current appears to the observer to flow round the wire in a counterclockwise direction.* If the observer now look at the other end of the bar, he will of course see the south-seeking end, and in his new position the current will now appear to him to flow round the wire in the same direction as that in which the hands of a clock go (or clockwise). The relative magnetic polarity of the iron bar and the direction of the current, as indicated by the arrows, are shown in Fig. 20.

The magnetic polarity of the end of an iron bar round which a current is flowing does not depend on whether the current is flowing from the left to the right-hand end of the bar, as in the first of Fig. 20, or from the right to the left-hand end, as in the last of Fig. 20; but merely on the direction the current flows *round* the bar. Now, in spite of the difference of the winding of the wire on the first and last of Fig. 20, it will be found that in both cases, if the bar be looked at

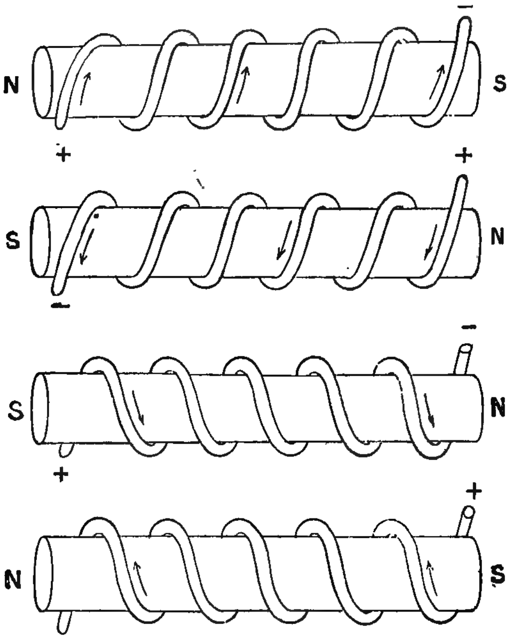


Fig. 20.

end-on from the right, the current is clockwise, whereas if the bar be looked at end-on from the left the current is counterclockwise.

Perhaps the simplest method for remembering the connection between the magnetic polarity of an iron bar and the direction in which a current circulates round it is that, if a current circulates round the bar in the direction in which the handle of a corkscrew (Fig. 21) is turned when the corkscrew is screwed down or up, the point of the screw will move towards the north-seeking magnetic end of the iron bar.

Example 10.—A compass needle is supported under a telegraph-wire running north and south. How will the needle deflect if a strong current flow through the wire towards the south?

Answer. — The north-seeking end of the needle will turn towards the east.

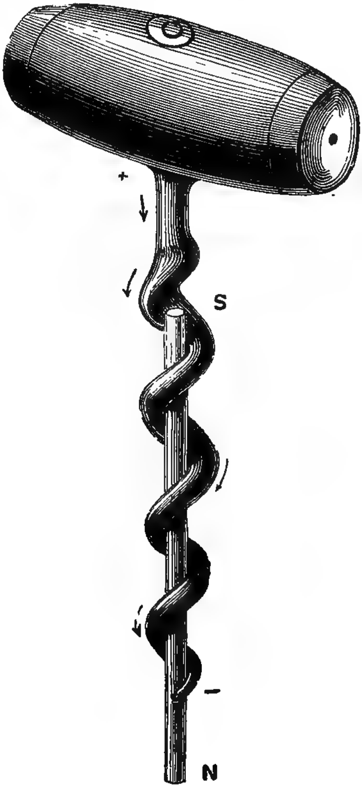


Fig. 21.

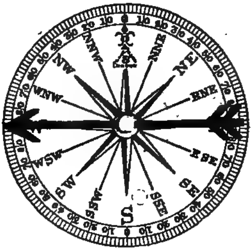
Example 11.—A flat vertical conductor is fastened against a wall, and in front is suspended a magnetic needle pivoted so as to turn on a vertical plane parallel to the wall. The north-seeking end of the needle is weighted so that the needle stands vertically when no current is flowing. Which way must a current flow in the conductor to make the upper end of the needle point to the right?

Answer.—Downwards.

Example 12.—Draw an arrow on the movable card of a compass, so that when the compass is placed above a horizontal conductor conveying

a strong current the arrow will indicate the direction of the current.

Answer.—



8. **Objection to the Usual Mode of Constructing Voltameters.**—The sulphuric acid voltameters, as usually pictured in books, which are still the only forms obtainable at some shops, are extremely unsuitable for practical use, as it is troublesome, after the tubes in which the gas is collected are full of gas, to fill them with liquid again for a new experiment.* The apparatus shown in Fig. 16, page 32, is very convenient when it is required to collect the oxygen and hydrogen separately, but it has the inconvenience that, the platinum plates being small and far apart, it requires the employment of several galvanic cells to make the gas come off quickly. For, although the quantity of gas produced in a given time by the same current is practically independent of the shape and size of the plates, the ease with which this current can be generated depends very materially on the size of the plates and their distance apart, and if we wish to produce chemical decomposition quickly, we ought to have the plates large and very near together, and the liquid employed ought to contain something like 33 per cent. of strong sulphuric acid

* The improved forms of voltameters described in § 9 have been adopted by certain instrument-makers since the first appearance of this book.

by weight, the mixture having a specific gravity of about 1·25 at 15°C.*

9. Description of Practical Forms of Sulphuric Acid Voltameters.—In Fig. 22 is shown a very convenient form of voltameter, designed by the author, consisting of a glass vessel closed at the top with an indiarubber stopper I, and containing moderately dilute sulphuric acid. The two platinum plates P are held together by indiarubber bands, but prevented from touching one another by small pieces of glass tubing put between the plates at the top and bottom, or to save the expense of thick platinum plates, two pieces of thin platinum foil may be used, stuck at the bottom with bicycle, or other suitable, cement, to a piece of glass tube, the weight of which causes the two pieces of foil to hang vertically, and therefore at the same distance apart all the way down. Wires coated with gutta-percha, to prevent

* The following table gives the specific gravities of several mixtures of pure sulphuric acid and distilled water :—

TABLE II.

Percentage H_2SO_4 by weight.	Percentage H_2SO_4 by volume.	Specific gravity at 15°C.
100	100	1·843
90	83	1·822
80	68·3	1·734
70	55·9	1·615
60	44·9	1·501
50	35·2	1·398
40	26·6	1·306
30	18·9	1·223
27	16·7	1·20
26	16·0	1·19
25	15·3	1·18
23·5	14·3	1·17
22	13·2	1·16
21	12·6	1·15
20	12·0	1·144
15	8·7	1·106
10	5·7	1·068

their being corroded by acid being spilt over them, or better still, platinum wires go from the plates, one to the "*key*" K (which is raised up above the general level of the apparatus to prevent its being corroded by drops of acid), and the other wire to one of the terminal binding screws seen in the figure. On pressing down K, the current produced by a generator attached by wires to the two binding screws, seen at the right-hand side of the figure, is allowed to pass through the apparatus. The gas which is generated is unable to escape from the vessel when the pinch-cock *c* is closed, and accordingly forces the liquid up the graduated tube *t*. This tube passes airtight through the indiarubber stopper I, reaches nearly to the bottom of the vessel, and terminates at the upper end in a thistle funnel F, so that if the current is by accident kept on for a longer time than is necessary to cause the liquid to rise to the top of the graduated tube, the liquid collects in the funnel instead of spilling over. This tube is also sloped so that the rise of liquid in the tube may increase the pressure of the gas in the upper part of the voltameter as little as possible.* The

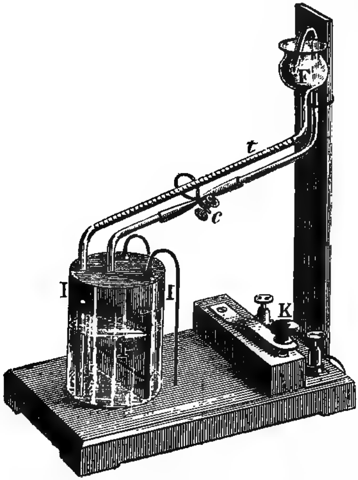


Fig. 22.—Improved Form of Sulphuric Acid Voltameter.

* If the vessel be full of liquid so that there is no gas between the top of the liquid and the indiarubber stopper I at the commencement of the experiment, the error arising from the compression of the gas produced by the rise of liquid in the tube *t* may be neglected.

second tube might be simply terminated with a piece of indiarubber tubing closed with a spring pinch-cock, *c*, on opening which the gas is allowed to escape, and the liquid runs back out of the tube *t*. If this is done suddenly, however, there is a tendency for small particles of the liquid to be jerked out of the lower tube. To prevent these particles being thrown on to the stand of the apparatus, the tube is carried up, and its end is bent over into the thistle funnel F.

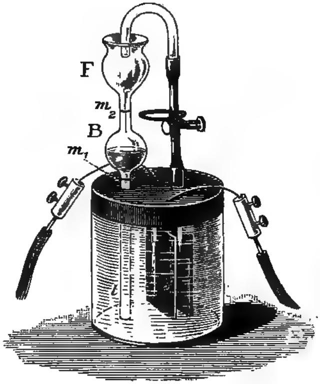


Fig. 23.—Later Form of Sulphuric Acid Voltameter.

Instead of observing the distance the liquid travels up the graduated tube *t* (Fig. 22) in a given time, we may notice the time it takes to travel from a certain fixed mark at one end of the tube to another fixed mark at the other. In other words, instead of measuring the *volume* of gas produced in a *given time*, we may measure the *time* taken to produce a *given volume*. And since for different currents the times taken for the same volume of gas to be produced must be inversely as the volumes of gas produced in the same time, we can deduce the former by employing a tube which has not been subdivided into equal volumes, but only has two marks on it.

With that method of using a voltameter to measure currents there is no necessity for the tube to be long, since it can be conveniently expanded into a bulb B (Fig. 23), and great sensibility can be combined with compactness by the bore of the tube being made small at the places where are the reference marks m_1 and m above and below the bulb.

The spring pinch-cock *c* should *not* be left squeezing the indiarubber tube of the voltameter (Figs. 22 and 23) when the instrument is out of use, for constant pressure on the sides of the tube causes it to acquire a permanent set and prevents it from regaining its circular form when the pinch-cock is removed.

10. Relative Advantages of Voltameters and Galvanometers.—The disadvantage of employing a voltameter for the practical measurement of currents is, that it requires a strong current to produce any visible decomposition in a reasonable time. Even the current of one ampere, which is about three times that used in an ordinary 8-candle incandescent lamp, would require about two hours, fifty-eight minutes, and forty-two seconds to decompose one gramme of dilute sulphuric acid, whereas the weak currents used in telegraphy, and, still more, the far weaker currents used in testing the insulating character of specimens of guttapercha, indiarubber, &c., might pass for many days through a sulphuric acid voltameter without causing any noticeable amount of chemical decomposition. Indeed, not to mention the enormous waste of time, and the difficulty of keeping the current strength which it was desired to measure constant all this time, the leakage of the gas which would take place at all parts of the apparatus that were not hermetically sealed,* would render such a mode of testing quite futile. Hence, although the voltametric method is the most direct way of measuring a current strength, and although it is constantly employed for measuring the large currents now used industrially, still the very fact that the amount of chemical decomposition produced in a given time by a certain current is independent of the shape or size of the instrument, makes it impossible to increase its sensibility. Consequently some other

* A glass vessel is said to be hermetically sealed when any opening that previously existed in it has been closed, by heating the glass round the opening until it becomes soft and sticky, and pressing the edges together.

apparatus must be employed for practically measuring small currents, and the law of the apparatus—that is, the connection between the real strength of the current and the effect produced in the apparatus—must be experimentally ascertained by direct comparison with a voltameter.

But if we are going to compare together the indications of the two instruments produced by various currents, the second instrument cannot be much more sensitive than the first; what advantage, therefore, can arise from employing an instrument as unsensitive as a voltameter? This leads us to the fact that it is very much more difficult to increase the sensitiveness of voltameters than of "*galvanometers.*" * We might increase the magnitude of the indications of a voltameter, such as that shown in Fig. 22, by using a tube *t* of very small bore, or by putting several such voltameters in series, and collecting the gases given off by each into one vessel; but we cannot by either of these means succeed in constructing a voltameter which possesses anything like the sensibility that can be very easily given to a *galvanometer*.

The indications of any measuring instrument may be increased in three distinct ways. As an illustration, let us consider an ordinary spring-balance, like the one attached to the apparatus B in Fig. 7, page 11. We may, in the first place, use a microscope, or we may fit the balance with a wheel and pinion, or employ some other magnifying arrangement to render the extension of the spring more apparent; or the electromagnet may be so constructed, either by employing more iron or by putting more convolutions of wire round its limbs, so that the pull on the "*keeper*" or armature *a* (Fig. 7), caused by passing a given current round the coils of

* While a "*galvanoscope*" is the name given to an instrument used for ascertaining whether a current is flowing, or merely which of two currents is the stronger, a "*galvanometer*" is the name given to an instrument by means of which the relative strengths of currents can be compared. Any galvanoscope when calibrated becomes a more or less sensitive galvanometer.

wire, is increased; or, lastly, we may use a weak spring in the balance, so that, for a given pull on the keeper, the movement of the index may be large.

Each of these three methods can be applied with great success to galvanometers. In the first place, the sensitiveness may be increased by using a long pointer, and the pointer may be made light, and therefore easily moved, by forming it of a very fine glass tube or of a narrow strip of some light substance like aluminium. But the best of all methods, and therefore the one employed with very sensitive galvanometers, consists in using *a ray of light* several feet long, but, of course, quite weightless. The sensibility of a galvanometer can also be made large by winding the bobbins with very many turns of very fine wire (*see* page 16); also by placing the bobbins very near the suspended needle. Friction can be diminished by suspending the little magnet with a thin fibre of *untwisted* silk. And lastly, by employing a very weak "*controlling magnet*" or by putting it at some distance from the galvanometer, the "*torque*"* required to turn the needle can be reduced to a very small amount, and therefore a considerable deflection can be produced by an extremely weak current.

And so successful have been the various attempts to increase the indications of galvanometers that it is now possible to measure accurately an electric current which is so small that it would have to flow for a *million years* through a voltameter before it produced as much chemical action as a current of one ampere could produce in a single hour.

Now, experiment shows *that a galvanometer of a particular shape and size, and with a definite magnetic needle, acted on by a definite controlling force, produced, say, by the earth's magnetism, or by some fixed permanent magnet, has a perfectly definite law connecting the*

* *Torque* is the tendency that any system of forces has to cause a body to turn, so that *torque* bears the same relation to turning that a *force* has to motion in a straight line.

magnitude of the deflection with the strength of the current producing it, although the absolute value of the current in amperes necessary to produce any particular deflection can be increased, or diminished, by using thick wire and fewer turns, or fine wire and more turns, to make a coil of the same dimensions. If, for example, with a particular gauge of wire employed to fill up the bobbin it requires $2\frac{3}{5}$ times as many amperes to produce a deflection of 40° as it requires to produce a deflection of 20°, then if a much finer gauge of wire be employed to fill the bobbin there will still be required $2\frac{3}{5}$ times as many amperes to produce a deflection of 40° as are required to produce a deflection of 20°. But in the second case $\frac{1}{1000}$ of an ampere may be all that is required to produce the 20° deflection, whereas five amperes may be required to produce the same deflection in the first. The law of the instrument remains the same, although its sensibility has been increased 5,000 times by using finer wire to wind on the bobbin.

Thus, while we may take advantage of the absolute character of the amount of chemical action to furnish us with our "*standard current meter*," we can avail ourselves of the variation that can easily be made in the deflection of a galvanometer needle corresponding with the same current, to furnish us with instruments of greater and greater degrees of delicacy.

11. Meaning of the Relative and the Absolute Calibration of a Galvanometer.—Two distinct things are required to be known with reference to a particular galvanometer—first, the law connecting the various deflections with the *relative* strength of the currents required to produce them; secondly, the *absolute* values of the currents—that is, the number of amperes required for the same purpose—or, what is sufficient if the first has been ascertained, the number of amperes required to produce some one deflection. The first is sometimes called the "*relative calibration*," the second the "*absolute calibration*" of the galvanometer.

A galvanometer with its bobbin wound with thick wire may be compared directly with a voltameter, and the relative calibration of the galvanometer determined; then if the same space on the bobbin be wound with any other gauge of wire the relative calibration of the galvanometer will be the same, and therefore known, provided that neither the length of the suspended magnet nor the law of the controlling force is in any way altered. Or if a galvanometer wound with thick wire be compared with a voltameter, and its absolute calibration determined, and if, further, the law of change of sensibility with gauge of wire has also been ascertained experimentally, then the absolute calibration of the same galvanometer, when wound with any gauge of wire, filling the same space, will be known without further experiments, provided that the length of the suspended magnet and the magnitude of the controlling force remain unaltered.

12. Experiment for Calibrating a Galvanometer Relatively or Absolutely.—Fig. 24 shows a voltameter, V, connected up with a galvanometer, G, and a "*set of resistances*," each consisting of a coil of wire with its ends

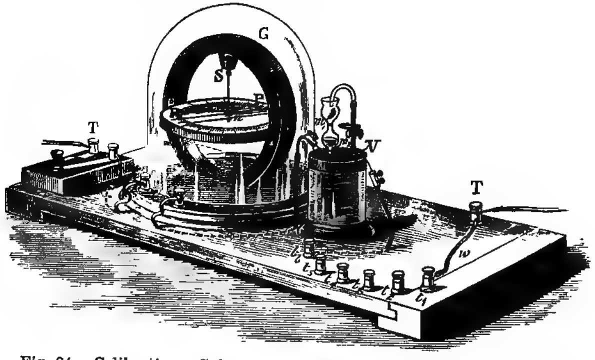


Fig. 24.—Calibrating a Galvanometer by Comparison with a Voltameter.

connected with two successive terminals, t_1, t_2, &c. These coils are wound on bobbins, and are placed underneath the base board to which the whole of the apparatus is fixed, and by means of which it can be bodily carried from place to place (from the laboratory to the lecture-room, for example, for demonstration to a class). The magnitude of the current is altered by joining the wire, *w*, to the various terminals, t_1, t_2, t_3, &c., on the base board. T, T are the main terminals, or binding screws, to which the wires coming from the current generator are attached.

It may be noticed that in the particular experiment shown in Fig. 24 it is quite unnecessary to know the length or gauge of the wire that has been wound on the various bobbins; nor is it at all necessary that all the coils should be made of the same length or thickness of wire, since, whatever resistance be inserted in the circuit, the current that passes through the voltameter is the same as the current that passes through the galvanometer, so that the variation in strength of the current is known from the voltameter observations, and not from the length of wire that has been introduced into the circuit. Indeed, the resistances in this experiment may be dispensed with altogether when there is any other easy mode of altering the current strength by using, for example, different numbers of "*cells*" or a different kind of battery to produce the current, but in practice this result is generally most easily attained by the use of a set of resistance coils.

The calibration might be performed by observing for a number of different currents the rise of the liquid in the graduated tube of a voltameter in a given time, and the corresponding steady deflection of the needle, or of the pointer, of the galvanometer. But more accurate observations can be made if, instead of observing the different lengths of the tube through which the liquid rises in a given time corresponding with the different currents used, the times be noted during which the liquid

rises through a given volume—viz. that between the two marks m_1, m_2 of the voltameter tube (Figs. 23, 24). A calculation can then be made of the rate at which gas is evolved by the current, and from this the strength of the current in amperes can be found. Thus, let V be the volume, in cubic centimetres, of the bulb B (Figs. 23, 24) between the marks m_1, m_2, and suppose the strength of the current be such that it takes s seconds for the liquid to rise from the mark m_1 to the mark m_2, then the number of cubic centimetres of gas generated by the current in every second is

$$\frac{V}{s} \text{ cubic centimetres.}$$

Now, one ampere liberates 0·1733 cubic centimetre of gas at standard temperature and pressure, and if we suppose the actual temperature to be 18°C., and the height of the barometer to be 750 millimetres, it follows from the expression given on page 29 that the number of cubic centimetres of gas liberated per second by one ampere is

$$0{\cdot}1733 \times \frac{76}{75} \times \frac{273 + 18}{273} = 0{\cdot}187.$$

Hence the current is—

$$\frac{V}{0{\cdot}187\, s} \text{ amperes.}$$

When the actual current in amperes producing any given deflection is ascertained in this manner, the galvanometer is said to be calibrated *absolutely.*

If, however, the volume V of the bulb B be unknown, or if the tube of the apparatus shown in Fig. 22 be divided into portions having equal volumes, but of *unknown* value in cubic centimetres, or, what is approximately the same thing in the case of a well-drawn tube, if the divisions merely mark off equal lengths of the tube, then the result of the experiment will merely give the *relative* calibration of the galvanometer.

The galvanometer, which is seen more in detail in Fig. 25, consists in this case of a vertical circular coil of

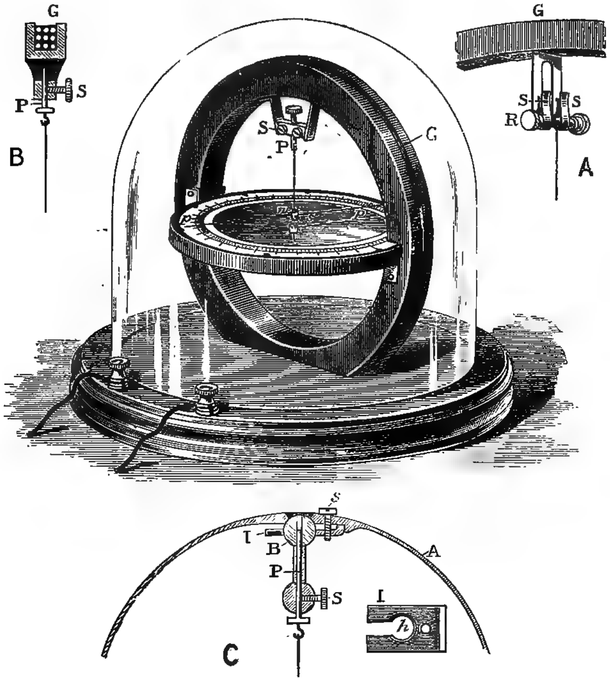


Fig. 25.—Tangent Galvanometer, showing Various Modes of Supporting the Fibre.

With A the needle can be moved sideways by sliding the roller R in the spring clips S, S, and can be raised or lowered by turning this roller. With B the pin P is held by a single set screw S instead of between two brass plates, as shown in the complete galvanometer. With C the pin P is held by a set screw S in a support made with a ball top B. This fits in the hole *h* in the plate I and forms a ball and socket, so that the needle can be accurately centred. The ball and socket joint is clamped to the semicircular support A with the screw *s*.

wire, G, at the centre of which a very short magnetic needle, *n s*, is suspended by a fibre of *unspun* silk, the

height of the needle being adjusted by raising or lowering the pin, P, which is finally clamped in position by the screw S. Other methods of supporting the fibre so that it may be easily raised or lowered, and in the case of two of them moved horizontally, are shown in Fig. 25. Rigidly attached to the needle is a long, very light pointer, *p p*, made of thin aluminium, or brass wire, or, best of all, of a fine thread of glass, the position of which is read off on a horizontal scale fixed just below the centre of the coil. If the end of the pointer be close to the scale, the error due to "*parallax*" will be avoided—that is, the error arising from looking at the pointer sideways, and so causing its end to appear to be over a part of the scale a little to the right or a little to the left of its true position. As this, however, is liable to lead to one or other of the ends of the pointer touching the scale, if the instrument be not very well made and carefully levelled, it is better to avoid parallax by fastening the scale (which in this case takes the form of a mere circular ring) to a disc of looking-glass, and by the observer always taking care, when taking a reading, to hold his head so that the pointer exactly hides its reflection in the looking-glass underneath it. If that condition be fulfilled, the distance of the scale from the pointer is immaterial.

The particular form of galvanometer shown in Figs. 24 and 25 is the one whose theory is fully described in the next chapter, but it is to be observed that any form of galvanometer could be calibrated relatively or absolutely by comparison with a voltameter in the way just described without its being necessary to know anything about the way in which the galvanometer is constructed.

13. Graphically Recording the Results of an Experiment.—The results of this experiment, and indeed of all experiments, are best recorded graphically by points on a sheet of squared paper,* that is, paper subdivided

* Prior to the commencement of the courses at the Finsbury Technical College, in 1879, squared paper was practically used in England only for the recording of results of original experiments.

into a number of small squares, by a large number of straight lines drawn at right angles to one another. The distances of the points measured horizontally from O Y (Fig. 26) should be taken to represent the deflections on the galvanometer G, and the distances of the same points measured vertically from O X the corresponding amounts of gas produced in a given time—that is, the corresponding values of the current.

It may be asked how distances along a line can represent the angular deflections on a galvanometer, or the amount of gas produced in a given time. What is meant is this: the line O X is subdivided into a number of equal divisions by the ruling on the squared paper; one, or any convenient number, of these subdivisions is taken arbitrarily to stand for 1°, then any deflection is represented by this number of divisions that we have arbitrarily taken to stand for 1°, multiplied by the number of degrees of the deflection. Similarly one, or any convenient number, of the divisions along O Y is taken arbitrarily to stand for one cubic centimetre of gas liberated per minute, then any number of cubic centimetres liberated per minute in a test will be represented by the number of divisions along O Y that has been taken to stand for one cubic centimetre per minute, multiplied into the number of cubic centimetres liberated in that test per minute.

If the galvanometer is being calibrated only relatively the unit volume of gas may be that of unit length of the tube or its whole volume, and in that case a convenient number of divisions along O Y will be taken to represent one such volume of gas liberated per minute.

And as these results, rather than the training of the experimenter, were the most important part of the investigation, the paper was very accurately divided, and sold at a high price totally out of the reach of students. It became, therefore, necessary to have squared paper specially made, cheap, and at the same time sufficiently accurately divided for students' purposes; and such paper, machine-ruled, can now be obtained at between a farthing and a halfpenny per sheet, or at about one-twentieth of the cost of the older squared paper.

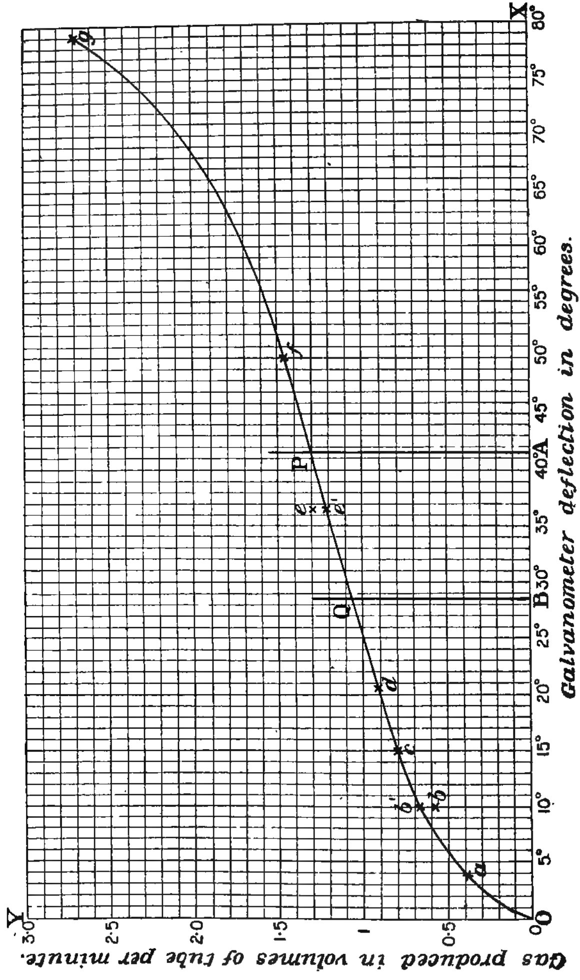

Y
X
3·0
2·5
2·0
1·5
1
0·5
O
5°
10°
15°
20°
25°
B 30°
35°
40° A
45°
50°
55°
60°
65°
70°
75°
80°
a
b
b'
c
d
Q
e
e'
P
f
g
Galvanometer deflection in degrees.
Gas produced in volumes of tube per minute.

Fig. 26.

In the same way a curve may be drawn to record the height of the barometer from hour to hour, or the variation in the price of some commodity from day to day, or the depth of the sea from point to point along some section of the ocean, and generally to give a picture of the way in which any two things vary with one another.

In selecting the scale—that is, in determining the number of divisions along O X or along O Y—that is to be taken to represent 1° deflection, or unit volume of gas liberated per unit of time in the preceding experiment, we must remember that it is desirable that the curve, which we are about to draw, shall be as large as possible, since the larger it is the more accurately we can draw it. The scale should, therefore, be so selected that the maximum deflection of the galvanometer that has been used in the experiment should be represented by nearly the whole of O X, and the corresponding maximum quantity of gas developed in the given time by nearly the whole of O Y, since with this arrangement the curve would occupy nearly the whole of the sheet of squared paper. For example, suppose that the length O X is divided by the ruling of the paper into 170 equal divisions, and O Y into 100, and suppose that the maximum galvanometer deflection was 79°, and that when that deflection was produced the liquid ascended from the zero mark at the bottom of the tube to the top mark in 22 seconds, then, if one minute be the fixed time decided on, the most suitable scales for distances measured along O X and along O Y would be selected as follows:—

$$\frac{170}{79} = 2{\cdot}15 \text{ divisions along O X per } 1^\circ.$$

$$\frac{60}{22} = 2{\cdot}7 \text{ volumes of the tube per minute.}$$

$$\frac{100}{2{\cdot}7} = 37 \text{ divisions along O Y per volume of the tube per minute.}$$

2·15 divisions per 1° would, however, be a little awkward to employ when deflections of 17°, 29½°, &c., had to be represented; 2 divisions per 1° would therefore be better. 37 divisions along O Y, to represent one volume of the tube per minute, would just enable the maximum rate of liberation of the gas, corresponding with 2·7 volumes of the tube in the minute, to be represented by the whole of O Y; but 37 divisions for unit rate of liberation would be a little awkward to employ when other rates had to be represented; probably, therefore, 30 divisions along O Y, to stand for the unit rate, would be more convenient.

Having obtained a sufficient number of points by experiment, a curve should be drawn connecting these points. Such a curve can be drawn by bending an *elastic* piece of wood, and holding it so as to pass as nearly as possible through all the points that are plotted on the squared paper to record the results, and then using the bent piece of wood as a ruler, along which to draw a line. A better way is to bend a piece of soft brass strip, bit by bit, until the edge of the strip passes through the average position of the points.

Unless, however, the experiment has been performed with great accuracy—to attain which requires, not merely the careful attention of those engaged in making the experiment, but a certain amount of practice in experimenting—it must not be expected that a curve so drawn will pass through all the points; some of them, *b*, are sure to be a little too low, meaning that the deflection on the galvanometer has been read too high, or that the rise of liquid in the graduated tube has been read too low, from, perhaps, an error having been made in taking the time, or from the current not having been kept on for a sufficient time before the pinch-cock *c* (Figs. 22, 23, 24) was closed for the gas to have commenced to come off regularly. Some of the points *e* (Fig. 26), on the other hand, are sure to be too high, meaning that the deflection on the galvanometer has been read too low, or the rise of

liquid in the graduated tube too high; or it may be that the experiments were fairly well made, and that b and e are merely plotted incorrectly, and so do not represent the results of the experiment.

14. Practical Value of Drawing Curves to Graphically Record the Results of Experiments.—It may be asked, But is it not possible that the points b and e, although not on the curve, may be quite correct? The answer is, No, because experience makes us quite sure that the connection between the deflection of the galvanometer G and the current strength must be a *continuous* one, and, therefore, that the points correctly representing the true connection must all lie on an *elastic curve*, or on such a curve as can be obtained by bending a thin piece of wood or steel, and, consequently, that if no mistake has been made in plotting the points b and e, some mistake must have been made in taking the observations. But what is even more important, we are also sure that the points b' and e' on the curve, obtained by drawing lines through b and e respectively parallel to O Y, give far more accurately the relative strengths of the currents producing respectively the two deflections in question, than the currents obtained directly from the experiment itself. *Drawing the curve, then, corrects the results obtained by the experiment.* But it does something more than that—*it gives, by what is called "interpolation," the results that would have been obtained from intermediate experiments correctly made;* that is to say, it tells us what would be the relative strengths of the currents that would produce deflections intermediate between the deflections that were actually observed. For example, suppose it be required to know the strength of current which will produce a deflection of $41\frac{1}{2}^\circ$, for which deflection no experiment has been made, compared with that which will produce a deflection of, say $28\frac{1}{2}^\circ$, for which deflection also no experiment has been made, then all that is necessary is to draw a line parallel to O Y, through the point A in O X corresponding with $41\frac{1}{2}^\circ$, similarly to draw a

line parallel to O Y, through the point B in O X, corresponding with 28½°, and observe the lengths of the lines between O X and the points P and Q, where they cut the curve, then the strength of the current which produces the deflection 41½° on this particular galvanometer bears to the strength of the current that produces the deflection 28½° the ratio of the length A P to the length B Q.

If the curve is an absolute and not merely a relative calibration curve, then the scale on which it is drawn will be known, and therefore the number of amperes corresponding with either A P or B Q.

The method of plotting the results of experiment on squared paper, and drawing a curve through them to graphically record the result, has a third important use in that, just as a map gives a better idea of the shape of a country than pages of description, *a curve enables us to see at a glance the general character of the result obtained.*

For example, suppose that the results obtained in some particular calibration of a galvanometer are:—

Deflection.	Relative Strength of Current.
10	24
17·3	41·5
22·8	54·7
29·5	70·8
37·4	89·7

no exact notion of the law of the galvanometer can be obtained by a glance at these figures; but if they be plotted on squared paper a straight line (Fig. 27) is obtained, and from this we see at once that this particular galvanometer has, in some way or other, been so constructed that the angular deflection of the needle is directly proportional to the strength of the current.

In the great majority of cases the angular deflection of the needle of a galvanometer is not proportional to the current strength, and a calibration curve is then needed to show the connection between them. After a little experience the eye becomes accustomed to the peculiarities of curves, and a glance at the calibration curve is

then sufficient to convey much information about the instrument to which it refers. It is always difficult for anyone to grasp the meaning of a table of figures, even if it be as simple as that just given, but the curve which represents them is much more readily understood, and its chief characteristics can also be more easily remembered.

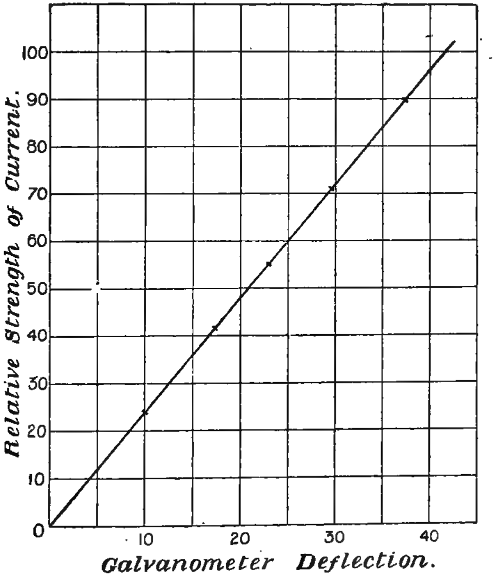


Fig. 27.

The curves are rendered more expressive if they are always plotted so that the horizontal distances, or "*abscissæ*," represent the values of the thing easily observed—for example, the angular deflections of the needle of a galvanometer, the hours of the day, the days of the week, &c.; and the vertical distances, or "*ordinates*,"

represent the values of the variable quantity which it is desired to record—for example, the relative strengths of the current producing these observed deflections, the heights of the barometer, the price in pounds of some commodity, &c.

It might at first sight appear to be a matter of indifference which of the two quantities was plotted horizontally; so also the north on a map might be at the bottom or at either of the two sides. But, just as convention has led to maps being always drawn with the north at the top and with the east to the right hand, so by common agreement the values of the previously unknown quantity are plotted *vertically*, and the values of the quantity which is assumed to vary regularly in a known manner are plotted *horizontally*. Hence in graphically recording the temperature at different hours of the day, temperature is plotted vertically and time horizontally; or in drawing a curve to indicate the depth of the Atlantic at different points between England and America, depth is plotted vertically, and distance measured, from either England or America, along the surface of the sea horizontally.

15. To Construct a Galvanometer Scale from which the Relative Strengths of Currents can be at once Ascertained.—Galvanoscopes, and even cheap galvanometers, are frequently constructed with scales divided simply into degrees, so that it is generally impossible by the mere inspection of the deflections produced by different currents to determine the exact relative strengths of these currents. If a calibration curve has been drawn from the results of previous tests, the relative strengths of any currents can, of course, be ascertained by using the curve to interpret the meaning of the galvanometer deflections. Constant reference, however, to a calibration curve or to a table of values leads to much waste of time, and therefore, when a galvanometer is to be used under the same general conditions, it is better to construct a scale with the graduation marks so drawn that the

relative strengths of currents are directly proportional to the deflections they produce as measured by the numbers on this specially constructed scale.

Such a scale may be constructed as follows :—Ascertain from the calibration curve (Fig. 26) the angles in degrees measured along O X, which correspond with currents whose relative strengths are proportional to 10, 20, 30, &c.; that is, with the currents which produce amounts of gas per minute, measured along O Y, proportional to these numbers. Then, by means of a protractor (Fig. 28), set off these angles on a blank scale and mark at the ends of the lines so drawn the numbers 10, 20, 30, &c., obtaining the result seen in Fig. 29. The angular spaces between 0 and 10, 10 and 20, 20 and 30, although of different sizes, correspond with equal additions to the current, and therefore must now each be divided into the same number of parts, and we obtain finally the scale shown in Fig. 30.

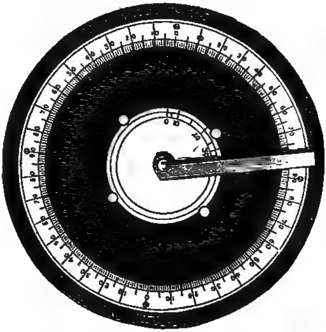


Fig. 28.—Protractor used in Subdividing a Galvanometer Scale.

As this scale is only intended to give the relative strengths, and not the absolute strengths of currents in amperes, the numbers 5, 10, 15, &c., or 2, 4, 6, &c., might have been put instead of 10, 20, 30, &c. If, however, the first of the larger angular spaces be taken as 10°, then it is convenient to make that angular space correspond with a current called 10, since the subdividing of this angular space into ten parts leads to the smaller angular spaces being about 1° each and, therefore, of convenient sizes.

In subdividing the larger angular spaces into the smaller ones a certain amount of discretion must be employed. If the ten parts into which the space between 10 and 20 on the scale was divided were made exactly equal, and also the ten parts into which the space between 20 and 30 were also made exactly equal, each of the

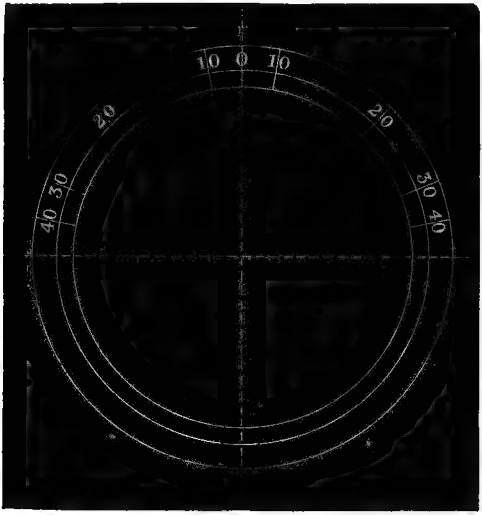


Fig. 29.

former would be much larger than each of the latter. Hence at the mark 20 there would be a sudden jump in the length of the space between two division marks; but that must be wrong, since as we pass along the scale the division marks must either be equidistant, or, if the space separating two adjacent ones varies, the variation must be a *gradual* one. We must, therefore, not simply divide the large angular space between 10 and 20 into ten equal parts, and the smaller angular space between

20 and 30 also into ten equal parts, but the lines must be judiciously drawn so as to make the spaces vary *slowly* in length.

At the point b' on the curve (Fig. 26, page 52) corresponding with the mark 10 on the scale the curve is bending over so as to become flatter than it was near the

Fig. 30.—Galvanometer Scale Divided so that the Readings Represent the Relative Strengths of the Currents.

beginning; hence, as we approach the mark 10 from 0, the spaces separating pairs of division marks on the scale ought slowly to increase. At the point d on the curve, corresponding with about $13\frac{1}{2}$ on the scale, this change of curvature ceases; the curve becomes a portion of a straight line, and remains a straight line up to a deflection of about 45°, corresponding with about the mark 20 on the scale; the division marks between $13\frac{1}{2}$ and 20 should, therefore, be drawn at equal distances. At about

the point f the curvature begins again to change, in this case growing steeper; consequently the division marks ought to begin to close up. This closing-up of the lines on the scale must go on up to the end of the scale corresponding with the point g on the curve, so that the length of a space near 50 on the scale is somewhat less than a space near 40, which is itself somewhat less than a space near 30, and so on.

Note to page 33.—The latest view of the electrolysis of dilute sulphuric acid is that the current splits the acid up into hydroxyl (OH) and *ionic* sulphur dioxide, according to the formula

$$2(H_2SO_4) = 4\,(OH) + 2\,(SO_2),$$

then that the hydroxyl resolves itself into water and oxygen according to the formula

$$4\,(OH) = 2\,(H_2O) + O_2,$$

while the sulphur dioxide acts on the water, forming sulphuric acid and liberating hydrogen according to the formula

$$2\,(SO_2 + 2H_2O) = 2\,(H_2SO_4) + 2\,(H_2).$$

CHAPTER II.

GALVANOMETERS AND AMMETERS.

16. Distribution of Magnetism in a Permanent Magnet.—Two magnets, when placed near together, are found to exert a force on one another which is an attraction or repulsion according to the way in which one is situated relatively to the other. Also, if a piece of soft iron is placed near a magnet it is magnetised and attracted towards the magnet, and if the iron is allowed to touch the magnet a force is required to pull it away again. The strength of the force needed to detach it, however, varies with the part of the magnet touched, and this fact is expressed by saying that the magnetism of the magnet is more concentrated at some parts than at others. The distribution of magnetism in an ordinary bar-magnet may be conveniently measured with the apparatus shown in Fig. 31, where M M is the magnet and B the piece of soft iron, which may be pear-shaped

or made like a ball, with a little ring to attach it to the thread. There are several ways in which the force required to detach the ball from the magnet may be measured, but the one that has been found very

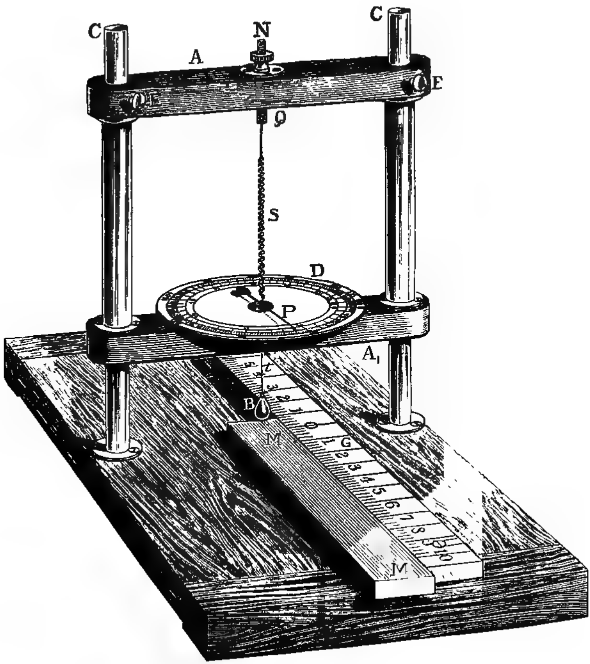


Fig. 31.—Apparatus for Measuring the Distribution of Magnetism.

convenient in practice is to support the ball by a thread, t, attached to the end of one of the "*magnifying springs*," s, devised by Professor Perry and the author for magnifying a linear motion. This spring, a portion of which is seen enlarged in Fig. 32, is shaped like a narrow

shaving curled up into a cylinder of small diameter, and, unlike an ordinary spiral spring, it has the peculiarity that *for a small increase in length along the axis there is large rotation of one end of the spring relatively to the other, the angle of rotation being directly proportional to the axial extension and to the force which is applied to produce this extension.*

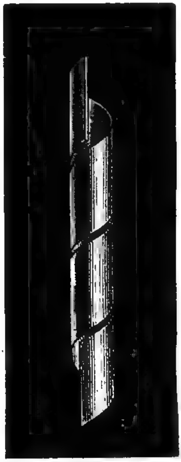


Fig. 32.—Magnifying Spring.

To the lower end of the magnifying spring S (Fig. 31), there is attached a light pointer, P, and the upper end of the spring is soldered to a screw, Q, which can be raised or lowered by turning the nut N. The screw itself is prevented from turning by a pin fastened to the bar A, sliding in a vertical saw cut in the side of the screw Q. The graduated dial D is carried by a metal disc, at the centre of which is soldered a piece of metal tube, which fits fairly tightly into a hole in the lower bar A_1. Thus the dial can be easily turned by hand, so as to bring the zero under the pointer P, when the magnet M M is removed, and the only force stretching the spring is the weight of the iron ball B. Then, if the ball be pulled down until it touches the magnet, and the nut N be then slowly turned until the ball is just pulled away, the angle through which the pointer P has then turned from the zero is a measure of the force exerted to detach the ball.

The flat graduated bar G is fastened to the base board by screws; but as these pass through slots cut in the wooden bar G at right angles to its length, this bar can be moved and screwed in any desired position, and so used as a guide, to slide the magnet M M against, when

it is desired to make a series of measurements at points along a particular line on the magnet.

When we wish to measure the force along the edge of the magnet, or along a line on the face of a very thick magnet, it is desirable to begin by loosening the screws E, E, which clamp the bar A to the vertical uprights C, then raising the bar A to clamp it higher up. In fact, the motion of the bar A up or down may be regarded as a *coarse adjus'ment*, and the turning of the nut N as a *fine adjustment*. As parallax will be diminished by keeping the distance between the dial and the pointer small, it is desirable to raise the bar A_1 when the bar A requires to be moved up. To make one adjustment sufficient it is convenient to connect the bars A and A_1 by tubes sliding on the uprights C C, as seen in the figure, so that the whole of the measuring apparatus can be moved up and down together when the screws E, E are loosened.

Before making a test the face of the magnet and the lower surface of the iron ball should be rubbed with fine emery-cloth and then wiped free from dust, and *it is to be carefully remembered that all magnetic experiments on the force of detachment are much affected by the character of the surfaces of contact.* It is therefore desirable to make each experiment several times.

If experiments be made at points equidistant from one another all along, say, the central line of the magnet, it will be found that the force exerted by the magnet on the ball is very large towards each end, rapidly diminishes as we approach the centre, and becomes practically nought at the middle of the magnet. If similar experiments be conducted along a line parallel to the long edge of the magnet, but much nearer to one edge than the other, similar results will be obtained, but the forces at the ends of the magnet will be even greater than before. If the magnet be "*uniformly magnetised,*" the attraction of the iron ball will not indicate any difference between the forces at two points similarly situated relatively to

the two ends of the magnet; but if we approach our bar magnet, M M, to a suspended compass needle, we find that the north-seeking end of the compass needle is attracted by one end of the bar magnet and repelled by the other, and so for the south-seeking end of the compass needle.

Hence, although the forces exerted on a piece of *soft iron* at points symmetrically situated relatively to the two ends of a uniformly magnetised steel bar are the same in every respect, the forces exerted by the two ends of the large magnet on one end of a compass needle are opposite in character.

17. Magnetic Poles.—The experiment just described shows that the magnetic force exerted by a magnet on a piece of soft iron is greatest near the ends of the magnet, although every part of the magnet exerts some force. Now it is found that the longer a magnet is compared with its breadth, the more concentrated is the magnetism towards its ends; and when the magnet is, like a needle, very long and thin, all the force which it is capable of exerting is due to the action of the magnetism present at the two extreme points of the needle. These points at which the magnetism is concentrated are called the "*magnetic poles*" of the magnet, and the line joining the poles is called the "*magnetic axis*" of the magnet.

18. Why Magnetic Needles tend to Point North and South.—If a piece of magnetised steel, $n\,s$ (Fig. 33), be balanced in a paper stirrup, P, and be suspended by a fibre of *unspun* silk so that the steel wire or bar is free to turn horizontally, it will be found that its axis points towards a magnetic pole held near it. This is because the poles of the suspended bar are acted on by forces due to the neighbouring pole. This action is an attraction in the case of one of the ends of the suspended bar, and a repulsion in the other. In the same way, due to the "*magnetic field*" exerted by the earth, the poles of any magnet are acted upon by a magnetic force which is directed northwards for one and southwards for the other pole. This force always exists, whether the magnet be

fixed or movable, and whether it is pointing northwards or in any other direction; but if the magnet be free to move easily, so that it can be turned by the application of a comparatively small force, and if it be acted on by the earth's magnetism alone, the axis of the suspended magnet is found to place itself north and south, and always with one particular end of the magnet towards

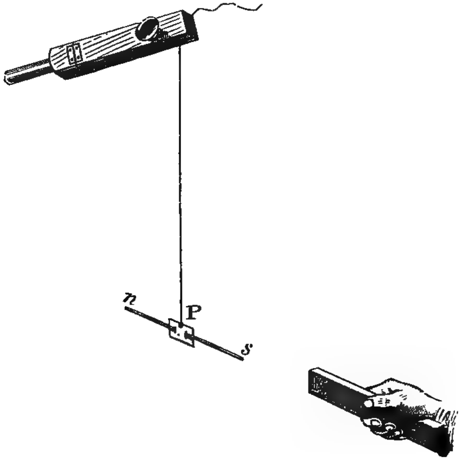


Fig. 33.

the north, which is therefore called the "*north-seeking*" end or pole of the magnet.

If this test be made with various magnets, first with one and then with the other, care being taken that the suspended magnet be not near any of the others, the north-seeking end of each magnet can be successively found and marked for future reference. Now, if any one of the magnets be suspended, and the ends of one of the other magnets be brought respectively near the ends of the suspended magnet, it will be found that marked poles

or unmarked poles repel one another, while a marked and an unmarked pole attract one another. Hence we are led to the general rule that *similar poles repel, dissimilar attract.*

19. Why a Galvanometer Needle has a Given Deflection for a Given Current.—A body moves, or changes the character of its motion, only when a mechanical force is exerted upon it. A magnetic needle which is free to turn is found to deflect whenever a wire through which a current of electricity is flowing is brought near it, unless the wire be placed exactly at right angles to the suspended needle and the current be flowing in one particular direction along the wire. Hence this deflection, which always occurs except in the case just mentioned, must take place, because in some way or other a conductor carrying an electric current exerts a force on the poles of a magnetic needle.

If there be no other magnetic force acting on the needle than that due to the current, the needle, if free to turn, will place itself at right angles to the wire conveying the current, the north-seeking or marked pole (as was seen in § 7, page 34, Chapter I.) coming towards the observer if the current flows in such a direction as to form part of a counterclockwise circuit round the needle.

If, in addition to the forces acting on a needle due to the current, there be other forces due to one or more magnets, weights or springs, &c., then the needle, if free to turn in any direction, will place itself along the resultant of all the forces. And consequently the law of a particular galvanometer—that is, the law connecting the strength of the current with the magnitude of the deflection—depends on the way in which the direction of the resultant force varies with the current flowing through the galvanometer coil.

If, instead of allowing the magnetic needle to deflect more and more as the current is increased, it is always held in the same position relatively to the coil by the application of a force, the law of the instrument becomes

very simple, for as long as the current does not become so strong as to alter the magnetism of the needle *the strength of the current is directly proportional to the force that has to be exerted to keep the needle at rest relatively to the coil; and this simple law is true whatever be the size and shape of the coil, and whatever be its position relatively to the magnetic needle.*

But with small currents the force exerted on an ordinary magnet is too feeble to enable the current to be easily measured by measuring a force, and consequently with galvanometers the needle is generally allowed to deflect, and the magnitude of the current is ascertained from the amount of the needle's deflection.

If the magnetic field controlling the motion of the needle and tending to bring it back into the zero position be that produced by the earth, the force acting on the end of the magnet will be the same both in magnitude and direction for all positions of the magnet within a limited space, say that of an ordinary room. Such a field is called a "*uniform magnetic field*," and it can be produced not only by the earth, but by a magnet put far away. The magnetic field, for example, produced inside, say, the space of a cubic inch by a magnet put a few feet away, will be practically uniform. A very weak magnet put at a distance of an inch may produce as powerful, or even a more powerful, effect than a strong magnet at a distance of several feet. The former, however, will not produce a uniform field throughout a space of even a cubic inch. The condition, then, for *the magnetic field throughout a space being a uniform one is that the linear dimensions of the space are small compared with the distance of the magnet from the nearest point of the space in question.*

And in exactly the same way, as long as we are dealing with a space of, say, a few thousand cubic feet, the gravitational force with which a body is pulled to the earth is constant both in magnitude and direction. But when the body is removed from England to America this is no longer the case.

20. Mapping out Lines of Force.—The apparatus seen in Fig. 34 enables us to map out the field produced by

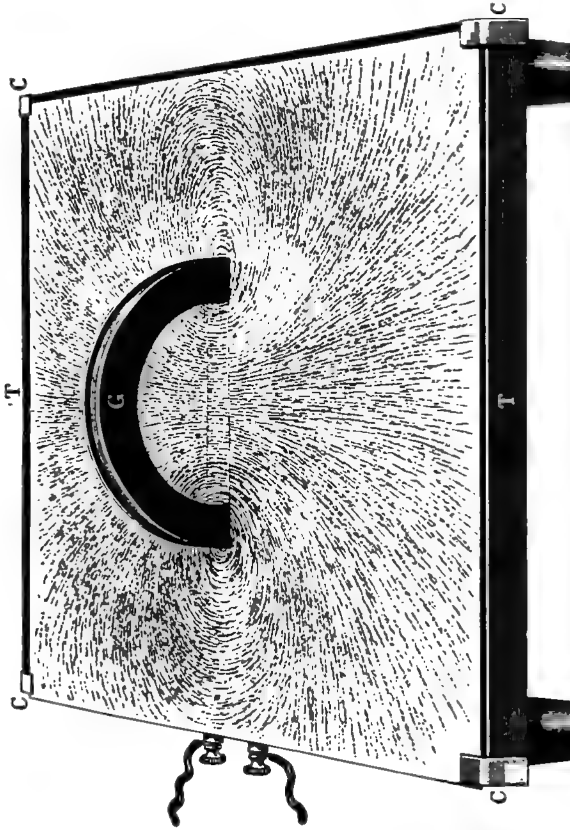


Fig. 34.—Lines of Force due to a Circular Coil carrying a Current.

a current flowing round a circular coil, G, by scattering iron filings over paper laid on a horizontal table, T T,

passing through the centre of the vertical coil, and by gently tapping the paper to assist the filings in over-

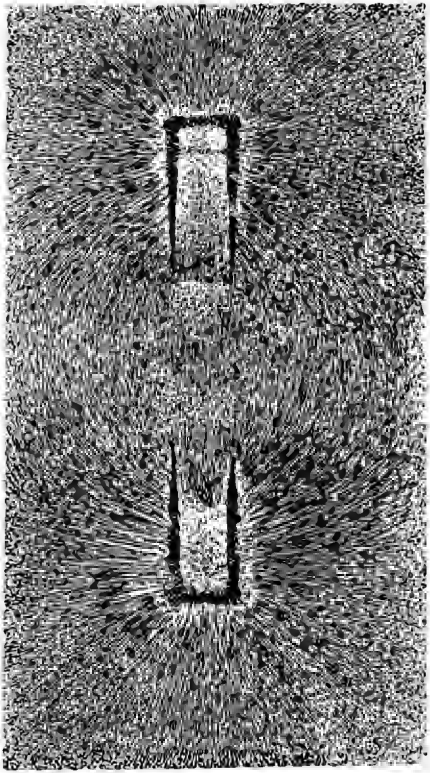


Fig. 35.—Lines of Force with a Bar Magnet.

coming friction The paper itself is held firmly to the table by the spring clips, c, c, c, c.

There are various easy ways of fixing these curves marked out by the iron filings, and so enabling a record

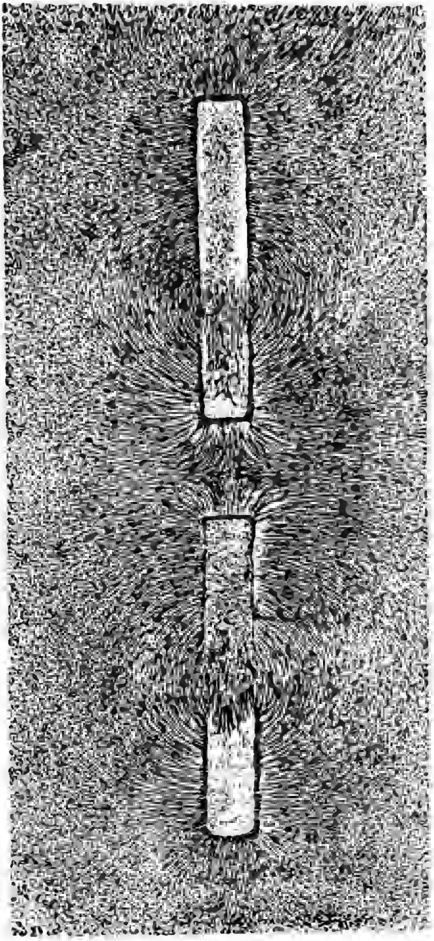


Fig. 38.—Lines of Force with Two Bar Magnets; Like Poles near one another.

to be kept of the "*lines of force*," from which we can at once see the position in which a little compass needle will place itself when put anywhere in the magnetic field. One of the simplest is to soak the paper in wax before using it; then, after the filings have been lightly

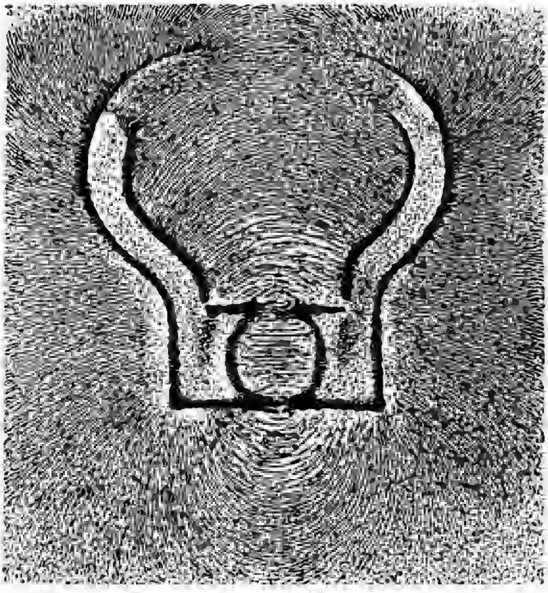


Fig. 37.—Horse Shoe Magnet with Curved Iron Pole Pieces.

scattered and the paper gently tapped in order to assist the filings in taking up their proper positions, to warm the paper with the flame of a Bunsen gas-burner moved quickly over it. The wax is thus melted, and the filings stick to it when it becomes cool and hard again.

Figs. 35, 36, 37, and 37*a* show the lines of force thus obtained with a straight magnet, with two straight

magnets placed end on with poles of the same name near one another, and with two horse shoe magnets. The horse-shoe magnets have fitted to them curved pole pieces of soft iron, and with the second one there is in addition a cylinder of soft iron placed between the pole

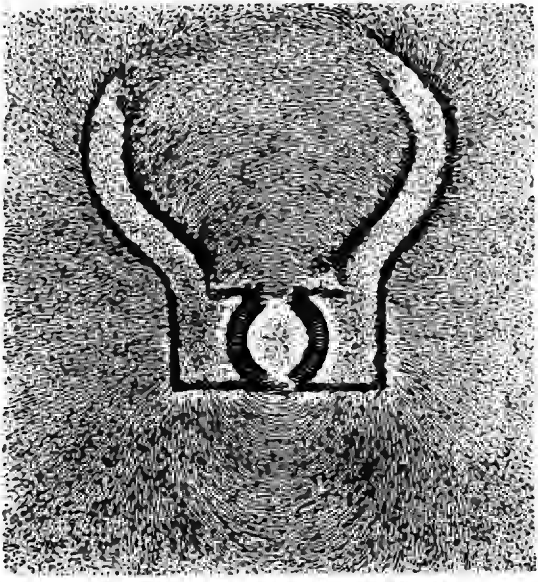


Fig. 37*a*.—Horse Shoe Magnet with Curved Iron Pole Pieces; the Magnet has also an Iron Cylinder between the Poles.

pieces to render the lines of force more or less radial, a result of great value in certain cases (*see* Moving Coil Ammeters, § 38, page 144).

For instructions regarding sifting and using iron filings *see* the Appendix, page 565.

The direction of the *lines of force* may in this and in other similar cases be traced out by using a small

compass needle, *n s* (Fig. 38); for at any particular spot where this little compass may be put the needle places itself so that its axis is a tangent to the line of force at that spot.

A sheet of paper having been placed on the horizontal table, and fixed by means of the spring clips, the little compass is placed at some particular spot, and as soon

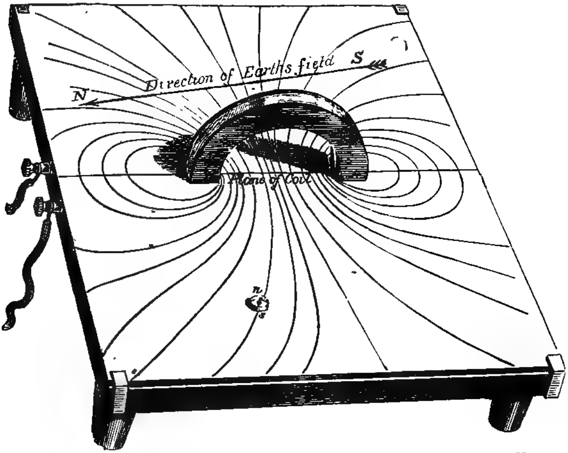


Fig. 38.—Mapping Out the Lines of Force with a Compass Needle.

as the needle has come to rest a point is marked with a pencil close to each end of the needle *n*, *s*. The compass is then removed and these two points joined with a straight line; next, the compass is placed a little farther on, so that the *n* end of the needle is close to the point formerly occupied by its *s* end. A second little line is now drawn joining points 2 and 3, and thus by drawing a number of such adjacent little lines we have a line of force marked out by a large number of its chords.

The compass method of tracing out lines of force is,

of course, a much more lengthy one than that of using iron filings, but it gives far more accurate results, since the friction resisting the compass needle taking up the right position is very small compared with that between the filings and the paper on which they are scattered. Also, unless the filings be scattered extremely sparsely, the magnetism induced in them sensibly disturbs the magnetic field, so that they indicate, not the magnetic field due to the coil alone, but the magnetic field due to the coil as disturbed by the presence of a large number of little magnets.

Further, it is important to remember, when mapping out a field due to a magnet, or to a coil carrying a current, and especially when the delicate compass method is employed, that the result can only be correct when no other magnet is near enough to produce a disturbance. Close to the magnet, or coil, under test the disturbance will be small, unless the disturbing cause be very near or very powerful; but at some distance from the magnet, or coil, under test the force which is being examined is itself so small that its direction and magnitude may be seriously altered, unless care be taken to eliminate all disturbing magnetic actions such as that of the earth, &c. To test whether this condition is fulfilled remove the magnet whose field is to be examined away to some distance, or stop the current passing through the coil if it be the magnetic field due to a coil that is being investigated, and examine whether the compass needle, when placed anywhere in the area under examination, shows no tendency to place itself in one position more than another—that is, shows that it is not acted on by any directive force.

In order to arrive at this state of things it is clear that the earth's magnetic force, which is present everywhere, and the magnetic action set up by any iron pipes, rails, &c., in the neighbourhood, must be neutralised by magnets or currents judiciously disposed.

In obtaining the lines of force seen in Fig. 38 no precaution was taken to neutralise the disturbing action

of the earth's field, the direction of which is shown by the arrow. Hence the lines of force in the further parts of the figure are twisted somewhat in a northerly direction, while in the nearer portion they are bent southwards, the effect of which is clearly seen at the left-hand lower corner.

If the main disturbance be that due to a uniform magnetic field such as is produced by the earth, a very convenient method of neutralising it over an area of two

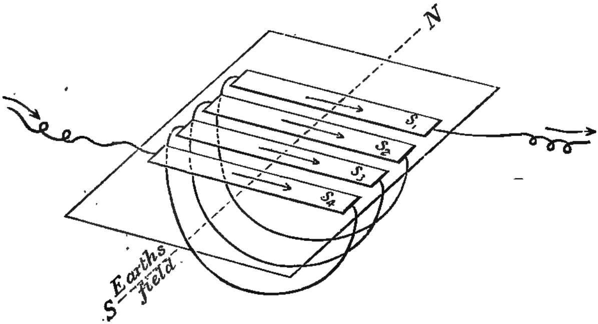


Fig. 39.—Arrangement for Neutralising a Uniform Magnetic Field.

or three square feet consists in placing a uniform sheet of copper just on or under the area in question and sending a current through it in such a direction and of such a strength as to set up a uniform magnetic field exactly equal and opposite to the disturbing one.

To avoid the use of a strong current, which would be necessary if we desired to employ a large current sheet, a set of strips, s_1, s_2, &c., of copper (Fig. 39) may be joined up in series, the whole current passing through them all in succession.*

* When the disturbance is due to the earth's field alone, the current must flow from west to east beneath the paper, and if the sheet is laid on the table beneath the paper, or at any rate is not more than an inch or two below it, the current strength must be about 0·73 ampere per inch width of sheet.

21. Comparing the Relative Strength of the Different Parts of a Magnetic Field.—Not merely does the position of rest of a pivoted compass needle show the direction of the tangent to the line of force at the particular point, but the square of the number of vibrations made by the needle when set swinging gives a measure of the "*strength of the magnetic field*" at that point—that is, tells us what is the force exerted at that part of the field on each pole of the little needle. The actual number of vibrations per minute made by a needle depends on three things: (1) the strength of the magnetic field; (2) the strength of the poles of the needle, and (3) its "*moment of inertia*."* For a given needle which is not put into so powerful a field that its strength is altered (2) and (3) are fixed; consequently such a needle may be used to measure the relative strength of different parts of the field.

If the magnetic field which is to be explored be a somewhat strong one, it will be difficult to accurately time the rapid vibrations of an ordinary compass needle. It is better, therefore, to increase its *moment of inertia* by adding mass to its two ends, which can be conveniently done by selecting two leaden shot of about equal mass, making a cut in each, and slipping one over the point of the needle at each end. The needle is then balanced by moving one or other of the shot nearer to, or farther from, the centre of the needle, and the shot can be secured in position by slightly squeezing them with a pair of pliers. A compass needle with such weighted ends—the whole, however, much enlarged—is seen in Fig. 40.

When such a weighted needle is used to explore the field produced by a current flowing round a large circular coil, like that seen in Figs. 34 and 38, it is found that at

* The *moment of inertia* of a body about any axis is found by imagining the body divided up into a large number of very small parts, and taking the sum of the products of the mass of each part into the square of its distance from the axis.

all points distant from the centre of the coil by not more than about one-fifth of its radius the number of vibrations per minute made by the needle is practically the same. And, since the mapping of the lines of force shows that within this little region round the centre of the coil the lines of force are straight and all perpendicular to the plane of the coil, we see that within this region the magnetic field due to the current flowing round the coil is a nearly uniform one.

Consequently if a needle not longer than about one-tenth of the diameter of the coil be suspended at the

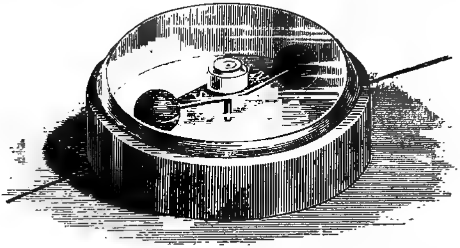


Fig. 40.—Weighted Compass Needle for Measuring the Strength of a Magnetic Field. (About two and a half times full size.)

centre of the coil, and if the controlling force be that due to the earth or to a distant magnet, the needle will be acted on by two nearly uniform magnetic fields, and, from what has been already said, it will place itself along the resultant of these two fields.

If the magnitude and direction of any two forces acting at one point on a body be known, the resultant force—that is, the force that may be regarded as acting in place of the other two—can be found by the rules of mechanics. For example, let N (Fig. 41) be one of the poles of a suspended magnetic needle, and let N P, N Q be drawn in the directions in which the two forces due to the action of some controlling magnet and the current flowing round some coil act on this pole. Further, let

the lengths of these two lines, N P, N Q, bear to one another the same proportion as the magnitudes of these two forces. Then N R will represent the resultant in magnitude and direction.

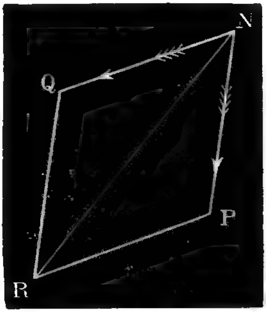


Fig. 41.

In a similar way a single resultant, S R′ (Fig. 42), may be found for the forces acting on the other end of the magnetic needle N S. The needle is, therefore, acted on by two forces, N R, S R′ (Fig. 42), which may, or may not, be equal to one another, which may, or may not, be parallel to one another, and which may, or may not, change in either magnitude or in direction as the needle moves under the action of these two forces.

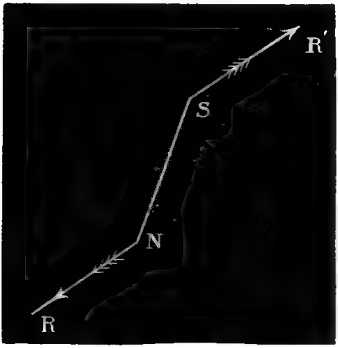


Fig. 42.

In the special case when the controlling and the deflecting fields are both uniform, N R and S R′ are parallel to one another, and do not alter in magnitude or direction as the needle deflects. They will not, however, be equal to one another unless, in addition to the two fields being uniform, the poles N and S of the needle have exactly the same strength. When N R and S R′ are parallel in consequence of both the magnetic fields being uniform, the only position in which

the needle can come to rest is that in which the four points R, N, S, R′ lie in one straight line which is parallel to N R′ or S R′; and this result, it is to be observed, is arrived at whether the poles of the needle are, or are not, equal to one another.

When the controlling field is *uniform*, if N P (Fig. 43) represents the direction and magnitude of the controlling force acting on one end of the needle for some one position of the needle, it will represent the direction and magnitude of the controlling force for all positions of the needle.

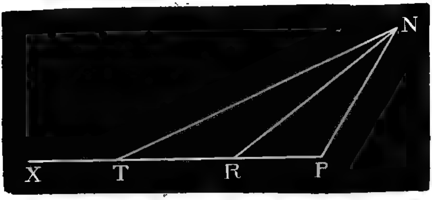


Fig. 43.

Similarly for a *uniform* deflecting field, if P X represents the direction of the deflecting force due to any current acting on one end of the needle for some one position of the needle it will represent the direction of the deflecting force produced by any other current for any other position of the needle.

To ascertain the length along P X, which represents the magnitude of the deflecting force, produced by a particular current passing round the galvanometer coil, we must measure P R proportional to that current, and, since the deflecting field is a uniform one, the length P R remains unchanged as the needle moves. Hence N P and P R represent the directions and magnitude respectively of the controlling and deflecting forces, whatever be the position of the needle, and, therefore, the only position in which the needle can come to rest under the action of these forces is along a line parallel to N R.

When no current is passing, the position of the needle will, of course, be along a line parallel to N P,

and if the current that causes the needle to place itself parallel to N R be one ampere, the angle P N R is the deflection for one ampere. To find the deflecting force for two amperes a point, T, must be taken in the line P X such that P T is twice P R; then for two amperes the needle will place itself parallel to P T, and the angle P N T will be the deflection for two amperes, &c.

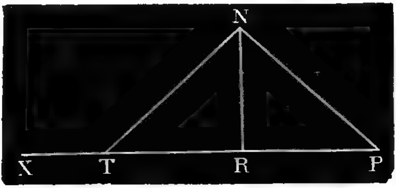


Fig. 44.

Now it is clear that the ratio of the angle P N T to the angle P N R will be very different as the direction of P X relatively to N P varies—that is, the ratio will depend on the angle which the needle makes with the plane of the coil when no current is passing. If, for example, P X were situated relatively to N P as in Fig. 44, the angle P N T might be exactly twice the angle P N R, whereas if P X and N P were as in Fig. 43 the angle P N T would be less than twice the angle P N R.

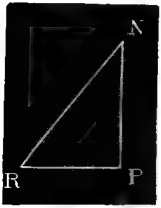


Fig. 45.

22. Tangent Galvanometer.—The relation between the angular deflections and the currents that produce them becomes very simple in one special case, and that is when the angle N P R is a right angle—that is, when the magnetic axis of the needle is parallel to the plane of the coil when no current is passing. For in that case (Fig. 45)

$$\frac{\text{P R}}{\text{N P}} = \text{tangent P N R},$$

since to find the tangent of an angle P N R we let fall a perpendicular from any point, R, of one of the lines, N R,

bounding the angle on to the other line, N P, and take the ratio which the side opposite to the given angle bears to the side adjacent to the angle.

Further, N P is a constant when a given magnetic needle is controlled by a given uniform magnetic field,

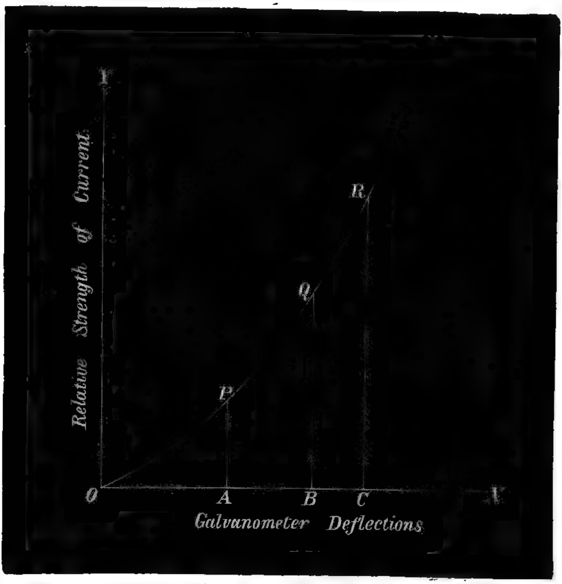


Fig. 46.—Calibration Curve of a Tangent Galvanometer.

and P R is proportional to the current flowing round the galvanometer. Hence we see that the tangent of the deflection of a needle will be proportional to the current passing round a coil when the four following conditions are fulfilled:—

(1) *The needle is controlled by a uniform magnetic field.*

(2) *The diameter of the coil is large compared with the length of the needle.*

(3) *The needle is suspended sufficiently near the centre of the coil that the field which is produced by the current passing round the coil is a uniform one in the neighbourhood of the needle.*

(4) *The axis of the needle is parallel to the plane of the coil when no current is passing.*

When these four conditions are all fulfilled the calibration curve of the galvanometer, when tested by comparison with a voltameter, as described in § 12, page 46, will be found to be of the shape shown in Fig. 46; and if any three points, P, Q, R, be taken on this curve, it will be found that the lengths A P, B Q, C R, parallel to O Y, bear to one another the ratios of the tangents of the angles represented by O A, O B, and O C respectively. Such a galvanometer (seen in detail in Fig. 25, p. 49) is, therefore, called a "*tangent galvanometer*," and it may be henceforth used without reference to any voltameter for the comparison of current strengths, as they will be simply proportional to the tangents of the angles through which the magnetic needle is deflected.

23. Adjusting the Coil of a Tangent Galvanometer.—We have next to consider how we can adjust the coil of a galvanometer so as to be sure that its mean plane is parallel to the axis of the needle when no current is passing. Owing to the coil having a certain breadth, it is impossible to see the needle when looking down on to the coil; indeed, it is for this reason that the long light pointer attached to the needle is placed at right angles to the needle. It would not be right to assume that because the instrument has been so turned that the pointer points to the zero on the scale, therefore the plane of the coil is parallel to the magnetic axis of the needle, for even if the scale has been attached to the instrument so that the line of zeros is at right angles to the plane of the coil, it does not follow that the pointer itself is at right angles to the needle. The two may

even have been placed at right angles to one another by the maker, and yet the pointer may have been bent subsequently, so that they are not at right angles at present; or no experiment may have been made by the maker to test this, as he is aware that the user will probably make a test and adjust the pointer for himself.

The test for parallelism of the axis of the needle with the mean plane of the coil may most simply be made as follows:—Turn the instrument until the pointer points to 0°, send any convenient current through it, and observe the deflection, then reverse the direction of the current *without altering its strength*, and observe the deflection on the other side of the scale. If these deflections are exactly equal, then the plane of the coil is parallel to the axis of the needle when the pointer points to 0°, and the instrument is properly adjusted. But if, on the other hand, one deflection is, say, 47° to the left, and the other, say, 44° to the right, the pointer is not at right angles to the magnetic axis of the needle, supposing, of course, that the scale has been so fixed that the line of zeros is exactly at right angles to the plane of the coil. Next, turn the instrument a little about its centre in the direction *opposite* to that in which the needle moved when the greater deflection was obtained. The pointer will now, of course, not point to zero; let it stand at 1° to the left. Again send a current, first in one direction, obtaining a deflection, say, 46° to the left, and in another direction, when it gives a deflection of, say, 45° to the right. Now remembering that the pointer started from 1° to the left, the true deflections of the needle are respectively, $46° - 1°$, or 45° to the left, and $45° + 1°$, or 46° to the right. Hence, the fault is now on the other side, or the left deflection is smaller than the right, and we have, consequently, turned the instrument too much. Turn, therefore, the coil round a very little in the opposite direction, so that when no current is passing through the instrument the pointer stands at, say, ½° to the left, and

send as before reverse currents of equal strength, obtaining apparent deflections, $45\frac{1}{2}°$ to the left and $44\frac{1}{2}°$ to the right, which, corrected for the initial zero error, correspond with equal deflections of $45°$ to either side.

The instrument will now be correct when it is so placed that for no current the pointer stands at $\frac{1}{2}°$ left, and it can be so used, but not, however, with the tangent scale described in the next section. To enable us to employ the side of the dial graduated in tangents, as well as to avoid having to remember the $\frac{1}{2}°$ left error, do not alter the position of the instrument, but bend the pointer until it points to $0°$ for the same position of the instrument in which it previously pointed to $\frac{1}{2}°$ left. The instrument will now behave as a correct tangent galvanometer when the pointer stands at $0°$ for no current.

We have spoken of reversing the direction of the current without altering its value. This may be done by causing the current to pass through any galvanoscope, the law of which may be quite unknown; and taking care that the deflection of the needle of this galvanoscope after the current has been reversed is the same in amount as it was before the current was reversed; indeed, if we reverse the connections of the galvanoscope at the same time that we reverse the connections of the battery, or other current generator employed in the experiment, it will not be even necessary to know that the coil and needle of this auxiliary galvanoscope are symmetrical, or that the strength of a current producing a deflection to the right is the same as that of a current producing the same deflection to the left.

24. Scale for a Tangent Galvanometer.—The scales of tangent galvanometers are frequently simply divided into degrees, and a reference has constantly to be made to a table of tangents to enable the galvanometer to be used. A better plan is to divide the scale, not into equal divisions, but into divisions the lengths of which become smaller and smaller as we depart from the zero or undeflected position of the needle, in such a way that

the number of divisions in any arc is proportional, but not necessarily equal, to the tangent of the angle corresponding with that arc. Or the scale may, as shown in Fig. 47, be divided into degrees on one side, and on the tangent principle on the other.

Such a tangent scale can be most easily constructed in the following way:—Draw a tangent (Fig. 48) F A F to a circle, and starting from the point A of contact of this tangent line with the circle, mark off A B, B C, C D, &c., in both directions all equal to one another. Then join the centre O of the circle with the points B, C, D, &c. by straight lines cutting the circle in the points 1, 2, 3, &c.; then the numbers 1, 2, 3, 4, &c., will be respectively proportional to the tangents of the angles A O 1, A O 2, A O 3, &c.

$$\text{For tan. } \mathrm{AO1} = \frac{\mathrm{AB}}{\mathrm{OA}};$$

$$\text{tan. } \mathrm{AO2} = \frac{\mathrm{AC}}{\mathrm{OA}};$$

$$= \frac{2\,\mathrm{AB}}{\mathrm{OA}};$$

$$\text{tan. } \mathrm{AO3} = \frac{\mathrm{AD}}{\mathrm{OA}};$$

$$= \frac{3\,\mathrm{AB}}{\mathrm{OA}};$$

and so on.

In fact it follows, from what we have already seen in § 21, page 82, that if the galvanometer needle, when no current is passing through the coils, points in the direction O A, the force perpendicular to O A required to deflect it till it points along any line, such as O D, is proportional to the length A D; and as the force exerted by the current in a tangent galvanometer is proportional to the strength of the current, and is at right angles to O A, which is the

direction in which the uniform controlling field acts on the needle, it follows that currents proportional to the

Fig. 47.—Scale for a Tangent Galvanometer.

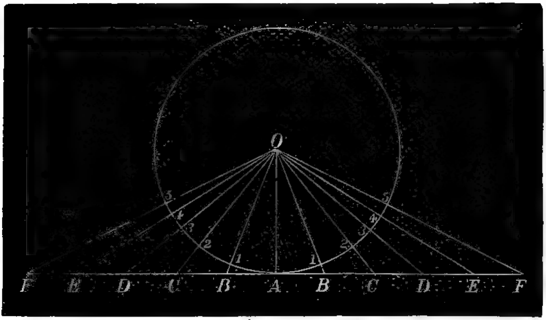


Fig. 48.—Constructing a Scale for a Tangent Galvanometer.

lengths A B, A C, A D, &c., will deflect the needle till it points respectively along O B, O C, O D, &c.

Beginners are apt to think that, because the divisions on such a tangent scale are very much crowded together in the higher part of the scale, the value of a current can be more accurately ascertained by taking a reading on the degree side, and then finding the value of the tangent in a table of tangents, than by reading it off on the tangent scale. But this seeming greater accuracy is quite delusive, since what has to be ascertained in either case is the tangent of the angle, not merely the angle, and although on the degree side of the scale the angle can be read much more accurately than can be its tangent, or a number proportional to its tangent, on the other side, this only indicates that the error of a tenth of a degree in a large angle, although a much smaller proportional error than a tenth of a degree in a smaller angle, produces a far greater proportional error in the tangent. For example, if $20^\circ{\cdot}1$ be read instead of 20°, the error is $\frac{1}{200}$, whereas if $85^\circ{\cdot}1$ be read instead of 85°, the error is only $\frac{1}{850}$, or less than a quarter of the preceding error. But the tangents are in the first case 0·3659, and 0·3640, the error in the tangent, therefore, is $\frac{19}{3640}$, or about $\frac{1}{192}$, whereas the tangents in the second case are 11·66 and 11·43, so that the proportional error is $\frac{23}{1143}$, or about $\frac{1}{50}$, which is nearly four times as great as before. Hence in this case, when the proportional angular error is diminished to one quarter, the corresponding proportional error in the tangents is increased four times. The crowding together of the divisions on the tangent scale at the higher readings is, therefore, a correct indication of the inaccuracy likely to occur in taking readings in that part of the scale.

It can be shown that if one current strength has to be measured by a tangent galvanometer, the result, other things being the same, will be most accurate when the deflection produced is 45°; or if two currents are to be measured, the measurements will be most accurate when the deflections are as nearly as possible at equal distances on the two sides of 45°.

From a consideration of what precedes, it follows that the deflection produced by a given current passing through a tangent galvanometer is *not* altered by varying the strength of the magnetic needle of the galvanometer, or by varying its length, provided that the needle is not made so long as to render the tangent law untrue for the particular galvanometer. For altering the strength of the needle alters the deflecting and the controlling forces in exactly the same proportion, so that the direction of the resultant of these two forces remains unchanged. So, also, altering the length of the needle does not change the direction of the resultant force.

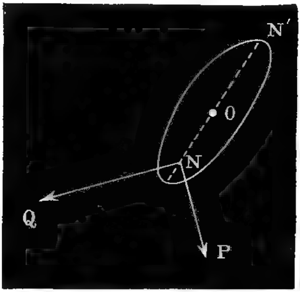


Fig. 49.

Hence the only advantages gained by using a strongly magnetised needle are, first, that it moves more quickly to the deflected position when a current is sent through the galvanometer, and returns more quickly to the zero when the current is stopped; secondly, that the friction at the pivot on which the needle turns, or the rigidity of the silk fibre supporting the needle, introduces less error in a measurement.

25. Tangent Law.—The conditions under which the tangent law is true, may be stated most generally thus:—

If any body N N′ (Fig. 49), turning about an axis at O, be acted on by two forces whose directions lie in a plane at right angles to this axis and intersect at a point N, the tangent of the angle made by N O with one of the forces, P, will be proportional to the magnitude of the other force Q when:—

(1) *The controlling force,* P, *is constant in magnitude, but not necessarily in direction.*

(2) *The deflecting force,* Q, *acts at right angles to the controlling force.*

In the tangent galvanometer these conditions, as already explained, are necessarily satisfied by the construction of the apparatus without any adjustment being necessary when the deflecting force is varied. So in the apparatus seen in Fig. 50, where both the controlling and the deflecting force are produced by weights, the above conditions will also be automatically fulfilled for any position of the rod N N′, if N N′

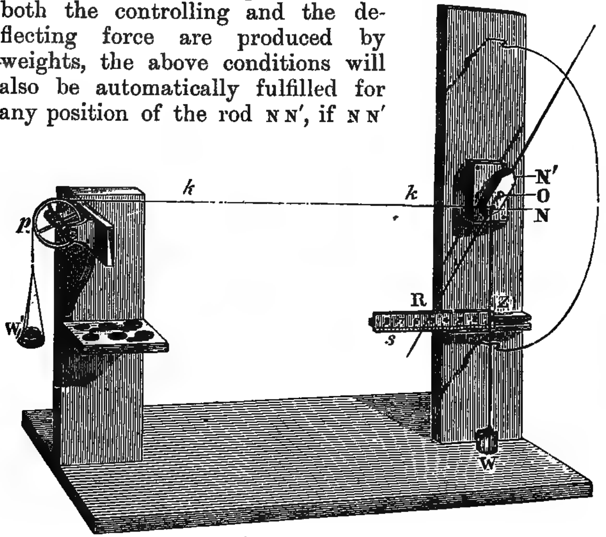


Fig. 50.—Simple Mechanical Apparatus for Testing the Tangent Law.

be short, and if the pulley *p* be placed far away from N N′ in such a position that the thread *k k* is horizontal. Any weight W′ put into the scale pan, will therefore be proportional to the tangent of the angle which N N′ makes with the direction of the controlling force.

This tangent is proportional to the length Z R if the scale *s s* be initially adjusted so that its zero line Z coincides with the pointer attached to N N′ when the only

force acting on N N′ is that due to W the controlling force. For the required tangent is the ratio of Z R to O Z, and O Z is, of course, a constant.

With the apparatus illustrated in Fig. 51, which is a more accurate, but at the same time a more expensive one than that shown in Fig. 50, the pulley *p* is near the rod N N′. Hence an adjustment is necessary to keep the

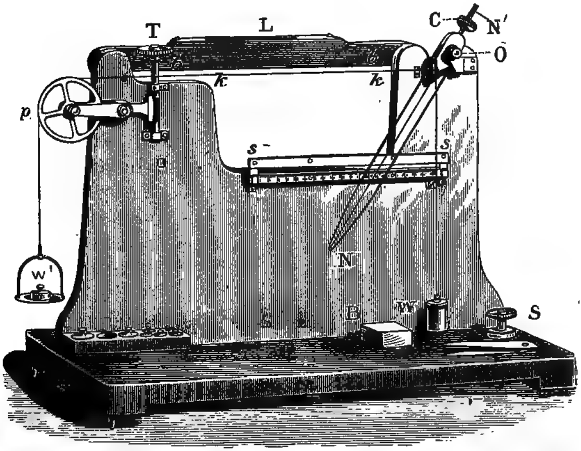


Fig. 51.—Improved Mechanical Apparatus for Testing the Tangent Law.

thread *k k* always horizontal, that is, at right angles to the direction of the controlling force. This adjustment is made by turning the tangent screw T, and the simplest way of insuring that the pulley *p* has been raised or lowered sufficiently to keep the thread *k k* horizontal, when the rod N N′ is deflected, is to commence the experiment by turning the levelling screw S, until the level L shows that the bar *b b* is perfectly horizontal; then, after putting each of the different weights W′ into the scale pan, to turn the screw T until the thread *k k*

appears by eye to be parallel to one of the edges of the bar *b b*.

As N N′ is not symmetrical above and below the axis O in the apparatus shown in Fig. 51, and, therefore, is not self-balanced, we must, before any measurements are commenced, screw the counterpoise weight C in or out, until the rod remains balanced in any position when the controlling and the deflecting forces are both nought. These forces are easily made nought by resting the weight W on the block of wood B, and by taking the thread *k k* off the pulley *p* and resting the scale pan on the base-board of the instrument. The scale *s s* is adjusted as before, so that when the controlling weight W alone acts on N N′, the zero line of the scale coincides with the position taken up by the pointer, only this adjustment can now be made very accurately by using as the pointer the wire stretched along the centre of the moving arm, and ensuring coincidence by observing when the image of this wire seen in the mirror which is attached to the scale coincides with the zero line Z.

The controlling and deflecting weights may of course be interchanged, in which case the rod N N′ will remain horizontal instead of vertical, when the controlling force alone acts on it, and the tangent of the angle, which is proportional to the magnitude of the deflecting force, will be measured on a vertical scale. Or, if a set square be used, or some similar method be employed for ascertaining when the two threads attached to N N′ are exerting forces at right angles to one another, the direction of neither force need be vertical nor remain constant as the rod N N′ deflects. Both the weights W and W′ may be attached to threads passing over pulleys, and all that it will be necessary to do before taking a reading will be to adjust one or other, or both of these pulleys, so that the two threads make a right angle with one another at the needle. That being done, the magnitude of that one of the forces which has been varied will be proportional to the tangent of

the angle that the line N O makes with the direction of the other force.

26. Variation of the Sensibility of a Tangent Galvanometer with the Number of Windings, and with the Diameter of the Bobbin.—A tangent galvanometer, whose bobbin contains only one turn of wire, is not suitable for measuring very weak currents, as it is not sufficiently sensitive. In order to obtain a delicate tangent galvanometer, the bobbin must be wound with many turns of fine wire, and the greater the number of turns employed, the smaller will be the current needed to produce a given deflection on the instrument. The exact way in which the sensibility of a tangent galvanometer is dependent on the number of windings may be experimentally tested by means of the apparatus shown in Fig. 52, and this may also be used to ascertain the variation in sensibility produced by changing the diameter of the bobbin on which the wire is wound.

g g is a flat box, containing a scale fastened to its bottom. Within this is a short needle carrying a long, light glass pointer, and suspended, by a piece of unspun silk, from a vertical brass rod, *r*, which slides up and down in a metal sleeve attached to the centre of the piece of glass forming the cover of the box *g g*, or, better, carried by the curved bridge-piece attached to the sides of the box. The box *g g* is so shaped as to allow the needle and pointer to swing freely through an angle of about 45 degrees in either direction from the central position. C C is a bobbin of large diameter, and such that its centre is at exactly the same height above the base-board B B as is the centre of the suspended magnetic needle. *c c* is a smaller bobbin, of which the diameter is exactly half that of the larger bobbin, but still large compared with the length of the suspended magnet. The centre of the smaller bobbin is also on the same level as the suspended magnet when the base-board *b b* of the smaller bobbin is placed on that of

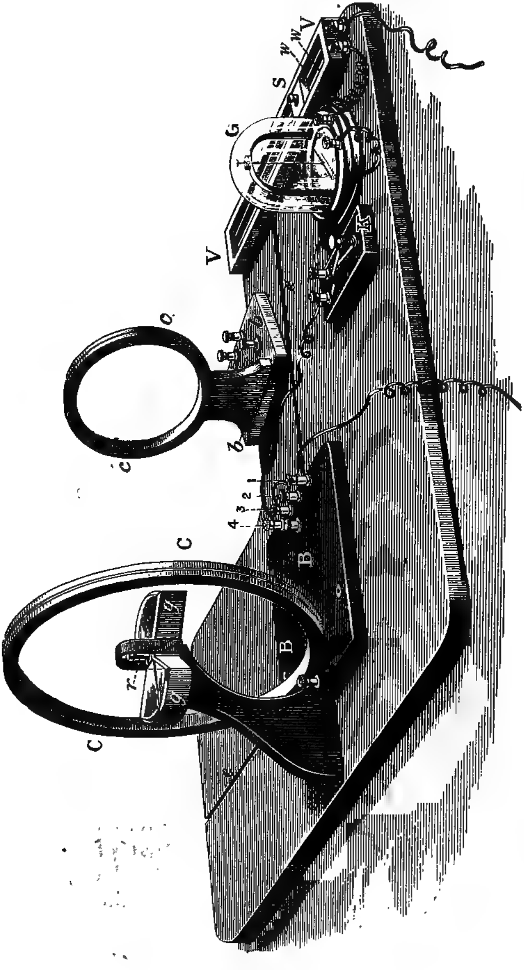


Fig. 52.—Apparatus for Testing the Strength of the Magnetic Field produced at Different Points by Different Coils.

the larger. On the larger bobbin C C are wound two distinct coils of insulated wire, one consisting of twelve convolutions, and having its ends attached to two of the binding screws, 1, 2, the other of four convolutions, and having its ends attached to the other two binding screws, 3, 4. If the binding screw 2 at the end of the first coil be joined by a piece of wire, as shown in the figure, to the binding screw 3 attached to the beginning of the second, the current will go $12 + 4$, or sixteen times round the bobbin; whereas if the wire connect the end of the first coil, 2, with the end of the second, 4, and the current enter and finally leave the bobbin by the two binding screws 1, 3, attached respectively to the beginnings of the two coils, then the current will go twelve times round the bobbin in one direction and four times in the other, or practically $12 - 4$, or eight times round the bobbin. Now, experiment shows that if a current of *constant strength* be passed successively first four, then eight, then twelve, then sixteen times round the bobbin, and if this is kept in a fixed position during the experiment, the tangents of the corresponding deflections produced will be as four to eight, to twelve, to sixteen, that is, simply proportional to the number of times the current passes round the bobbin.

In this experiment the current, after passing from the battery through the coils C C, and the key K, is led through the current indicator G, and through a variable length of wire *w w* back again to the battery. The stretched wires are, for safety, placed in a grooved box, V V, attached to the board which carries the whole apparatus, and the length of wire included in the circuit can be increased or diminished by adjusting the screw clamp S, which is so arranged as to slide along the groove. In this manner the strength of the current can be altered, and if the insertion in the circuit of a greater or less number of coils on the bobbin C C, or any other cause, tends to vary the magnitude of the current, the screw clamp S must

be moved until the deflection of the galvanoscope G returns to its original value, thus showing that the current has the same magnitude as at the commencement of the tests.

These experiments prove that the sensibility of a tangent galvanometer is proportional to the number of turns of wire used on its bobbin. We may next proceed to investigate the effect of the size of the bobbin by experiments made on the small coil *c c*. The diameter of this coil is only one half that of C C, and there are four convolutions of wire wound upon it. When experiments are made it is found that, if the two bobbins *c c* and C C are placed so as to be in the same plane, and so as to have their centres coincident with that of the suspended magnet, the tangent of the deflection produced by any current flowing round the smaller one is twice as great as the tangent of the deflection produced by the *same* current flowing four times round the larger bobbin; and also, if the same current pass four times round the smaller bobbin in one direction, and eight times round the larger in the opposite direction, that no deflection is produced whatever the current may be.

From this we learn that the tangent of the deflection produced by a current, that is, *the sensibility of the instrument is directly proportional to the number of convolutions of wire, and inversely proportional to the diameter of the coil.*

In order, therefore, to get a sensitive instrument we should use coils of small diameter, and wound with many turns of wire, and it might be imagined that a tangent galvanometer intended for the measurement of very weak currents should be made in this way. As a matter of fact, however, the coils of good tangent galvanometers are always large in diameter compared with the length of the suspended needle; and the number of turns of wire used in winding is always limited by the consideration that the depth and width

of the channel in which the wire is wound must not exceed a certain fraction of the diameter of the coil. These restrictions are only imposed in order to ensure the fulfilment of the tangent law, and need not be considered when there is no necessity for the tangent of the galvanometer deflection to be strictly proportional to the current.

An instrument which is to be used as a tangent galvanometer must, however, be so constructed that all the conditions mentioned in § 22, page 84, as necessary to ensure the fulfilment of the tangent law are complied with. Now when the needle in the box *g g*, Fig. 52, is deflected, its poles move away from the coil C C, and the force exerted by the current in this coil is less, after the needle has moved, than before. The tangent law will not hold good unless the change produced in this way is small enough to be neglected. In order to test this point, the apparatus shown in Fig. 52 is arranged so that each of the coils C C, *c c*, can be moved either in its own plane or perpendicular to its plane. To facilitate this two grooves, *e e*, are made in the base-board, and cylindrical pegs are placed through holes suitably made in the base B B of the coil C C and work in these grooves, so that the coil can be slid along either in or perpendicular to its own plane. The grooves are graduated, and the alteration produced by moving the coil a given distance from the needle can be noted for each of the bobbins C C and *c c*.

Experiments of this kind will show that as the bobbin is moved the deflection alters, and that the change produced for the same amount of motion is proportionately greater for the small bobbin *c c* than for the large one C C. For example, if the coil be moved parallel to itself, and so that its axis passes through the centre of the needle, it will be found that the tangent of the deflection of the needle for a given current is proportional to

$$\frac{r^2}{\sqrt{(r^2 + x^2)^3}}$$

where r is the mean radius of the coil and x the distance from the mean plane of the coil to the centre of the needle. Now it is clear from this formula that for a given change in x there will be a greater change in the value of this fraction the smaller r is.

It thus becomes apparent that any error due to want of proper centering of the needle of a tangent galvanometer, or to the actual movement of its poles when it is deflected, must prove far more serious when the bobbins are small than when they are large; and for this reason instruments in which the tangent law is to be accurately relied upon are invariably constructed with large bobbins.

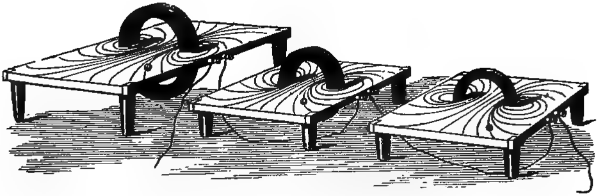


Fig. 53.—Mapping out the Fields produced by the Same Current Flowing Round Different Coils.

The law and variation of the sensibility of a tangent galvanometer with the number of windings and the diameter of the coil can be ascertained, and the practical importance of using a large coil for a tangent galvanometer experimentally illustrated, by employing three coils similar to the one used in § 20, instead of the apparatus shown in Fig. 52. These three coils (Fig. 53) are joined up in series, so that the same current necessarily passes through each, the diameter of the large coil is double that of either of the smaller, and the number of windings on the large coil and on one of the smaller are equal, and double that on the other small coil. For example, the large coil may conveniently be 30 centimetres in

diameter, and have 100 convolutions of wire on it, while the two small coils are each 15 centimetres in diameter, and have respectively 100 and 50 convolutions on them.

By means of compass needles (Figs. 38 and 40) the fields of the three coils can be explored, both as regards the direction of the lines of force, and the strength of the field at different points, and, from the results so obtained conclusions can be drawn as to change of the force at the centre of a coil produced by varying the size of the coil or the number of windings and the rate of variation of the force as we depart from the centre for each of the two sizes of coils.

The three coils must, of course, be placed far enough apart that the fields do not act on one another, and to save time the fields for the different coils may be mapped out simultaneously by different explorers.

Example 13.—A tangent galvanometer is made with two coils of equal diameter, the first consisting of 500 convolutions of wire, the second of one convolution. If a current of 0·25 ampere sent through the first cause a deflection of 45°, what current sent through the second in the opposite direction, while the same current was still flowing through the first, would cause the deflection to become one of 10°?

Let x be the unknown number of amperes:

$$\text{Then } \frac{500 \times 0{\cdot}25 - x}{500 \times 0{\cdot}25} = \frac{\tan. 10^\circ}{\tan. 45^\circ}$$

Answer.—103 amperes.

Example 14.—A galvanometer is about to be constructed of two coils: the first, 6 inches in diameter, consists of 350 convolutions of wire; the second has two convolutions only. A current of 0·4 ampere sent through the first causes a deflection of 30°. What must be the diameter of the second coil, in order that a current

of 80 amperes in the opposite direction, sent through it, while 0·4 ampere is still flowing through the first, may cause the deflection to become 5°?

Let x be the diameter of the second coil.

Since the effect of the current is directly proportional to the number of convolutions, and inversely proportional to the diameter—

$$\frac{\dfrac{0{\cdot}4 \times 350}{6} - \dfrac{80 \times 2}{x}}{\dfrac{0{\cdot}4 \times 350}{6}} = \frac{\tan. 5^\circ}{\tan. 30^\circ}$$

Answer.—8 inches nearly.

Example 15.—A tangent galvanometer is about to be constructed of two coils: the first, 7 inches in diameter, consists of 600 convolutions of wire; the second is to be 5·5 inches in diameter. A current of 0·1656 ampere sent through the first causes a deflection of 40°. Of how many convolutions of wire must the second coil consist, in order that while 0·1656 ampere is still flowing through the first, a current of 65 amperes flowing through the second may cause the deflection to become 8°?

Answer.—One convolution.

27. Values in Amperes of the Deflections of a Tangent Galvanometer controlled only by the Earth's Magnetism.—The sensibility of any galvanometer depends not merely on the bobbin, but also on the strength of the controlling field. If this controlling field be altered by bringing up a magnet, then even if the magnet be so placed that the position of rest of the needle for no current be unchanged, still the force and therefore the current required to turn the needle through a given angle will be altered. For let the controlling force N P be increased to N P′ (Fig. 54) so that the zero position of the needle is the same, but the needle is held in that position with a greater force, then in order

that the angle P′ N R′ may remain of the same value as before, the deflecting force P R must be increased to P′ R′, that is, in the same proportion as the controlling force. If the current has the same value as before, so that P′ R″ is equal to P R, then the angular deflection of the needle instead of being P N R′ will be reduced to P′ N R″. Even if the controlling field be merely that due to the earth, this will alter from place to place, and from year to year; so that a tangent galvanometer requiring a current equal to 1 ampere to produce a deflection of 45 degrees in some particular town, will generally need a somewhat different current to produce the same deflection if moved to another town, and even if kept in the same position the calibration will be found to gradually alter with time.

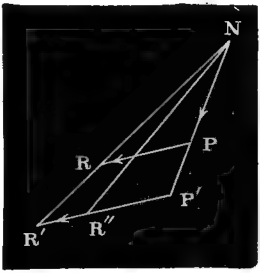


Fig. 54.

When the needle of a tangent galvanometer is supported in such a way that it turns in a horizontal plane, and when the controlling force is *entirely* produced by the "*horizontal component of the earth's magnetic force*,"* the following formula connects the current A in *amperes*, passing through the coil, with the deflection d in degrees, the radius r of the coil in *inches*, and the number of convolutions N of wire on the bobbin

$$A = K \frac{r}{N} \tan. d,$$

where K is a constant which depends on the place where the galvanometer is situated, and on the year. The

* The *horizontal component of the earth's magnetic force* is that component of the earth's force which exerts a directive action on a compass needle, or needle turning in a horizontal plane.

following table gives the values of K, for London, for the middle of the years 1891 to 1900 :—

TABLE III.

Year.	Value of K.	Year.	Value of K.
1891	0·7378	1896	0·7420
1892	0·7386	1897	0·7429
1893	0·7395	1898	0·7437
1894	0·7403	1899	0·7446
1895	0·7412	1900	0·7454

When the controlling force acting on the needle of a tangent galvanometer is due to the presence of a distant magnet, placed so that the needle is parallel to the plane of the coil when no current passes, the preceding formula holds true, but the constant, K, must be determined experimentally.

If the value of K for the earth's field alone be accurately known for the particular place and the particular year, then the value of K for any other controlling field may be ascertained by employing the principle described in § 21, page 79. Remove all magnets, pieces of iron, &c., so that the needle of the tangent galvanometer is acted on by the earth's field alone, and count the number of oscillations, n_1, say, that the needle makes in any convenient time. Replace the controlling magnet, or magnets, as desired, and again count the number of oscillations, n_2 say, that the needle makes in the same time, then the K for the earth's field alone must be multiplied by n_2^2/n_1^2 to obtain the K to be used in the preceding formula for the particular combination of controlling magnets in question.

Example 16.—How many amperes would deflect the needle of a tangent galvanometer 60° in the year 1891, the controlling force being the horizontal component of the earth's magnetism at London, and the galvanometer

having a bobbin 5 inches in radius, wound with six convolutions of wire?

The number of amperes is $\frac{0{\cdot}7378 \times 5 \times \sqrt{3}}{6}$.

Answer.—1·065 ampere.

Example 17.—Through what angle would 0·598 ampere deflect the needle of a tangent galvanometer with a bobbin 7 inches in radius, wound with five convolutions of wire, in the year 1898, the controlling force being the horizontal component of the earth's magnetism at London?

$$0{\cdot}598 = \frac{0{\cdot}7437 \times 7 \times \tan. d}{5}$$

$$\therefore \tan. d = \frac{5 \times 0{\cdot}598}{0{\cdot}7437 \times 7}$$

$$= 0{\cdot}5743$$

$$\therefore d = 29°50'.$$

Answer.—29°50′.

Having found tan. d, d may be found either by looking in a table of tangents or in the following way :—

Take a sheet of squared paper, and on it select two axes, or lines of reference, O X, O Y, at right angles to one another. Choose any number of the divisions on your paper to represent unity, taking care that there are more than 100 of these larger divisions along O X, and at least 58 along O Y. These numbers are chosen because the tangent of the angle required is approximately given by the ratio $\frac{57{\cdot}4}{100}$. Along O X mark off O A, equal to 100 of the divisions, then on the line through A, parallel to O Y, mark off A B as nearly as possible equal to 57·4 of the divisions. Join O B. Then B O A is the angle d.

$$\text{For } \tan. \text{B O A} = \frac{\text{B A}}{\text{O A}}$$

$$= \frac{57{\cdot}4}{100}$$

$$= \tan. d.$$

The angle d may now be found by means of a protractor.

Example 18.—If the horizontal component of the earth's magnetism in 1893 at London be the controlling force in a tangent galvanometer, the bobbin of which is 11 inches in diameter, how many convolutions of wire must be wound on the bobbin in order that a current of 1·017 ampere may give a deflection of 45°?

Answer.—4 convolutions.

Example 19.—If the horizontal component of the earth's magnetism in 1895 at London be the controlling force in a tangent galvanometer, the bobbin of which is wound with eight convolutions of wire, what must be the radius of the bobbin in order that a current of 0·384 ampere may give a deflection of 50°?

Answer.—3·48 inches.

Tan. 50° may be found either in a table of tangents or in the following way:—

Take a sheet of squared paper; on it take axes O X, O Y, at right angles to one another; with a protractor make the angle B O X, equal to 50°, and produce O B as far as the paper will allow. Let A B be the farthest line from O, parallel to O Y, which cuts B O. Then $\tan. 50° = \frac{AB}{OA}$.

Count the number of divisions and fractions of division in A B and O A, and divide the one by the other.

If the angle be large, great care must be taken to lay it down accurately with the protractor, since a small error in a large angle will introduce a large error in the tangent.

Example 20.—About how many times the horizontal component of the earth's magnetism must the controlling force be in a tangent galvanometer, having a bobbin 5 inches in radius wound with six convolutions of wire, in order that a current of 20 amperes may make a deflection of 45°?

Answer.—About 32 times.

Example 21.—The needle of a tangent galvanometer when acted on by the earth's field alone makes one oscillation in 1·3 second, whereas, when the controlling magnet is placed in position, it makes one oscillation in 0·433 second. If the coil be half a foot in radius, and be wound with twenty turns of wire, what current will produce a deflection of 30° in 1898 at London?

Answer.—1·16 ampere.

It is not necessary that the coil of a tangent galvanometer should be circular, but in order to obtain the straightness of the lines of force in the neighbourhood of the axis, as seen in Figs. 34, 38, and 53, and not merely for points actually on the axis, of which we could only avail ourselves by using an infinitely short magnet, the diameter of all parts of the coil must be large. Hence, if an elliptic or other non-circular coil were used, its smallest diameter would have to be large, and consequently its largest diameter unnecessarily so.

28. Magnetometer.—The following is an important example of the use of the tangent law. Let *n s* (Fig. 55) be a small compass needle, suspended so as to be free to turn in a horizontal plane, and first let it be acted on only by a uniform magnetic field, produced by the

Fig. 55.—Principle of the Magnetometer.

earth for example. Along a horizontal line passing through the centre of the needle, and perpendicular to the position taken up by the needle when acted on by the uniform field alone, let a magnet N S be placed, the distance of the nearest end N of this magnet from the needle being considerable compared with the length of the needle itself.

We have then the exact conditions for the tangent law to be true, and, consequently, the tangent of the

angle through which the needle $n s$ turns in taking up the deflected position $n' s'$ is a measure of the magnetic force exerted by the large magnet N S at the spot occupied by the little magnet.

Such an arrangement constitutes what is called a "*magnetometer*," and it may be used to test the strength of different magnets of the same length, put successively so as to occupy the position N S; or, by making a single magnet, N S, occupy different positions along the line A B, and measuring the tangent of the different deflections of the needle $n s$, we can find out the force produced by a given magnet at different positions along its axial line A B.

29. Calibrating any Galvanometer by Direct Comparison with a Tangent Galvanometer.—The necessity that a galvanometer in order to obey the tangent law should have its coil very large, compared with the length of the needle, prevents a tangent galvanometer from being very sensitive. It also renders a tangent galvanometer unportable, for if the needle were made very short instead of the coil being made large, its movement would be seriously impeded by the mass of even a very light pointer attached to it. If the indications of an instrument are to be unaffected by moving it from one place to another, as well as unaffected by the proximity of a mass of iron like a fireplace, iron water-pipes, &c., the controlling magnet must be powerful, must be attached to the galvanometer, and have its poles close to the needle. Hence, a uniform controlling field which, as has been seen, is one of the conditions for the tangent law to be true, cannot be attained in such an instrument.

The law of such a galvanometer must, therefore, be obtained experimentally, and a very convenient way of performing the calibration is to compare the deflections of the instrument under test with those of a tangent galvanometer, when the same currents pass through both apparatus, which may be arranged as in Fig. 56.

G is the standard tangent galvanometer, D the

galvanometer, which, if rough and portable, is sometimes called a "*detector*," requiring to be calibrated. V is a **V**-shaped tube containing two zinc rods dipping into a small quantity of a solution of zinc sulphate, and is used for varying the strength of the currents passing through G and D by altering the distance between the bottoms of the zinc rods. The wires coming from the current-generator are attached to the terminals T T, and a key

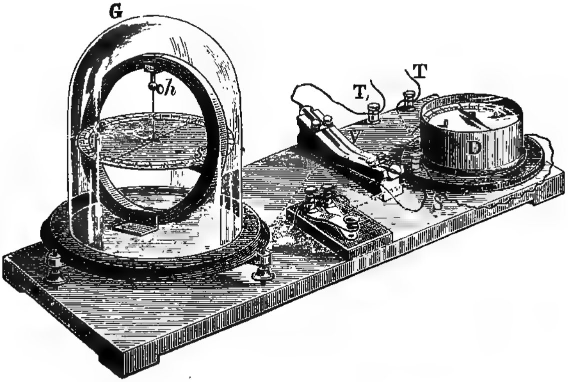


Fig. 56.—Calibrating a Detector by Comparison with a Tangent Galvanometer

placed between G and D enables the current to be made or broken. As the same current passes through G and D, it is quite unnecessary to know the value of the resistance introduced by V; all that has to be done is to observe a number of corresponding deflections of the needles of G and of D, then, since the true value of the current is proportional to the tangent of the deflection in G, a calibration curve can be drawn for D, in which horizontal distances represent the observed angular deflection of the needle of D, and vertical distances the relative strengths of the currents producing these deflections. If the number of amperes producing any particular

deflection in G be also known, then D will be calibrated absolutely.

It frequently happens that, on account of the great increase in sensitiveness produced by putting the wires conveying the current close to the needle, a rough galvanometer with a few turns of wire is more sensitive than a tangent galvanometer with many turns. Under such circumstances it would be difficult to compare them, as a large deflection on D would only correspond with a small one on G, and a smaller deflection on D would not produce a deflection on G large enough to be read at all accurately. This difficulty may, however, be overcome by putting a piece of wire S (Fig. 56), to act as a bye path, a "*shunt*" as it is called, between the terminals of D, which shunt allows a portion of the current to pass through it instead of through D. As, however, for the same shunt the same *fraction* of the total current is, as we shall see later on (§ 96, page 289), always shunted past D, the sensibility alone of D, and not the law connecting current strength with deflection, is altered by using such a shunt. The use of a shunt, therefore, alters the absolute but not the relative calibration of a galvanometer; consequently, if D is absolutely calibrated, the same shunt must always be employed when it is desired to use the absolute calibration curve of that galvanometer.

30. Pivot and Fibre Suspensions.—The galvanometers G and D differ also in another particular, namely, in the way in which the magnetic needle is supported. In D the little magnet has a jewel in its centre, and rests on a sharp pivot, as in an ordinary pocket compass; whereas in G the needle is supported by a fine fibre of *unspun* silk, the upper end of which is fastened in one of the ways illustrated in Fig. 25 (page 49), so that it can be lowered on to the card *s s*, on which the scale is engraved, when the instrument is being carried about, and raised again so as to be in the centre of the coil when the instrument is in use. The fibre suspension introduces far less friction to the motion of the needle than the best

jewel and pivot, and, in addition, costs far less; but with a fibre suspension it is generally necessary that the instrument should have levelling screws, such as are seen attached to G, Fig. 56, and that it should be levelled before being used.

A galvanometer needle should therefore be supported by a pivot when the instrument has to be moved about, and used quickly in different positions. But when the galvanometer is employed in a fixed position, and great accuracy is desired, the needle ought always to be suspended by a fibre of unspun silk. For the exact method of preparing a silk fibre and using it to suspend a needle *see* the Appendix, page 566.

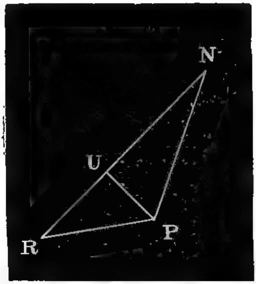


Fig. 57.

31. Sine Law.—Another interesting case of the combination of the controlling and deflecting forces (Fig. 41, § 21, page 81) occurs when the controlling force, N P, is constant in magnitude, but not necessarily in direction; and when the direction of the deflecting force, P R, instead of making a constant angle with the controlling force N P, as in the tangent galvanometer, makes a constant angle with the direction of the deflected body.

As already has been explained, equilibrium of a needle, whose two ends are symmetrical with the coil, and which is controlled by a uniform magnetic field, will occur when the four points R, N, S, R′ (Fig. 42, page 81) are in one straight line. Hence, the condition that the deflecting force P R (Fig. 41, page 81) makes a constant angle with the needle, is equivalent to saying that the angle P R N is a constant.

Now, if a perpendicular, P U, be let fall from P on to N R (Fig. 57), the sine of P N R, that is the sine of the

angle the needle makes with the direction of the controlling force, equals the ratio of PU to NP, while the sine of PRN equals the ratio of PU to PR. Hence

$$\frac{\text{PR}}{\text{PN}} = \frac{\sin.\ \text{PNR}}{\sin.\ \text{PRN}};$$

but by hypothesis, PN is constant in magnitude, so also is sin. PRN, therefore

PR is proportional to sin. PNR.

But since PR, the deflecting force, makes a constant angle with the needle, the needle must make a constant angle with the coil. And this is the condition we saw in § 19, page 70, which causes the deflecting force to be proportional to the current. Hence we know that the deflecting force is proportional to sin. PNR, also that the deflecting force is proportional to the current, therefore the current must be proportional to sin. PNR, that is to the sine of the angle the needle makes with the direction of the controlling force.

Generally, then, we may say that if a body, turning on an axis, be acted on by two forces in the plane in which the body is free to turn, the deflecting force will be proportional to the sine of the angle between the body and the controlling force if :—

(1st) *The controlling force is constant in magnitude, but not necessarily in direction.*

(2nd) *The angle between the direction of the deflecting force and the deflected body is kept constant.*

If then there be a rod NN′ (Fig. 58)—short or long—turning about a pivot O, and acted on by a weight W (and we apply various deflecting forces by placing weights W′ in the scale pan), and, if further, after the application of each weight, W′, we alter something so as to bring the angle between the rod and the direction of the deflecting force—that is, between NN′ and NQ—always to the same value, these deflecting forces will be proportional to the sines of the angles that NN′ makes with NP, the direction of the controlling force W.

There are three distinct ways in which, after the application of different deflecting forces, W′, the angle between N N′, and N Q, the direction of the deflecting force, can be brought always to the same value.

(1st) By altering the direction of the deflecting force without altering its magnitude.

(2nd) By altering the direction of the controlling force without altering its magnitude.

(3rd) By altering the magnitude of the controlling force.

The last method is inadmissible, as constancy in the magnitude of the controlling force is a condition for the sine law being true. But either methods (1) or (2), or a combination of them, may be employed.

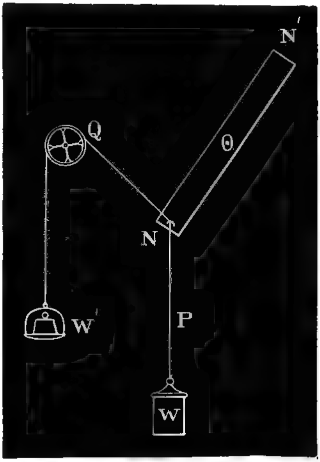


Fig. 58.

The apparatus seen in Fig. 59 is arranged for utilising the first of the methods, by causing the angle between N N′ and the direction of the deflecting force, to have the same value in any one set of experiments. On inserting different weights W′ in the pan, the pivoted rod N N′ will be pulled more or less to one side, and the angle between it and the deflecting force will be altered. But, if for each weight the screw T be turned, and the arms O D and E Q be revolved together round a centre O, until the pivoted rod N N′ has the same position relatively to O D, it will have the same position relatively to N Q, the direction of the deflecting force.

When making a set of measurements, the first thing to do is to remove the silk thread, *k k*, off the pulley *p*, and place the scale pan on the base board of the apparatus; then rest the weight W on a block of wood, or hold it in the hand, so that both the deflecting and

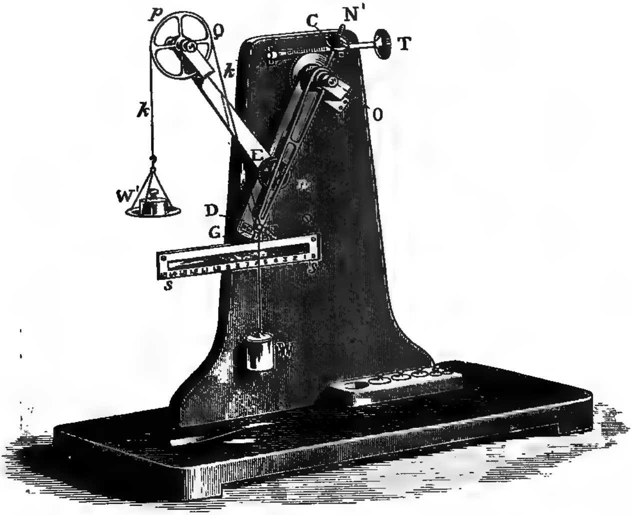


Fig. 59.—Apparatus for Mechanically Testing the Sine Law. Adjustment Made by Altering the Direction of the Deflecting Force.

the controlling forces are removed. The counterpoise weight C is then screwed in or out until the rod N N′ remains balanced in any position.

Next the controlling weight W is allowed to pull the rod N N′ vertical, and the scale *s s* is adjusted, so that the silk thread supporting W, and the reflection of this thread in the piece of looking-glass attached to the scale, are seen coinciding with the zero on the scale.

A weight W′ having been placed in the scale pan, the screw T is turned until the reflection of a projecting point at the lower end of N N′ (seen in a small piece of looking-glass G carried at the end of the arm O D) coincides with a scratch on this glass. This device enables N N′, after the insertion of each weight W′ in the scale pan, to be very accurately caused to have the same position relatively to O D, and therefore relatively to N Q, the direction of the deflecting force.

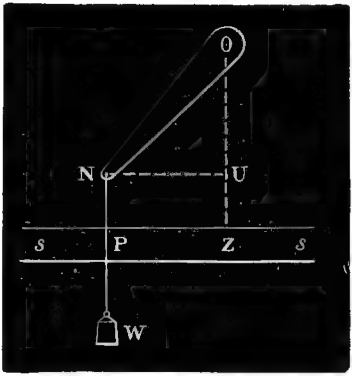


Fig. 60.

The angle which N N′ makes with the direction of the controlling force is N O U (Fig. 60), and its sine the ratio of U N to N O. But the length N O remains constant as N N′ turns about O, therefore the sine is proportional to U N or Z P, that is to the distance the silk thread supporting the controlling weight W has moved along the scale *s s* from its zero Z.

By loosening the nut *n* (Fig. 59), turning the arm E Q relatively to the arm O D, and tightening the nut *n* again, E Q can be fixed so as to make any angle with O D. Further, by sliding the screw and nut *n* along the slot, which is on the end of the arm O D, the end of the arm E Q can be fixed at various distances from O; and experiment shows that in whatever initial position the arm E Q may be fixed relatively to the arm O D, the distances Z P corresponding with a set of deflecting weights W′ are proportional to these weights, provided that after the insertion of each weight in the pan the screw T be

adjusted so as to bring N N′ into the same position relatively to N Q.

But the absolute value of the length Z P corresponding with a particular weight W′ put in the scale pan will vary with the position in which the arm E Q is fixed relatively to the arm O D.

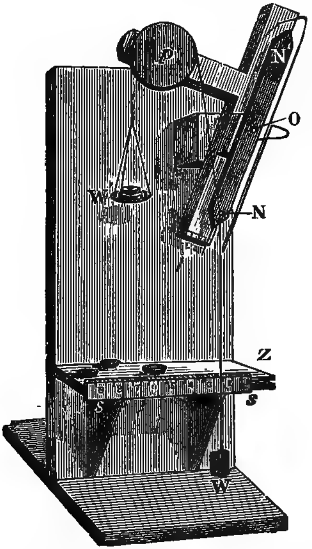


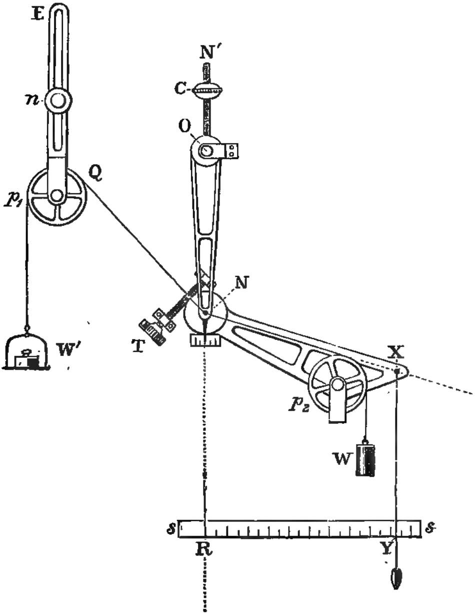
Fig. 61.—Simple Apparatus for Mechanically Testing the Sine Law.

If it be only desired to make a series of measurements with one angle between these two arms the apparatus may take the much simpler form illustrated in Fig. 61, which is also of a simpler and cheaper character in several other particulars.

As already explained (page 113), there is another method of keeping the angle between the deflecting force and the movable rod constant when different weights W′ are inserted in the scale pan, viz., by altering the direction of the controlling force. The apparatus indicated in Fig. 62 enables tests to be made of this method by using the sine principle. The pulley p_1 is fixed in one definite position relatively to the stand for one set of measurements, and after the insertion of each weight W′ in the scale pan the rod N O is made to take up a vertical position, and therefore to take up a fixed position relatively to N Q the direction of the deflecting force. This is effected, not by altering the position of the pulley p_1, but by turning the screw T,

which causes the pulley p_2 to alter the direction of the controlling force.

The scale *s s* has been previously adjusted, so that when the rod N O is acted on by W alone, and when the



ig. 62.—Apparatus for Mechanically Testing the Sine Law. Adjustment Made by Altering the Direction of the Controlling Force.

pulley p_2 has been turned on one side so that the force W exerts on N O is in a vertical direction, the silk thread supporting W and its reflection in the mirror, carried by the scale, are seen coinciding with the zero line of the scale. R Y will then be proportional to the sine of the angle N O makes with the controlling force,

and therefore will be proportional to the deflecting weight W′.

On loosening the nut n, the slotted arm E (Fig. 62) can be turned, also raised or lowered, so that the pulley p_1 can be fixed in a variety of positions relatively to the vertical rod N O, and the angle between N O and N Q can be made to have a number of fixed values. Now experiment shows that, whatever may be the value of this angle, provided that it is maintained at one fixed value during a set of experiments, the weights W′ are directly proportional to the lengths R Y—that is, are proportional to the sines of the angle which the rod N O makes with N X, the direction of the controlling force.

32. Employment of the Sine Principle in Galvanometers.—In *any* galvanometer in which the controlling force is produced by the earth's magnetism, or by any *distant* fixed magnet, this force will be constant in magnitude and direction, and independent of the needle changing its position; also the deflecting force produced by the current passing round the bobbin can be made to have an invariable direction relatively to the needle if the bobbin, or the framework of the instrument to which the bobbin is attached, be turned round after the deflected needle, until the needle and the bobbin occupy the same position relatively to one another for each value of the current passing through the galvanometer. That it is possible to attain this result arises from the fact, that, although the needle turns when the bobbin is turned, they do not move at the same rate. If the adjustment just described be always made, the sine of the angle through which the needle has been deflected from the position of rest which it occupied when no current was passing round the bobbin, will be the angle between the needle and the controlling force, and will, therefore, be directly proportional to the current strength.

Now, if the coil be placed so as to have *any* fixed position relatively to the needle, both when no current passes through the coil and when a given current passes

through it, then the angle through which the coil has to be turned from the first position to the second is the same as the angle which the needle makes with the controlling force; hence, in the so-called sine galvanometers, there is, in addition to the scale moving with the bobbin, an independent *fixed* scale, to show through what angle the coil has been turned. This, however, is not necessary, since, if, after the coil has been turned until it has the fixed position relatively to the needle, the current be interrupted without the position of the instrument being disturbed, then the needle will swing back, and, after a few oscillations, will take up its original undeflected position, the angle between which and its deflected position will be the angle of which the sine has to be taken.

For example, let the scale of a galvanometer be divided into degrees, and let it be fixed to a galvanometer coil, but the position of the coil relatively to the scale be not known; that is to say, the angle between the plane of the coil and the line passing through the zero and the centre of the scale may have any value provided that their value is fixed. To compare then the relative strengths of currents by using the sine principle we proceed as follows:—

Send one of the currents C_1 through the coil and turn the galvanometer round until the needle points to some definite point on the scale, the zero point for example. Stop the current and wait until the needle comes to rest at say α_1° degrees from the point on the scale which has been chosen. Perform the same operation with the currents C_2 and C_3, obtaining angles α_2° and α_3° respectively, then

$$C_1 : C_2 : C_3 : \text{sin. } \alpha_1^\circ : \text{sin. } \alpha_2^\circ : \text{sin. } \alpha_3^\circ.$$

As a current passing through a coil has usually the greatest effect on a magnetic needle suspended inside it when the axis of the needle is perpendicular to the axis of the coil, this is the fixed position of the coil relatively

to the needle usually adopted, and the one in which the pointer stands at 0° on the movable scale. But this particular position is not at all necessary for the fulfilment of the sine law, and therefore special precautions need not be adopted, as in the case of the tangent galvanometer (*see ante*, page 85), to ensure that the axes of the needle and of the coil are at right angles when the pointer stands at zero on the scale.

If the needle of a galvanometer be supported on a

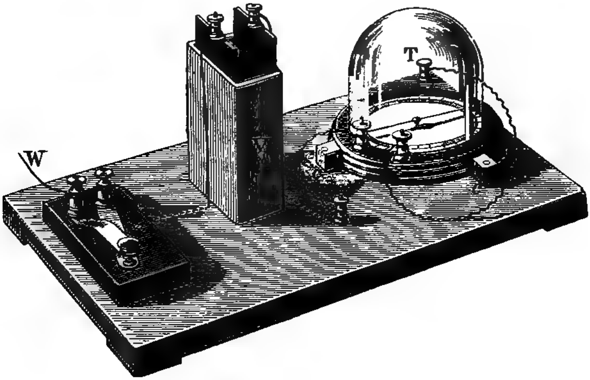


Fig. 63.—Calibrating a Galvanometer by the Sine Method.

pivot so that a slight motion of the galvanometer sideways does not shake the needle out of position, the first method (page 113) is the easier one to employ for causing the needle to take up the same position relatively to the deflecting force caused by different currents passing round the coil.

Fig. 63 shows an arrangement for calibrating a pivoted needle galvanometer in this way. Three little blocks of wood, of which two only, *c*, *c*, can be seen in the figure, are temporarily fixed so as to allow the galvanometer to be turned round without shifting its

position, a precaution of practically no consequence if the needle be pivoted and the controlling force be due to the earth's magnetism alone, but desirable if the whole or part of the controlling force is produced by a not very distant magnet. Of course the magnet must be so far away that neither the magnitude nor direction of its attraction on the suspended needle is altered by the turning of the needle; but this need not be very far, unless the needle employed is long. V is a vessel containing two zinc plates for adjusting the strength of the current by placing the plates at different distances apart, or, instead, a **V**-shaped tube containing zinc rods as seen in Fig. 56, page 109, can be more conveniently employed for this purpose. W is one of the wires leading to the current generator, and T is the terminal to which the other wire is attached.

To perform the calibration, the galvanometer is placed as it is intended to be subsequently used, and a current sent through it producing a deflection, d_1, say; then the galvanometer is turned round until the pointer points to some fixed mark on the scale, the current stopped and the angle a_1 observed through which the pointer moves from this mark. Next the galvanometer is replaced in its normal position, a second current sent through it, producing a deflection d_2, say; then the galvanometer is turned round as before until the pointer points to the same fixed mark, the current stopped, and the angle a_2 observed through which the pointer moves from the mark. This process having been carried out with various currents, it follows from what precedes that the currents which produce the deflections d_1, d_2, d_3, &c., when the galvanometer is kept fixed in its normal position are proportional to sin. a_1, sin. a_2, sin. a_3, &c.

In certain cases, even although the needle be suspended by means of a silk fibre, it may be possible to pivot the galvanometer, so that it may be easily turned, in a horizontal plane, about the axis of suspension of the needle without setting the needle swinging. Under

these circumstances the method of turning the coil so as to follow the needle may be conveniently employed. In Fig. 64 is seen the section of a galvanometer showing one method of pivoting.

The galvanometer is of the type described in § 33, page 125, and illustrated in Fig. 65, the section, seen in

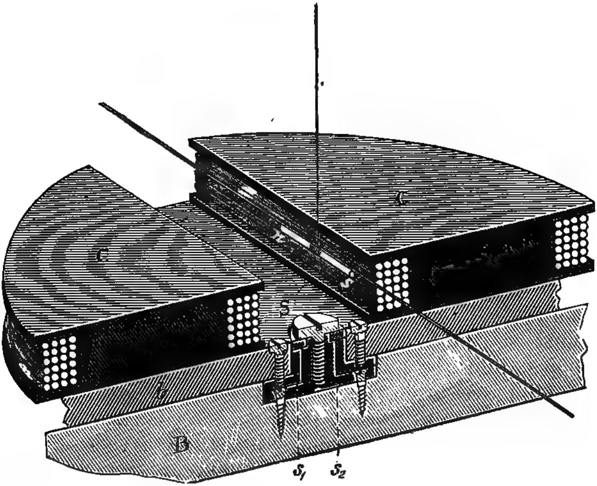


Fig. 64.—Section of Galvanometer with Silk Fibre Suspension Pivoted for Turning round its Centre.

Fig. 64, passing through the centre of the instrument transverse to the coils C, C, which are fixed to the base *b*. A screw, S, fastens the base *b* of the galvanometer to the base-board B, but leaves it free to be turned, the bearing surfaces being those of the brass pieces s_1 and s_2, attached respectively to *b* and B. The top of s_1 is filed flat, and a washer inserted under the head of the screw S prevents this screw from being tightened or loosened when the galvanometer is turned about it.

If the needle be suspended by means of a silk fibre and hang very close to the wires of which the coil is made, which is always the case with a very sensitive galvanometer, the instrument must be carefully levelled with levelling screws until the needle hangs free of the coil. In such a case it would be quite impossible to turn the instrument round unless its construction were specially designed for that purpose. If then the sine principle be applied to an ordinary sensitive galvanometer the second method (page 113) must be employed.

The controlling magnet must be put sufficiently far away from the galvanometer to produce a field that is uniform throughout the small space in which the needle moves, and must be sufficiently powerful that the entire control of the needle is practically produced by this magnet alone, that is to say, if the controlling magnet be placed at the distance l, say at which it is to be used, and with its axis in *any* radial line passing through the centre of the suspended needle, the needle must place itself in this radial line when no current is passing through the galvanometer.

The two conditions referred to at the beginning of the last paragraph can be most easily fulfilled if before commencing the experiment the magnetic field due to the earth—iron gas-pipes, &c.—be neutralised by a magnet suitably placed, and left untouched during the calibration.

A circle of radius l, and with its centre in the vertical line about which the needle turns, having then been drawn on the table, the controlling magnet is placed on the table with its centre at different points on the circumference of this circle, and with its axis in a radial line passing through the vertical line about which the galvanometer needle turns; then the currents C_1, C_2, C_3, &c., which are required respectively to bring the needle to some fixed position, are respectively proportional to the sines of the angles a_1, a_2, a_3, &c., which the galvanometer

needle makes with the axis of the controlling magnet in its different positions.

This novel method of applying the sine principle to the calibration of a sensitive galvanometer, the construction of which did not allow of the galvanometer being moved while in use, has been employed with success in the laboratories of the Central Technical College.

To calibrate a galvanometer by the employment of the sine principle requires the current in each case to remain constant long enough for the instrument to be turned round after the needle, or for the direction of the controlling force to be altered, until the needle and the deflecting force are brought to a fixed position relatively to one another. But when once the calibration curve has been drawn, a galvanometer so calibrated can, of course, be used to measure currents as transient as a galvanometer calibrated in any other way.

33. Construction of Galvanometers in which the Angular Deflection is directly Proportional to the Current.—We have already seen (page 84) that the current is proportional to the tangent of the deflection of the galvanometer needle, when neither the magnitude nor direction of the controlling force is altered as the needle moves into a new position on being deflected, and when, in addition, the direction of the controlling force is at right angles to the direction of the force with which the current passing round the coil acts on the needle.

In order, therefore, that the angular deflection may be directly proportional to the current, we must either cause the needle on being deflected to move into a position in which the current passing round the coil acts more powerfully on it, or into a position in which the controlling force becomes weaker; or we may arrange that both these results may be produced.

The first condition may be obtained in a rough way by employing the very defect of construction previously

referred to in the adjustment of the tangent galvanometer, which made the deflection on one side of the zero larger than that produced by the same current on the other—viz., not putting the coil so that its plane was parallel to the suspended magnet when no current was passing through the coil. The needle, when deflected to that side on which the greater deflection is obtained, will, instead of moving from a stronger to a weaker part of the magnetic field produced by the current, move at first into a stronger part, and then afterwards into a slightly weaker part. The effect of this arrangement is to make the proportional law connecting current and deflection true for a much larger deflection from the undeflected position of the needle than if we commenced with the needle parallel to the plane of the coil for no current. But this arrangement has the disadvantage that it can only be used for currents deflecting the needle to one side of the scale, for, if the current be flowing in the opposite direction, the defect of want of proportionality between current-strength and deflection will be increased.

This plan, by means of which the proportionality on one side of the scale is sacrificed to increase that on the other, has been employed by the author, and later on by MM. Carpentier and Deprez, and others, for making proportional galvanometers.

Another device for causing the strength of the deflecting field to increase as the needle deflects is employed in the galvanometer originally devised by Mr. Mather and Professor Walmsley, and in use for many of the experiments of the first-year students at the Central Technical College. This instrument, as illustrated in Fig. 65, consists of two coils shaped as shown, and fixed so that they are separated by a distance a little less than the length of the needle. The galvanometer is placed so that when no current is passing through the coils the needle hangs symmetrically between them; and when the controlling field is a uniform one, the current

is directly proportional to the angular deflection up to 45° or 50°. (*See* page 147 for plan used when coil moves.)

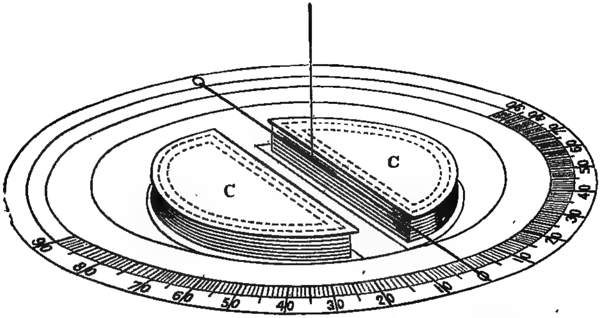


Fig. 65.—Mather and Walmsley's Proportional Galvanometer.

Even although the controlling magnet of a galvanometer be rather near the needle, the controlling field may be regarded as an approximately uniform one if the deflections of the needle be all *very small.* Similarly for *very small* deflections the controlling field may be regarded as approximately uniform whatever be the shape and size of the coil or of the needle. If then, in addition, the controlling magnet be so placed that when no current is passing the needle makes about the same small angle with the plane of the coil on one side of it that it makes with that plane on the other side for the greatest deflection employed, the distribution of the forces will

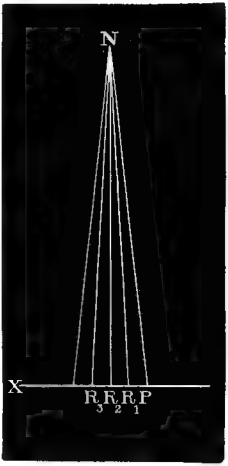


Fig. 66.

be as in Fig. 66, where N P represents the magnitude and direction of the controlling force, PR_1 the magnitude and direction of the deflecting force for current 1, PR_2 for current 2, PR_3 for current 3, &c., PR_2 being twice PR_1, PR_3 three times PR_1, &c.

Therefore the angular deflections of the needle for currents 1, 2, 3, &c., are PNR_1, PNR_2, PNR_3, &c., and, as these angles are all very small, and the base lines are proportional to the currents, it follows that the angular deflections are also proportional to the currents. Indeed for *very small* deflections this result will be nearly true, whether the angle N P X is a little less than, or a little more than, or exactly equal to a right angle; that is, whatever be the angle the needle makes with the plane of the coil provided that this angle is small.

34. Galvanometers of Invariable Sensibility.—Now that measuring currents in amperes has acquired the same sort of practical importance as weighing coals in tons or finding the number of cubic feet of gas passing through a pipe, it is necessary to have galvanometers which are portable, and whose indications are not affected by moving the galvanometer from one place to another, or by placing it near an iron pipe, a fire-place, or even near the powerful electromagnets of the *dynamo machines* which are employed for the mechanical production of electric currents.

An instrument of this type must be "*direct-reading*"; that is, the deflection of the pointer must indicate at once the current in amperes, for in commercial work there is no time to refer to a table of values, not to mention the risk that would be introduced by a table of values belonging to some other instrument being used by mistake.

Such instruments, by means of which the current can be read off at once in amperes without any calculation or reference to any calibration curve, are called "*ammeters*," and during the last twelve years so much

attention has been given to the design and construction of this class of electrical meter that it is now possible to measure a current with as much accuracy as a leg of mutton can be weighed in a pair of scales, or with a spring-balance, and with even greater facility.

The controlling force must necessarily be exerted in such a way that it is the same wherever the ammeter is placed; indeed, many ammeters are so constructed that the controlling force is not changed by laying the instrument on its side, or in any other position, so that a current can be read off equally well whether the ammeter is lying on a table, hung up on a wall, held in the hand, or used on board a ship rolling in a heavy sea.

There are three distinct ways in which the controlling force is exerted in ammeters.

(1) By means of a powerful permanent magnet placed inside the instrument and rigidly fixed to it.

(2) By means of a spring.

(3) By means of a weight.

The first two methods have the advantage that with their use the moving part of the ammeter can be balanced like a wheel in a watch, so that the instrument can be made to read correctly in any position; the former of these two has also the further advantage that as the control exerted by a powerful magnet close to the needle is very large, outside magnetic disturbances have little effect. But while a magnet or a spring can be made constant enough in their action for many practical purposes, their variation with time is of course greater than that of a weight, hence the third method of control is the one adopted when accuracy is of more importance than portability.

In the earlier editions of this book several ammeters were described, and their advantages and disadvantages compared. But the methods of constructing the coils and needles, and the various devices that are now adopted in applying the controlling force in one or other of the

three ways just referred to have become so numerous, that anything like a complete description of all the types of ammeters now in use and an examination of their relative advantages would alone fill a good-sized book.

Ammeters, besides differing in the methods used for exerting the controlling force, also differ in design, depending on whether the instrument is intended to measure currents of very different values, or only currents all of about the same value. In the former case the design should be such that the scale is equally or nearly equally divided, so that there is about the same distance between any adjacent pair of division marks, while in the latter the scale should be very "*open*"; that is, the division marks should be widely separated at the one part of the scale which is in constant use, and crowded together at those parts which correspond with currents which rarely have to be measured. Instruments with this latter type of scale are especially employed when an ammeter is used to indirectly measure "*electric pressure*" (*see* § 49, page 179).

In § 2 we saw that when a conductor conveying a current is placed near a magnet there is a force exerted between the conductor and the magnet, tending to make them move relatively to one another.

The force acts in such a direction that a wire carrying a current tends to move perpendicular to itself and perpendicular to the lines of force. It is only when the wire lies along the lines of force that the action between it and the magnet is nought, however strong be the current and however powerful the magnetic field. With any other position of the wire relatively to the direction of the magnetic field there is some force, and this force has its greatest value for a given length of conductor carrying a given current, and placed in a field of given strength, when the conductor is perpendicular to the lines of force.

By employing a very powerful magnet the force exerted on a wire, even when conveying a feeble current

can be made considerable, and this action has been employed by Maxwell, Lord Kelvin, Deprez, d'Arsonval, and others, to obtain galvanometers which are not only very sensitive, but the indications of which are very little affected by extraneous magnetic disturbance.

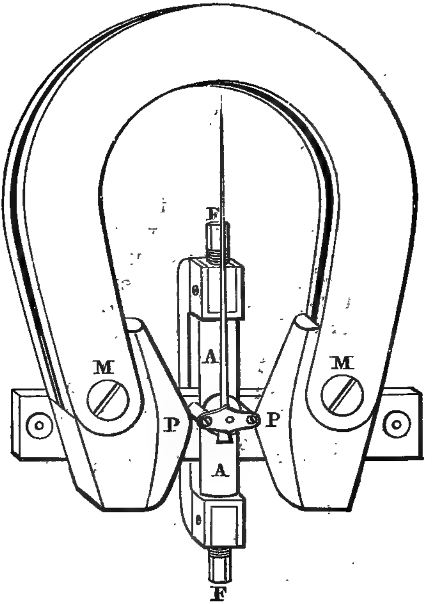


Fig. 67.—Framework of Ayrton and Perry's Permanent Magnet Ammeter.

EARLY FORMS OF AMMETERS.

35. Permanent Magnet Ammeter.—The earliest ammeter, having an equally divided scale so that the deflection in degrees was directly proportional to the current, was the "*permanent magnet ammeter*" devised by Professor Perry and the author in 1880. The coil

was wound on the two halves of a flat brass tube A A (Fig. 67), shown unwound in the figure, and inside this tube, at its centre, there was pivoted a small *soft-iron* needle, shaped like a flat ellipsoid, *n n* (Fig. 68) and controlled by a powerful permanent magnet, M M (Fig. 67). The weight of the needle and of the pointer *p* was accurately counterbalanced by a small weight, *w*, hence no controlling force was introduced by gravity, and the instrument could be used equally well in any position.

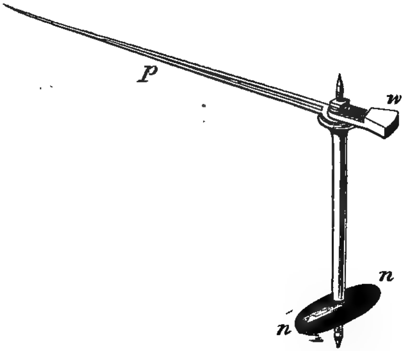


Fig. 68.—Needle, Staff, and Pointer of Ayrton and Perry's Permanent Magnet Ammeter.

On a current flowing round the coil there was exerted a greater or less force tending to place the axis of the needle along the axis of the brass tube A A (Fig. 67), while the controlling magnet M M exerted a force tending to place the axis of the needle along the line joining the tip of the soft-iron pole pieces P, P; the needle therefore placed itself in the direction of the resultant magnetic field.

In addition to giving the pole pieces the shape seen in the figure, the wire was heaped up somewhat near the ends of the coil, therefore not merely did the controlling force diminish as the needle deflected, but the deflecting force, for a given current, also increased. Thus, as explained in § 33, page 124, there were two causes tending to make the angular deflection vary in direct proportion to the current flowing, and, when sufficient care was exercised in

winding the coil, a straight line calibration could be obtained.

When, however, these instruments began to be manufactured in large quantities, the labour of modifying the winding of the coil by trial until direct proportionality

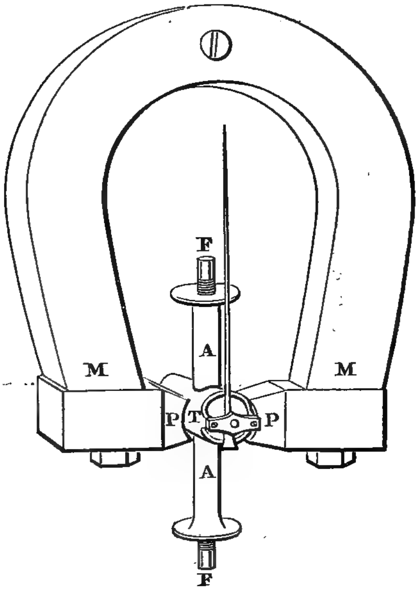


Fig. 69.—Ayrton and Perry's Permanent Magnet Ammeter.

was obtained became too great, and, instead of depending only on the shape of the pole pieces, of the needle, and of the coil for obtaining a straight line law, two soft-iron cores, F, F, screwing into the ends of the brass tube were added, and by screwing these cores in more or less the rate at which the deflecting force (for a given current) varied with the position of the needle could be altered.

Later on a third plan was employed, the soft-iron pole pieces P, P, (Fig. 69) were themselves made adjustable, and to prevent the controlling force produced by the pole pieces when withdrawn falling off too rapidly as the needle deflected, the ends of the poles were made concave instead of convex as before. These movable poles introduced the power of making another adjustment in addition to that effected by screwing the soft-iron cores F, F, in or out of the coil, and by means of these two

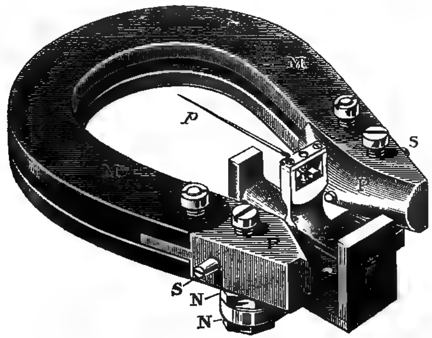


Fig. 70.—Ayrton and Perry's Permanent Magnet Ammeter. Latest Form.

adjustments not merely could the angular deflection be made nearly proportional to the current, but the deflection for the same current could be increased or diminished. Thus the sensibility of the instrument could be adapted to suit the engraved *direct-reading* scale instead of each scale having to be engraved somewhat differently to suit the sensibility of the instrument.

A fourth and still later method of effecting these two adjustments was carried out by Mr. Esson, illustrated in Fig. 70. This consisted in obtaining the straight line law by screwing in or out the soft-iron screws S, S, the ends of which formed the magnetic tips

of the pole pieces, while the right sensibility was arrived at by turning the brass nuts N, N, and so lifting the coil with the needle and pointer bodily more or less out of the magnetic field created by the permanent controlling magnet.

The Ayrton and Perry permanent magnet ammeter had an important advantage over the various types of soft-iron needle ammeters that are at present constructed, in that the deflection of the pointer indicated not merely the strength of the current but also its direction; for in certain cases, such as the charging of "*secondary*" cells, the supply of current to "*arc lamps*," a knowledge of the direction of the current is as important as the measurement of its strength.

To cause the deflection of the pointer to be exactly the same on the two sides of the zero when a current was reversed, an adjustment was necessary, but this was easily effected in the Esson method of construction by turning the coil about the brass screw which held it to the bar at the back of the magnet, this screw being placed so that a line drawn through it passed through the centre of the needles.

By employing a very short needle, and a very light pointer made of thin aluminium, corrugated to give it mechanical strength, the combination seen in Fig. 68 had only a small *moment of inertia*. This, combined with the very strong permanent magnet to produce the control, rendered the instrument very "*dead beat.*" Therefore, instead of the needle being set swinging, and only coming to rest after some time, when a change suddenly occurred in the current, the needle moved sharply into its new position, and all such changes even if quickly produced were accurately indicated.

The "*dead beat*" character of the *permanent magnet ammeter*, its extreme freedom from extraneous magnetic disturbance, its power to indicate the direction of the current as well as its strength, and the fact that this form of ammeter could be used in a horizontal or

vertical position, or even on board a rolling ship or on a rapidly-going train, led to many thousands of them being employed, in spite of the fact that their sensibility

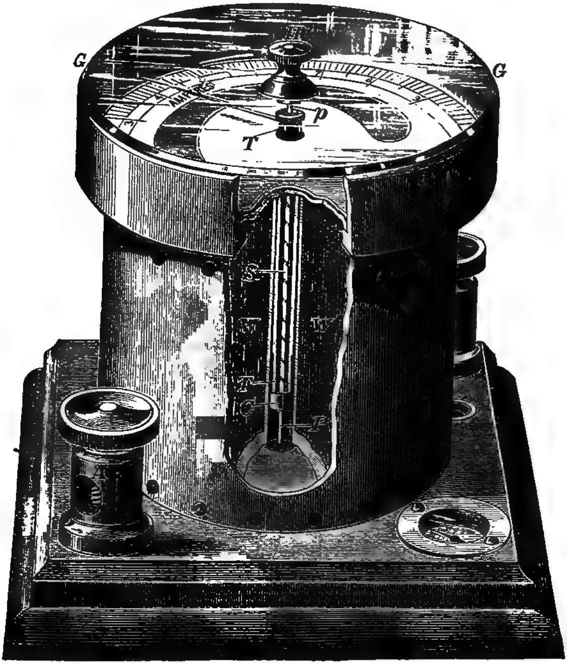


Fig. 71.—Ayrton and Perry's Magnifying Spring Ammeter.

gradually became greater as the permanent magnet grew weaker.

36. Magnifying Spring Ammeter.—Another form of ammeter devised by Professor Perry and the author in 1883 is seen in Fig. 71, and is called a "*magnifying spring ammeter*," because its action depends on the use of

this particular form of spring described in § 16, page 65. T T is a *thin* tube of charcoal iron, attached at its lower end to a brass cap C, terminated in a brass pin P, guided at the bottom in the way shown. To C is attached the lower end of the *magnifying spring* S (made of hard phosphor bronze), the upper end of which is attached rigidly to a brass pin *p*, passing through a hole in the glass top of the apparatus G G, and fastened by means of a screw and nut to the brass milled head H outside the glass top. This pin *p*, to which the upper end of the spring is attached, also serves as a guide for the top of the iron tube.

In the space W W there is wound a coil of insulated copper wire or a coil of copper strip, seen in section in the figure, the copper strip being insulated by a strip of paraffined silk being wound between the layers of copper. The ends of the coil are attached to the terminals. Now when a current is passed through this coil the iron tube is sucked down into it, and its lower end C, to which the spring is attached, receives a large rotatory motion, which is communicated directly to the pointer rigidly attached to the top of the iron tube. Parallax, in taking readings of the pointer, is avoided by the horizontal scale having a piece of looking-glass let in it in the well-known way.

The equality of the divisions throughout the whole scale depends on the fulfilment of three conditions :—

1st. Placing the iron tube just so far inside the coil that the downward pull for a given current varies but little with a *small* upward or downward position of the tube.

2nd. Employing the *magnifying spring* so that a large motion of the pointer is obtained with a very small downward motion of the iron tube.

3rd. Making the iron tube extremely thin so that it is "*magnetically saturated*" with a current passing round the coil smaller than the instrument is intended to measure. Hence, for all currents to be measured the

tube behaves as if it were a permanent magnet of constant strength.

Practically, then, the tube behaves as a permanent magnet in a fixed position. Hence, as explained in § 19, page 70, the force exerted is directly proportional to the current, and, since the rotation of the free end of a *magnifying spring* is directly proportional to the stretching force, it follows that the angular deflection of the pointer is directly proportional to the current. Practically, this is found to be true through a range of about 270°, excluding the first 15° corresponding with currents too small to saturate the iron, and therefore for the first 15° from zero the scale is not graduated.

Experiment shows that the iron tube should be placed with about two-thirds of its length inside the coil, for if an iron core be lowered into a coil round which a constant current is flowing the pull for a core and coil of the same length is found to increase until about two-thirds of the core is inside the coil, then it remains practically constant as the core is lowered through a certain distance, next the pull falls off with more or less rapidity, and finally becomes nought when the centre of the core coincides with the centre of the coil.

This instrument being direct-reading, has to be provided with an adjustment for sensibility, and this is obtained partly by the amount of wire or strip that is wound on the bobbin, and partly by means of a small movable bobbin, wound with a coil of fine wire of the same length as that employed in winding the main coil, joined up in parallel with the main coil. This movable coil slides up and down on the main bobbin, and by trial a position is found for it such that the readings on the dial are correct, and in that position this auxiliary coil is permanently fixed by the maker of the instrument.

The pointer will deflect in the same direction, no matter which way the current passes through the instrument, and owing to the softness of the iron used in

making the tube T T, and the smallness of its mass, there is but very little *residual magnetism* left in it; hence, the pointer indicates the *correct* strength of the current, no matter which way it passes through the instrument. To ascertain the direction of the current, a small compass needle is let into the base of the instrument, as seen in the front right-hand corner of Fig. 71, and, to avoid a reversal of the magnetism of this compass needle should a large current be suddenly sent through the ammeter, the little box containing the needle is made of fairly thick iron.

It might at first sight be thought that the indications of the *magnifying spring ammeter* would be easily affected by the presence of any neighbouring magnet. This, however, is not the case, and arises from the fact that the main motion of the iron tube is a bodily downward one, a motion of *translation* and not a motion of *rotation*. Now a motion of translation cannot be produced by a distant magnet, no matter how powerful it may be, whereas even a distant weak magnet can produce a rotatory motion and deflect the needle of an ordinary galvanometer. This experiment can be easily tried by floating a compass needle on a piece of cork in a basin of water and holding a magnet a foot or two away. The needle will turn so as to place itself along the lines of force, but if the magnet be far enough away the field is a uniform one over a small space round the needle (see § 19, page 70). The needle will not move bodily so as to approach the magnet no matter how strong the latter may be.

Hence, the magnetism of even a powerful dynamo machine two or three feet away from a magnifying spring ammeter will not affect its indications.

SOME MODERN FORMS OF AMMETERS.

37. Gravity Control Ammeters. — Unless it is required to use an ammeter in several different positions, the control at the present day is usually produced by a

weight, and, in order that the same type of instrument may be easily modified during construction, so that the scale may either be approximately equally divided, or, instead, made very open at some particular part, the

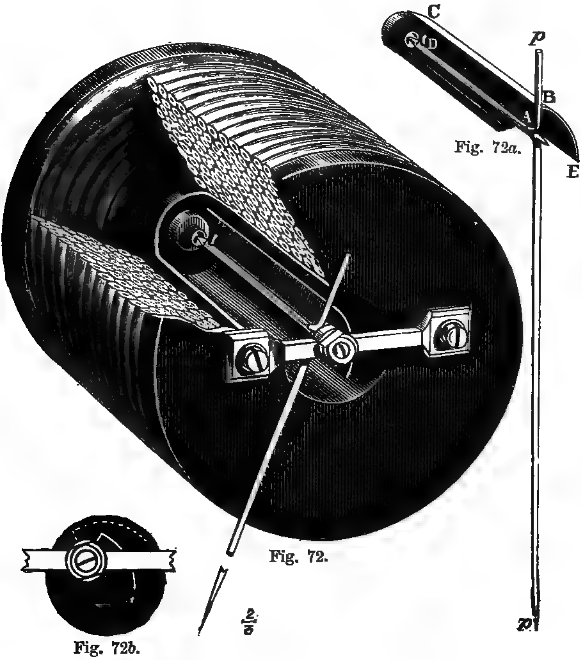


Fig. 72a.

Fig. 72.

Fig. 72b.

Figs. 72, 72a, and 72b.—Schuckert Gravity Control Ammeter, two-thirds of full size.

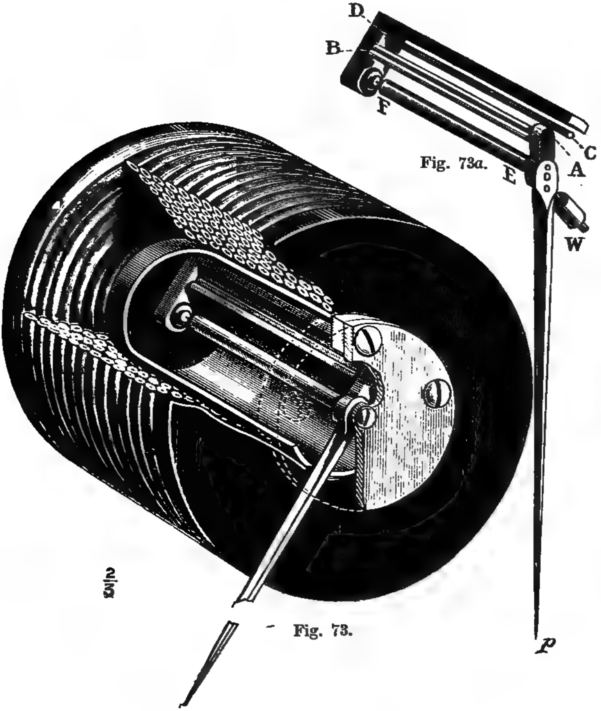
deflecting force is often produced by the attraction or repulsion of two or more pieces of soft iron, one being fixed and the other movable, and both magnetised by the current flowing through a coil surrounding the entire soft-iron system.

Figs. 72, 73, and 74 represent the working parts of three such "*gravity control ammeters*," each shown two-thirds full size; the coil of wire and brass tube inside the coil being partially cut away so that the needle may be clearly seen. Figs. 72*a*, 73*a*, and 74*a* show the needles, also two-thirds full size, withdrawn from the brass tube. In Figs. 72, 73, 74, and 74*a* the needles are seen in the positions they occupy when no current passes round the coil, while in 72*a* and 73*a* they are shown as if deflected by a current passing round the coil strong enough to turn the pointer into a vertical position.

The first of this group of figures, Fig. 72, illustrates the type of *gravity control ammeter* constructed by Messrs. Schuckert at Nürnberg, and much employed on the Continent. The needle is made out of a sheet of thin soft iron bent so that it has one plane face A B C D (Fig. 72*a*), passing through the axis of rotation A D, and a curved face, B E. The weight of the pointer *p p*, which is attached to the needle along the edge A B of its plane face, A B C D, is adjusted so that when no current is passed round the coil the needle and pointer hang as is shown in Fig. 72, and also by the continuous white line in Fig. 72*b*. If the axis of rotation A D were placed along the axis of the cylindrical coil, no deflection of the needle would be produced by any current passing round the coil, therefore the needle is pivoted *eccentrically*. And, since the magnetic field produced by a given current flowing round the coil increases in strength as we proceed from the axis of the coil outwards, the curved face of the needle is attracted into the position shown in Fig. 72*a* when the current is about half as strong as the ammeter is intended to measure, and into the position indicated by the dotted white line in Fig. 72*b* for a much larger current.

The controlling couple is produced by the weight of the needle and pointer, the weight of the pointer assisting

the deflecting force for currents smaller than that which brings the pointer to the vertical position, seen in Fig. 72*a*, and resisting the deflecting force for larger currents.



Figs. 73 and 73*a*.—Nalder Gravity Control Ammeter, two-thirds of full size.

Fig. 73 represents the gravity control ammeter constructed by Messrs. Nalder, and much used in Great Britain. In this instrument the needle consists of a bundle of three soft-iron wires, AB (Fig. 73*a*), fixed to

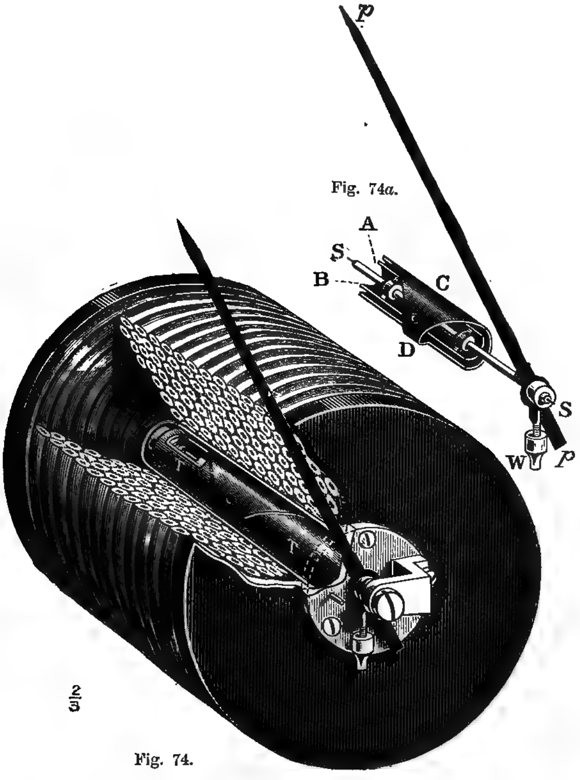
one end of the pointer A p which turns about the axis E F. When no current passes round the coil the counterpoise weight W hangs vertically and brings the bundle of three soft-iron wires, A B, close to a fixed soft-iron wire, C D, which is fastened to the framework carrying the back jewel in which the staff E F of the needle is pivoted at F. All these iron wires are magnetised in the same way when a current passes round the coil, with a north-seeking magnetic pole, say, at A and C, and a south-seeking pole at B and D. Consequently the bundle of iron wires A B is repelled from the stationary iron wire C D, with a force which is the greater the larger the current; the needle A B and the pointer A p are, therefore, caused to turn and lift the counterpoise weight W through an angle which measures the current flowing.

For a given needle the angle through which the pointer is turned by a given current, say 100 amperes, may be made as large as we like, either by winding the coil with a greater number of turns of finer wire, or by screwing the counterpoise weight along the screw pin which carries it, so that the weight is brought nearer the axis of rotation.

In Figs. 74 and 74a is seen the interior of the *latest* form of the *Evershed instrument.* Along the axis of a brass tube, T T (Fig. 74), there is pivoted a staff, S S (Fig. 74a), to which is attached concentrically the needle A B, made of soft iron in the shape of a half cylinder, and outside this brass tube T T there is slipped a little collar, C D (Fig. 74a), made of soft iron and shaped as shown. Fig. 74, therefore, shows the working parts as they would appear if the coil of wire were partially cut away, while Fig. 74a indicates the moving parts together with the iron collar C D, as if left in the position in which it is actually held by the brass tube T T after this brass tube has been removed.

When a current flows round the coil the curved iron needle A B is magnetically pulled round, against the

action of the controlling weight W, so as to fill up the space, more or less, where the iron has been cut away



Figs. 74 and 74a.—Evershed Gravity Control Ammeter, two-thirds of full size. Latest form.

from the collar C D, and the shape of the calibration curve of the instrument depends on the exact way

in which the iron has been cut away from the collar C D, and on the position of the collar relatively to the needle when the pointer is at zero. By turning this collar more or less round the brass tube T T (Fig. 74), on which it is somewhat tightly fitted, the shape of the calibration curve can be entirely altered. Hence, with exactly the same construction of instrument equal additions of current may be made to produce nearly equal increments in the angular deflection—that is, the scale may be a *uniform* one, or else the pointer, when at one definite part of the scale, may be caused to move through a considerable angle for a small change in the current compared with the change that is necessary to produce the same angular variation in the deflection at other parts of the scale. That is, the scale may be made very open at one particular part corresponding with that value of the current for which it is desired to measure a change with special accuracy.

With the shape of the collar which is shown in the figure, and with the position in which it is shown the scale is very open at about two-thirds of its length reckoning from the zero, and the particular current that produces this deflection can be varied by altering the number of convolutions of wire on the coil or by screwing the counterpoise weight W (Fig. 74*a*) along the screw pin that carries it, so as to fix this weight nearer to, or farther from, the axis of rotation S S.

The simplicity of construction of this new form of Evershed instrument, combined with the ease with which it can be made to have either a uniform or an open scale, renders it even superior to his older form, of which many thousands are in daily use.

38. **Moving Coil Ammeters.**—A very convenient, portable and accurate moving coil ammeter has been perfected by Mr. Weston, of Newark, America, and is seen in Figs. 75 and 75*a*, about one-third full size,

partially taken to pieces. The particular specimen of Weston instrument shown is called a "*milammeter*,"

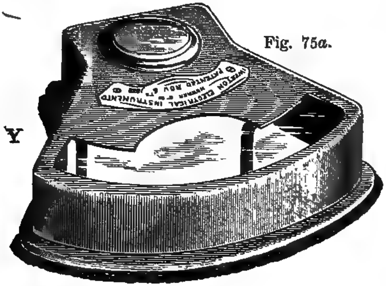


Fig. 75*a*.

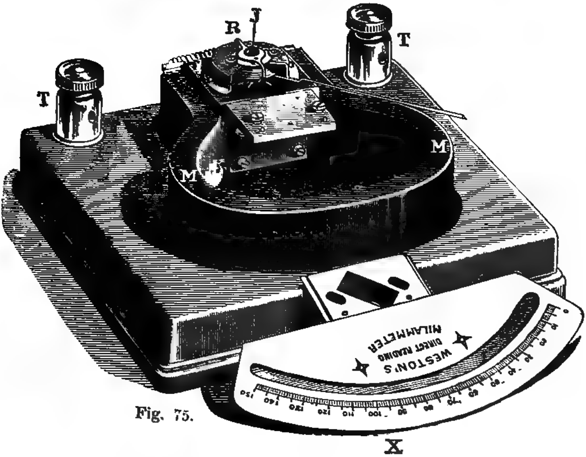


Fig. 75.

Figs. 75 and 75*a*.—Weston Milammeter, or Small Current Ammeter, one-third of full size.

because a current may be directly measured with it in "*milli-amperes*," or thousandths of an ampere. The moving coil C C, which can be easily seen in Fig. 75*b*,

where one end of the permanent magnet and a portion of one of the iron pole pieces are shown broken away, (this figure being about two-thirds of full size), consists of a number of convolutions of fine wire wound on a little flat aluminium frame, pivoted top and bottom in jewelled centres, J, and controlled by two spiral hair springs, S, S, made of non-magnetic material, the springs also serving to lead the current into and out of the moving coil. The inner ends of these two springs are attached to the steel pivots on which the coil turns, and the pivots themselves are soldered respectively to the two ends of the wire with which the coil is wound. The outer ends of these spiral springs, S, S, are fastened to the tips of two arms used for regulating the springs. One of these arms, R, is seen at the top of the instrument (Figs. 75 and 75*b*), and a portion of the lower regulating arm is just seen at the bottom of Fig. 75*b*. Flexible wires soldered to the other ends of these movable arms lead the current to and from the terminals of the ammeter T, T.

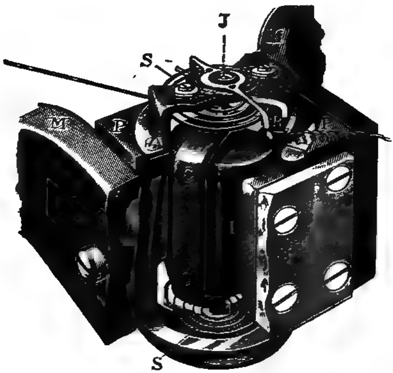


Fig. 75*b*.—Coil, Pole Pieces of Magnet, Springs, etc., of the Weston Milammeter. About two-thirds of the full size.

To the ends of the horse-shoe magnet M M there are attached curved iron pole pieces, P, P, and in the centre of the moving coil there is a stationary soft-iron cylinder, I, so that the magnetic lines of force passing from the poles of the magnet to the core are nearly radial (*see* Fig. 37, § 20,

page 75). Hence the force exerted on the coil, when a given current passes through it, is but little changed by the position of the coil, and therefore is nearly directly proportional to the current flowing even although the coil is deflected. The controlling torque exerted by the springs is proportional, or very nearly proportional, to the angle through which the coil is turned, therefore the angular deflection of the coil and of the pointer in degrees is very nearly directly proportional to the currents in amperes, as is seen from the divisions of the scale X being all nearly equal to one another. This scale, on which the current is read off directly in milli-amperes, is fixed over the curved portion of the permanent magnet M M, and the whole is protected by the cover Y (Fig. 75*a*) when the instrument is fitted together.

The clearance—that is, the space between the poles P, P, and the core I in which the coil turns—is very small, being in some of the Weston instruments only 0·04 of an inch. This enables the magnetic field to be made very strong, and also very constant in strength. The framework and the coil wound on it are made very light, the total weight of frame, coil, pointer and counterpoise being in some instruments only 2·25 grammes. This small mass, combined with strength of the magnetic field and the corresponding strength of the control springs, causes the instrument to be very quick in its action. Further, when the coil swings in the magnetic field, produced by the permanent magnet, "*eddy currents*," or "*Foucault currents*," are induced in the aluminium frame on which the coil is wound and flow in such a direction that the magnetic field acting on these *eddy currents* checks the motion of the coil. Hence any vibrations of the coil are rapidly damped, and the "*Weston ammeter*" is therefore very dead beat. Lastly, as the moving system is very carefully balanced, this ammeter can be used in any position.

The maximum thickness that can be given to the material of the spiral springs S, S, however, is very small,

otherwise they would become too rigid for the coil to be easily deflected. Hence, the current that can be sent through the coil of a Weston instrument without heating the springs is only quite small. For using this instrument to indirectly measure large currents *see* § 53, page 188.

A simple form of moving coil ammeter, seen in Fig. 76, is constructed by Messrs. Paul. It consists of a deep cylindrical permanent magnet, M, with a

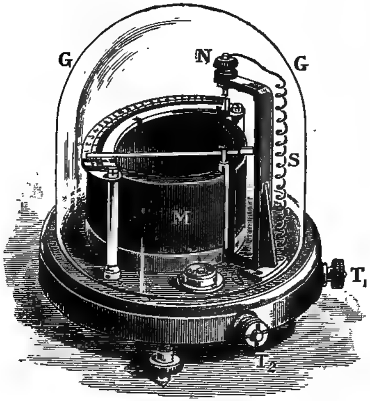


Fig. 76.—Ayrton and Mather's Moving Coil Ammeter, about one-fourth of the full size.

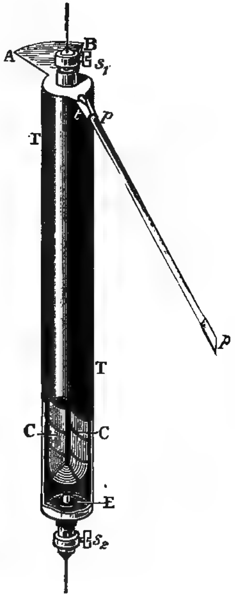


Fig. 76a.—Moving Coil, full size, of Ayrton and Mather's Ammeter.

very *narrow air gap.* In this gap is suspended, by means of a very thin strip of phosphor bronze, a coil wound in accordance with the principle developed by Mr. Mather and the author for obtaining the greatest deflecting torque with a given strength of field, a given current, a given number of windings of wire on the coil, and a given moment of inertia. The coil, which is shown

full size in Fig. 76*a*, has no stationary iron core in its centre, as in the Weston ammeter, but the bundle of wires which form one side of the coil C are nearly in contact with the bundle of wires forming the other side C, so that the cross-section of the coil has the form of two circles almost touching one another. The coil is contained in a thin tube, T T, made of silver partly to protect it from mechanical injury and partly in order that the instrument may be rendered dead beat by the *eddy currents*, which are induced in the good conducting silver tube when it swings in the magnetic field, damping the motion of the tube and quickly bringing it to rest.

One of the terminals of the ammeter T_1 (Fig. 76) is connected, by means of the spiral of wire S with the top of the phosphor-bronze strip, and the bottom of this strip is gripped in the screw-clip S_1 (Fig. 76*a*), to which one end of the wire wound on the coil C C is soldered. The other end of the coil of wire is soldered to a similar screw-clip, S_2, at the bottom of the silver tube, one or both of these clips being insulated from the tube itself by a collar of ebonite, E, and in the lower clip S_2 is gripped the upper end of a spiral made out of extremely fine phosphor bronze, the lower end of this spiral being attached to the terminal T_2 (Fig. 76).

Or instead of using the screw-clips, S_1 and S_2 (Fig. 76*a*), the bottom of the straight strip of phosphor bronze, which supports the coil, and the top of the phosphor-bronze spiral, which is used to make electric connection with the lower end of the coil, may be soldered in position. But in that case solder which melts at a temperature not much above that of boiling water should be employed, otherwise the elasticity of the phosphor bronze will be diminished by the heat employed in the soldering.

The pointer *p p* (Fig. 76*a*) is made of a narrow tube formed out of very thin aluminium sheet, the ends of this tube being squeezed flat on a vertical plane at the

place where it projects over the scale so that the deflection may be accurately read. The best way of fastening the pointer to the silver tube is as follows: Under the top clip S_1 there is screwed a piece of aluminium t A B cut out of somewhat thicker aluminium sheet than that used for making the pointer, and shaped as shown. The narrower end of this piece is rolled up into a little tube, t, into which the end of the tubular pointer is inserted and fixed in position with a touch of varnish.

On removing the glass shade G G (Fig. 76) which covers up the ammeter and protects it from dust and draught, the pointer can be accurately adjusted to zero by turning the nut N.

The final adjustment of the sensibility of a permanent magnet instrument can be conveniently made by slightly altering the strength of the field in the neighbourhood of the coil. This can be easily done by diverting more or less of the lines of force through a piece of iron, the number so diverted being varied by altering the distance between one of its ends and one pole of the magnet, with an adjusting screw, the other end of the iron being permanently in contact with the other pole of the magnet.

39. Moving Coil Ammeter with Magnetic Control.—It is important to observe the difference in the change of sensibility produced by a weakening of the horse-shoe magnet in the ammeters illustrated in Figs. 67, page 130, and 75, page 145. When the magnet is used to control the deflection of a soft-iron needle deflected by the current passing round a stationary coil the weakening of the magnet increases the sensibility of the instrument, whereas when the magnet deflects a coil carrying a current against the action of a weight or a spring the weakening of the magnet diminishes the sensibility.

By balancing one of these defects against the other in the following way, the author has constructed a type of permanent magnet ammeter the indications of which are practically unchanged for considerable alterations in

the strength of the magnet :—the deflecting torque is produced by a stationary magnet acting on a moving coil conveying the current to be measured, but, although phosphor-bronze strips are used to convey the current into and out of the coil, as in Fig. 76, they are made as thin as possible so as to exert but a very slight control. The main part of the control is produced by the attraction of the stationary magnet on some tiny hard steel magnets, which are inserted at various places inside the coil. Weakening the large stationary magnet therefore diminishes both the deflecting and the controlling torques in nearly the same proportion, and so leaves the sensibility of the instrument practically unchanged.

CHAPTER III.

DIFFERENCE OF POTENTIAL, AND RESISTANCE.

40. Difference of Potentials.—When a current of electricity is flowing through a wire, it has the same strength at all cross-sections of the wire. If, for example, the wire be cut *anywhere*, and a galvanometer be put in circuit, the galvanometer will always show the same deflection while the same current is flowing; or if several galvanometers, or ammeters, be placed at different parts of the same circuit, each instrument will be found to indicate the same current. In the same way, in the case of a water-pipe, the quantity of water passing every cross-section of the pipe per second is exactly the same as soon as the flow of water becomes ***steady.*** Just at the commencement, when, for example, some water has

entered at one end of the pipe, and none has flowed out at the other—when the pipe is filling in fact—the flow at different cross-sections may be different; so also, in many cases, just at the moment after completing an electric circuit, the current will differ at different cross-sections. But as soon as the flow in each case becomes a steady one this difference disappears, and the strength of the water current—that is, the number of gallons of water passing per minute (not, of course, the velocity of the particles of water)—is the same at all parts of the pipe, even if the pipe be broad at some points and narrow at others. In the same way the strength of the electric current flowing through a single circuit is "*uniform*" * at all parts of the circuit, independently of the thickness of the conductor, and of the material of which it is made.

But although the stream of water is the same at all parts of the pipe, the pressure per square inch of the water is by no means the same, even if the pipe be quite horizontal and of uniform cross-section. This pressure per square inch of the water on the pipe, which is the same as the pressure per square inch of one portion of the water on another portion adjacent to it, becomes less and less as we proceed in the direction of the flow. It is, in fact, this difference of pressure, or "*loss of head*" as it is sometimes called, that causes the flow to take place against the friction of the pipe, the difference of pressure at any two points, in the case of a steady flow through a horizontal pipe of uniform sectional area, being balanced by the frictional resistance of that length of pipe for that particular rate of flow.

Quite analogous with this there is, in the case of an electric current flowing through a conductor, a "*difference*

* *Uniform* refers to space, *constant* to time. The height of the houses in a street is generally not uniform, but it is constant so long as there is no change made in the height of the houses. If water be run out of a cistern the level at all parts of the surface of the water is uniform, but it is not constant, since it steadily falls as the water runs out.

of potentials" between any two points in the conductor, and this *difference of potentials*, or "*potential difference*," or "*P.D.*" as it may be shortly called, is needed to overcome the *resistance* of the conductor, or opposition that it offers to the passage of an electric current through it. In fact the analogy between *difference of potentials* and difference of fluid pressure is so

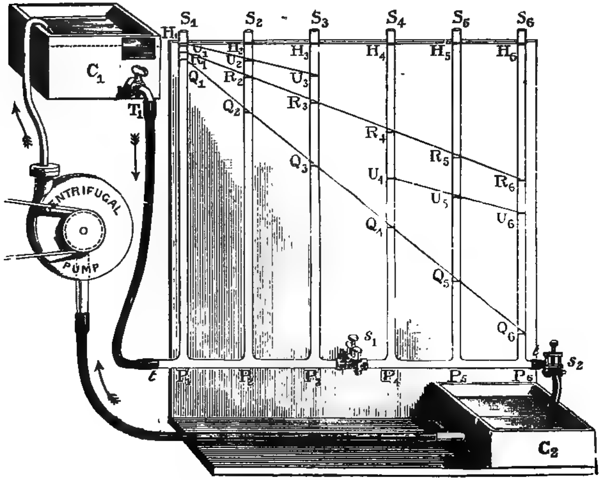


Fig. 77.—Apparatus for Testing the Distribution of Water Pressure.

marked that the name "*pressure*" is now frequently used to stand for *difference of potentials.*

The pressure per square inch of the water at any point of a tube conveying a stream can be ascertained by attaching a vertical stand-pipe to the tube, and observing to what height the water is forced up in this stand-pipe, and if at a number of points, P_1, P_2, P_3, P_4, P_5, P_6 (Fig. 77), in a glass tube, *t t*, conveying a stream of water, a series of vertical glass stand-pipes, S_1 S_2 . . . S_6, be fixed,

the height to which the water is forced up in them will show the distribution of pressure along the tube. If the tube $t\,t$ be straight and of uniform cross-section, and if the flow be a *steady* one, the tops of the water-columns in the stand-pipes will be found to all lie in one straight line, $Q_1 Q_2 \ldots Q_6$, therefore, if the length $P_1 P_2$ of the uniform tube be equal to the length $P_4 P_5$, the difference between $P_1 Q_1$, the height of water in the stand-pipe S_1, and $P_2 Q_2$, the height of water in the stand-pipe S_2, is exactly equal to the difference between $P_4 Q_4$ and $P_5 Q_5$. Also, if the length $P_1 P_4$ be three times the length $P_4 P_5$, the difference between $P_1 Q_1$ and $P_4 Q_4$ is equal to three times the difference between $P_4 Q_4$ and $P_5 Q_5$. Or, in other words, *when there is a steady flow of liquid through a uniform tube, the difference of pressure between any two points is proportional to the distance between these points.* And this is true whatever the inclination of the tube $t\,t$ to the horizontal, provided that the tube is straight and of uniform cross-section everywhere.

If the tap T_1 and the screw pinch-cock s_1 be fully open, and the screw pinch-cock s_2 be fairly open, the stream of water through the tube $t\,t$ will be rapid, and the slope of pressure—that is, the line $Q_1 Q_2 \ldots Q_6$ joining the tops of the columns of water in the stand-pipes —will be steep. If now the pinch-cock s_2 be screwed up a little so as to impede the passage of the water, the flow will be decreased, and the slope of pressure $R_1 R_2 \ldots R_6$ will be less inclined to the horizontal than $Q_1 Q_2 \ldots Q_6$.

As the pinch-cock s_2 is more and more screwed up the pressure line will become more and more horizontal until, when the flow is entirely checked, the line $H_1 H_2 \ldots H_6$ joining the tops of the columns of water in the stand-pipes becomes quite horizontal and at the same level as the water in the cistern C_1.

It will be noted that if there be any flow, the level of the water in the first stand-pipe S_1 is less than that in the cistern itself, which is seen through a little glass

window at the right of the cistern C_1. This is on account of the resistance offered to the flow by the tap T_1 and by the indiarubber tube $T_1\,t$. Similarly, if the pinch-cock s_1 be screwed up so as to check the flow between P_3 and P_4, there will be a sudden drop in pressure between P_3 and P_4, so that the tops of the water columns in the stand-pipes will now be in two different straight lines, $U_1\,U_2\,U_3$ and $U_4\,U_5\,U_6$, parallel to one another, but the latter $U_4\,U_5\,U_6$, much lower than the former.

As the pinch-cock s_1 is screwed up more and more the lines $U_1\,U_2\,U_3$ and $U_4\,U_5\,U_6$ will become more and more horizontal, but at a greater distance from one another, until, when s_1 is entirely closed, the former line will coincide with $H_1\,H_2\,H_3$, while the latter will sink down to the level of the tube $P_4\,P_5\,P_6$ itself.

In a very similar way the "*electric potential*" at different points of a wire conveying a current can be measured by apparatus which we shall presently describe, and if a number of measurements be made of the potential at different points of a circuit conveying a current, it will be found that the results are smaller and smaller as we proceed in one direction; and, further, if the conductor be all of uniform gauge, and made of the same material, and the electric current be a *steady* one, it will be found that the P.D. between any two points is proportional to the length of the conductor between these points (*see* § 42, page 167).

Electricity is put in motion, and a current of electricity is produced, as a consequence of the potential varying from place to place, just as a current is produced in water when subjected to pressures which are not uniform. In order to produce and *maintain* a current of either water, or electricity, work of some kind has to be done. Thus in Fig. 77 the current of water in the tube $t\,t$ will gradually diminish as the water passes from the upper to the lower reservoir, and will cease entirely as soon as the reservoir C_1 is empty. In order to maintain the current it is necessary to provide some

means of keeping up the level of water in the upper reservoir, and the simplest method of doing so is by means of a pump working at such a rate that water is raised from the lower vessel C_2 to the upper one C_1 just as fast as it flows from C_1 to C_2 through the tube *t t*. Exactly analogous with this pump in the water circuit is the "*voltaic cell*," or "*dynamo machine*," or other "*current generator*," in the electric circuit. A *current generator* does not create electricity any more than a fire engine creates water, it merely sets it in motion,

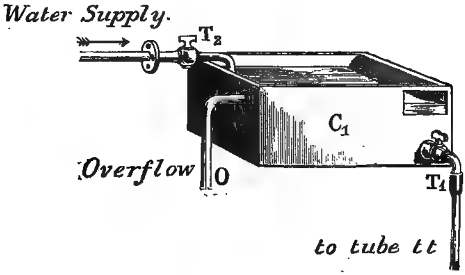


Fig. 78.—Alternative Arrangement of the Cistern for the Apparatus in Fig. 77.

and in either case work has to be done in keeping up the flow (*see* Chapter V. on Electric Energy and Power).

In the case of the water flow we may commence by filling the reservoir C_1 and maintain the level by allowing water to flow from the cistern of the building into the reservoir C_1 as fast as it flows out. Or, to save trouble, we may let the water run into the reservoir rather faster than it flows out through the tube *t t*, and allow the surplus to flow out through an overflow pipe O (Fig. 78). With the latter arrangement the level of the water in the reservoir C_1 will remain automatically constant whatever be the flow through the tube *t t*, provided, of course, that the tap T_2 be opened enough to cause the flow from

the house cistern into the reservoir to be never less than the flow out through the tube *t t*.

We shall see later on (§ 154, page 510) that an analogous arrangement not only can be employed but is very frequently employed in electricity when it is desired to maintain the P.D. between two points of a conductor, constant, in spite of its resistance and, therefore, of the current flowing through it being varied.

If the substance flowing were a gas, the distribution of pressure could not be measured by stand-pipes, since if

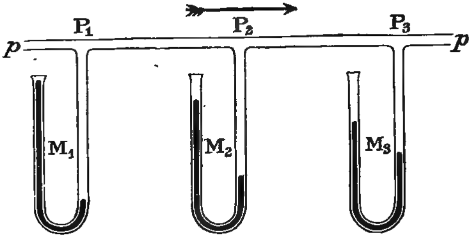


Fig. 79—. Apparatus for Testing the Distribution of Gas Pressure relatively to the Atmospheric Pressure.

the pipes were open at the top, the gas would flow out; or the outside air would flow in, and, if the pipes were closed they would all be filled with the gas itself, or with a mixture of gas and air.

The distribution of pressure along a pipe, *p p* (Fig. 79), conveying a stream of gas might be measured relatively to the outside atmospheric pressure by means of "*manometers*," M_1, M_2, M_3 attached at the points P_1, P_2, P_3 of the pipe, the difference of level of the liquid on the two parts of the tube of each *manometer* measuring the excess of the pressure of the gas at that part of the pipe over the atmospheric pressure. Or, if we desired that our measurements should be independent of the atmospheric pressure and merely indicate the pressure at

various parts of the pipe relatively to the pressure at one point P_4, then the *manometers* might be arranged as

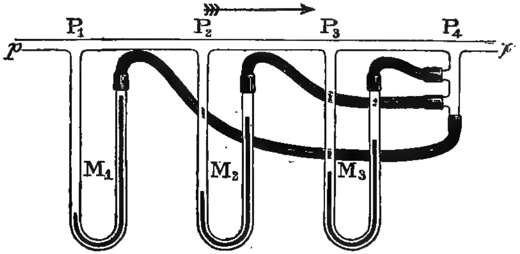


Fig. 80.—Apparatus for Testing the Distribution of Gas Pressure relatively to the Pressure at one Point of the Pipe.

in Fig. 80, in which case the difference of level of the liquid in the curved tube of any one manometer M_2 would show how much the pressure of the gas at the point P_2 of the pipe *pp* exceeded the pressure at the

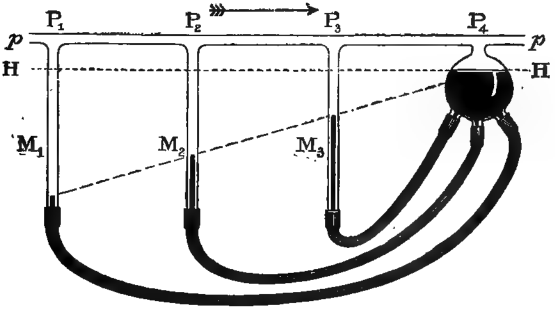


Fig. 81.—Simpler Apparatus for Testing the Distribution of Gas Pressure relatively to the Pressure at One Point of the Pipe.

point P_4. Perhaps the most convenient way would be to construct the apparatus as seen in Fig. 81, since then

the pressure of the gas at any point P_2 relatively to the pressure at P_4 would be at once seen from the distance the top of the column of liquid in the tube M_2 (Fig. 81) was below the horizontal line H H; and the difference of pressure of the gas at any two points P_2 and P_3 would be therefore measured by the difference in the depths below the horizontal line H H of the tops of the liquid columns in the manometers M_2 and M_3.

If the pressure of the atmosphere surrounding the apparatus in Fig. 79 were changed, then, although the flow of gas along the pipe *p p* might remain exactly the same, as well as the pressures at its two ends, the difference of level of the liquid in each of the manometers in this figure would change. But the level of the liquid in the manometers in Fig. 81 is wholly independent of the outside atmospheric pressure, and depends solely on the length, cross-section, shape, and internal character of the pipe *p p*, on the rate of flow and on the nature of the fluid flowing through the pipe. These manometers tell us nothing about the absolute pressure of the gas at the different points of the pipe through which it is flowing, but only the pressures relatively to the pressure at the point P_4.

41. Potential of the Earth Arbitrarily called Nought; Positive and Negative Potentials.—So in the same way the electric potential of a point in a wire through which a current is flowing is usually measured *relatively* to that of some other point of the wire. And even when one point of the wire is connected with the earth and the potentials of different points of the wire are measured above or below the potential of the earth, which is *arbitrarily* called *nought*, it is still but a relative measurement, for in thus taking the potential of the earth as the *potential level* to measure from, no assumption is made as to the earth having no charge of electricity on it; indeed, so far from that being the case, experiment shows that the electrical effects produced in the neighbourhood of the earth would be caused were

the earth electrified "*negatively*" or "*resinously*," that is, electrified in the same way as is a piece of ebonite after being rubbed with a piece of dry flannel, and oppositely electrified to a piece of dry smooth glass which has been rubbed with a piece of dry silk, the electrification of the glass being called "*positive*" or "*vitreous*" (*see* page 216).

So that measuring electrical potentials relatively to that of the earth is like measuring heights above the Trinity water-mark, or measuring longitude east or west of Greenwich, the place which is *arbitrarily* said to have zero longitude.

A similar convention is followed in the measurement of temperature, for in the centigrade scale the temperature of melting ice is called 0°, while in the Fahrenheit scale the zero is a temperature much below this, and one which is roughly that of a mixture of ice and salt. Now, although Fahrenheit is said to have called this temperature zero because he had an idea that it was the lowest temperature that could be produced artificially, no such assumption is at present made in calling this particular temperature 0°F.

In addition to a P.D. being said to exist between two points in a conductor through which a current is flowing, any two conductors are said to differ in potential when there is a tendency for electricity to pass from one of them to the other. This tendency may manifest itself in four ways:—

(1) By the production of a current (lasting, it may be, for only the fraction of a second) when the two conductors are touched together, or when they are electrically connected by means of a wire, or other conductor;

(2) By a "*brush discharge*" or an "*electric spark*" passing between the conductors when they are near together, and when the P.D. between them is high;

(3) By small light bodies, such as grains of dust, pieces of paper, pith, &c., being attracted backwards and forwards between the conductors

(4) By the conductors trying to approach one another, as if there were an attraction between them.

When different pieces of electrical apparatus are enclosed in a metallic box (a not infrequent arrangement as will be seen later on) the potential of the box itself is usually called nought, and the potentials of the different bodies inside it are measured relatively to that of the box by the methods subsequently described. This box in such a case is sometimes called, in electrical language, the "*earth,*" but it must not, therefore, be inferred that there is any metallic connection between the box and the ground; the box and all the apparatus inside it might, indeed, be up in a balloon, and still the joining of some part of the internal apparatus to the metallic box by a wire might be called "*earthing*" that piece of apparatus (*see* § 66, page 220).

A conductor is said to have a "*positive potential*" when on *earthing* the conductor a current flows from the conductor to the earth, and a "*negative potential*" when the current flows in the opposite direction. Also when two conductors, A and B, are in such a condition that, if joined with a conductor, a current would flow from A to B then, irrespectively of the actual signs of the potentials of A and B, as defined in the last sentence, the potential of A is said to be "*higher*" than that of B (*see* § 7, page 31, for the definition of the direction of a current). Further, if two bodies, whether conductors or not, differ in potential a *positively* electrified body, placed in their neighbourhood, tends to move *away from* the body having the *higher* potential *towards* the other body having the *lower* potential.

42. Electrometer. — The potential difference, or P.D. between one conductor and another, is measured by an "*electrometer.*" *Electrometers*, like galvanometers, are of two kinds, those in which the measurement is made by noting how much a needle is deflected against the action of a controlling force, and those in which we observe by how much the controlling force must be

increased to resist the motion of the needle and keep it in a fixed position. This latter, or "*zero type*" of electrometer, has an advantage over the former in that it supplies a simple definition of the measurement of difference of potentials; that is, it enables us to easily specify what we mean by one P.D. being two or three times another P.D.

For, just as the magnitude of any one of the pr. perties of a conductor carrying a current might be arbitrarily chosen as the direct measure of the strength of the current, so any one of the effects exhibited by two bodies differing in potential might be selected to measure the P.D. between them. But the particular effect which furnishes the simplest definition of the relative magnitude of various P.Ds. is the attraction between two conductors kept in a fixed position relatively to one another thus: let two conductors N, I (Fig. 82), be maintained at a certain P.D. relatively to one another, then N will be attracted so as to enter I. If now a force be applied to resist this attraction, and to keep N in a fixed position, relatively to I, this force may be *arbitrarily* defined as being directly proportional to the square of the P.D. between N and I, that is to say, *the* P.D. *may be defined as being directly proportional to the square root of the force of attraction between* N *and* I *when in a fixed position relatively to one another.*

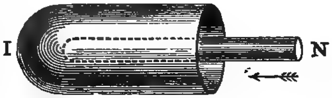


Fig. 82.

This definition leads to exactly the same result as the definition given of a P.D. in mathematical treatises on electricity, but it is much simpler to grasp and lends itself more directly to the understanding of the principles underlying the action of *electrometers*. It is only, however, during the last few years, thanks mainly to the energy and ingenuity of Mr. Mather, one of the staff in the Physical Department of the City Guilds Central

Technical College, that it has become possible to use the definition given above in the construction of *zero electrometers* suitable for measuring P.Ds. no larger than

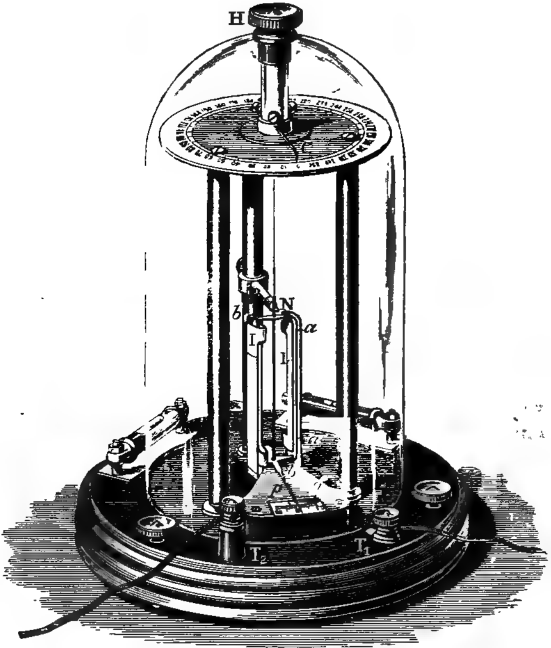


Fig. 83.—Ayrton and Mather's Zero Electrometer, or Zero Electrostatic Voltmeter, one-third of the full size.

those commonly employed in laboratories for sending currents through wires.

In the actual instrument, devised by Mr. Mather and the author, the moving part N, or the *needle* * as it is

* A magnetised sewing-needle having been originally used for the suspended magnet in a galvanometer, the name *needle* came gradually

called, takes the form of two thin narrow pieces of aluminium *a*, *a* (Fig. 83), joined together at the top and bottom by cross pieces, *b*, *b*, and supported by means of a thin strip of phosphor bronze from a head H, carrying an index C, which can be turned round over a graduated dial. The conductors, I, I, or the "*inductors*" as they are called, into which the two parts *a*, *a* of the needle are attracted, are shaped as shown, and, by means of a pointer *p*, carried from the bottom of the needle, the position of the needle can be observed. As usual, parallax is avoided by observing the reflection of this pointer *p* in a piece of looking-glass *g* fixed to the base of the instrument.

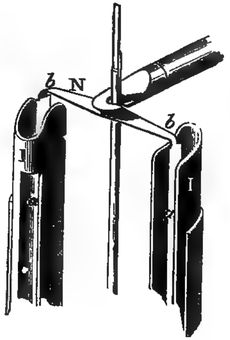


Fig. 83*a*.—Details of Needle and Inductors, rather larger than full size.

Any P.D. set up between the needle and the inductors is then measured by turning the head H until the pointer *p* (carried by the needle) is brought into the same position it occupied when the needle and the inductors had the same potential; the angle through which the index C has been turned is noted, and its square root taken. For this is the angle through which the strip carrying the needle has been twisted and, therefore, this angle measures the moment of the force, or the torque, that has been exerted on the needle.

The terminals T_1, T_2, are connected respectively with the needle and the inductors, and equality of potential

to designate the little magnet in a galvanometer, whether it was long and pointed like a sewing-needle, or short and blunt. And now the expression *needle* is employed for the suspended movable part of an electrical measuring instrument even when the shape of the moving system in no way resembles that of a sewing-needle, as in the electrometer shown in Fig. 83.

of these two bodies can be secured by connecting these terminals together with a piece of wire, thick or thin. For if there be any difference of potential, a momentary current will flow through this wire which will annihilate the P.D.

Further, if the terminals be joined respectively by wires with any two conductors A and B, momentary currents will flow, and the potentials of the needle and inductors will become respectively the same as those of A and B. In fact, we may say generally, that *if any number of conductors be touched together, or be joined by wires, and if no current be flowing between any of the bodies, the conductors and wires are all at the same potential.* To be strictly correct, this general proposition requires that all the conductors should be made of the same material, and be at the same temperature.

This last proposition can be stated briefly and completely thus :—*the potential of all parts of a conducting system composed of the same material at the same temperature and on which electricity is at rest is uniform.*

In order to ensure that the electric force exerted on the needle shall be wholly due to the P.D. between it and the inductors, and that no part of this force shall be caused by the attraction of external bodies, the interior of the glass shade is coated with a *conducting transparent* varnish devised by Mr. Mather and the author, the composition, and action of which are explained in § 58, page 200.

The spindle of the needle in the electrometer (Fig. 83) moves in guides top and bottom, the upper guide being clearly seen in Fig. 83*a*, which shows the top of the needle and of the inductors rather larger than full size; hence the instrument may be turned upside down, or carried about without its being necessary to clamp the needle, and without there being any risk of breaking the thin phosphor-bronze strip supporting it.

If now, in addition to sending a steady stream of water through the tube *t t* (Fig. 77, page 154), the water in the

tube be used as a conductor and a steady electric current be sent through it, the various P.Ds. between the pairs of points P_1 and P_2, P_2 and P_3, &c., can easily be measured with the electrometer just described by simply dipping wires, attached respectively to the terminals of the electrometer, into the water in the various pairs of stand-pipes S_1 and S_2, S_2 and S_3, &c. For, since there is no electric current in the water in a stand-pipe itself, there can be no P.D. between the different parts of the water in the same stand-pipe; hence the water in the stand-pipes can be used simply as extensions of the wires attached to the terminals of the electrometer. When the screw pinch-cock s_1 is fully open, so that the tube $t\,t$ is throughout of uniform bore, it will be found that the P.Ds. between the different pairs of points are related to one another in exactly the same way as are the differences between the water pressures for the same pairs of points.

Thus the distribution of potential along a uniform conductor conveying a steady electric current is exactly analogous with the distribution of fluid pressure along a uniform tube, through which flows a steady stream of liquid.

43. Ohm's Law.—But if instead of measuring the P.D. between different points along a conductor through which flows a steady current we measure the P.D. between two *fixed points* in a given conductor through which *different currents* are flowing, then the P.D. does *not* vary with the current in the same way that the difference of pressure between two points in a given tube varies with the stream of fluid flowing through it. Let us consider the second case first:—Keep the level of water in the reservoir C_1 (Fig. 77) constant in the way already described, open the screw pinch-cock s_2 a certain amount, the screw pinch-cock s_1 being fully open, and, when the stream has become steady, measure with the graduated glass the number of cubic centimetres of water that flow through the tube $t\,t$ per second, also the difference of pressure between two fixed points in the

tube P_1 and P_6 for example. Next open the pinch-cock s_2 a little more, and again measure the number of cubic centimetres of water per minute that flow out of the tube, as well as the difference between the height of the water in the stand-pipes S_1 and S_6. If such measurements be made for several different steady rates of flow, numbers

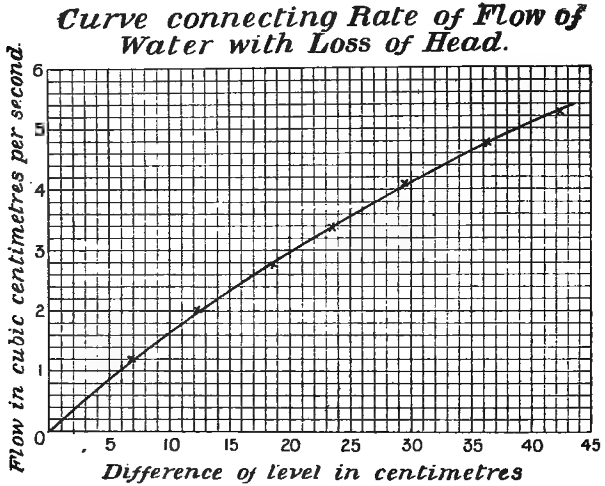


Fig. 84.

like the following will be obtained, and when plotted they give the curve seen in Fig. 84, concave to the axis along which difference of level is plotted.

Difference of Level in Centimetres.	Flow in Cubic Centimetres per second.	Ratio of Difference of Level to Flow.
6·9	1·20	5·75
12·4	2·00	6·20
18·7	2·78	6·73
24·0	3·39	7·08
29·5	4·09	7·21
36·2	4·76	7·60
42·1	5·26	8·00

If the numbers in the third column were all the same it would tell us that the ratio of the difference of level to the number of cubic centimetres flowing per second—that is, the ratio of pressure to current—was a constant for a given pipe. In that case the points on the curve in Fig. 84 would all lie in one straight line, and to double, treble, quadruple the current would require exactly double, treble, quadruple the pressure. But the numbers in the third column steadily increase as the current increases, and if we examine the numbers in the first two columns we find that to increase, for example, the flow from 1·20 to 4·76 cubic centimetres per second —that is, to make the current *not quite four* times as great—we have to increase the difference of level from 6·9 to 36·2 centimetres—that is, to increase the pressure *more than five* times.

The quantity of water, therefore, that flows per second through a given pipe does not increase as rapidly as the difference of pressure between two fixed points in it, or, in other words, we must *more* than double, treble the difference of pressure to produce twice, three times the flow, even although the tube through which the water flows remains absolutely unchanged. It might, therefore, have been expected that the same sort of inequality would be found in the ratio connecting the P.D. between two fixed points in a conductor and the current flowing through it.

But that is not the case, for if the conductor K (Fig. 85) remains at the same temperature, and be not changed in any way, experiment shows that the P.D. between two fixed points, K_1, K_2 in it, measured by the electrometer E (in the way described in § 42, page 165), is directly proportional to the current flowing through this conductor, the currents being measured relatively to one another by any suitable galvanometer G,* for which the law connecting current

* For the details of the construction of the galvanometer illustrated in Fig. 85, *see* § 38, page 148.

and deflection has been obtained by a relative calibration as described in § 12, page 46.

For carrying out these tests the current can be conveniently produced with a battery, B B, of what are known as "*dry cells*" (*see* § 140, page 457), or of "*accumulators*" (*see* Vol. II.); and its strength can be varied by

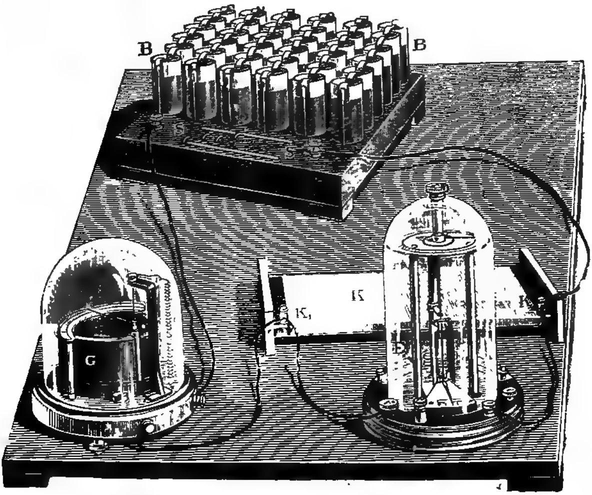


Fig. 85.—Apparatus for Testing Ohm's Law.

altering the number of cells employed. This alteration in the number of cells that are used in the different tests, can be easily effected by means of the mercury switch-board, S S, seen in front of the battery of cells in Fig. 85. The construction, and mode of using such a mercury switch-board, will be found described in § 162, page 539.

This experimental result, that *the ratio of the* P.D. *to the current, if steady, is absolutely constant for a given*

The following is a sample of the results obtained by the students at the Central Technical College, with the apparatus shown in Fig. 85 :—

Relative Strength of Current Measured by Galvanometer G.	Angular Twist given to Torsion Head of Zero Electrometer E.	Square Root of Angular Twist.	Ratio of Square Root of Twist to Current.	Difference between the Value of this Ratio and the Mean Value of the Ratio.
10·34	105°·8	10·28	0·995	− 0·002
13·8	190°·8	13·81	1·001	+ 0·004
17·2	291°·6	17·08	0·993	− 0·004
20·4	415°	20·37	0·999	+ 0·002
		Mean	0·997	$\pm$ 0·003

conductor at constant temperature, is known as "*Ohm's Law*," since it was first published by Ohm in 1827, although not exactly in the form here given. And it is important to notice that all experiments that have been made to test its accuracy, even when made with the most sensitive instruments yet constructed, have failed to detect any inaccuracy in this law.

It is sometimes stated that *Ohm's law* is self-evident, but that misconception has arisen first from the law being so extremely simple, and secondly from its wide use in electrical calculations having led people to gradually imagine that no connection between P.D. and current for a given conductor, other than direct proportionality, could exist.

So far, however, from this being the case, it is still possible that with very large currents, especially when flowing through conductors of great width, some slight want of proportionality between P.D. and current may be detected, even when the current is absolutely steady, and the conductor absolutely at the same temperature, and in every other respect in the same physical condition. At present, however, the law must be regarded as being rigorously true without exception.

44. Resistance.—Since the ratio of the P.D. to the current has a constant value for each conductor this ratio has been called by a special name—the "*resistance*" of the conductor, and gradually people have grown to think and speak about the *electric resistance* of a wire as being a definite property which belongs to the wire like its length and its cross-section.

If, however, the ratio of P.D. to current had been no more constant for a given conductor than is the ratio of pressure to flow for a given tube carrying a liquid stream, it is practically certain that this conception would not have come into existence. Therefore the mere statement that a definite wire has a definite resistance is in itself an assertion, although not of course a proof, that *Ohm's law* is true.

The analogy between the distribution of water pressure and of electric potential is a very useful one for students to use, as it enables them better to grasp the meaning of electric potential; but, like many analogies, it must not be carried too far; for not merely, as we have seen, is the ratio of difference of pressure to the quantity of a fluid flowing per second not constant for a given pipe, but any bend made in a straight pipe, even when the cross-section of the pipe is in no way decreased, causes a diminution in the flow for the same difference in pressure between its two ends; whereas bending a wire through which a steady electric current is flowing, has no effect on the electric stream. Even a sudden expansion in a pipe, that is an enlargement of the bore, for a short distance checks the fluid stream, whereas if the cross-section of a conductor be made larger for a short portion of its whole length, either no change whatever is observed in the current, or the change, if noticeable, is always an increase and never a diminution in the steady current flowing.

45. Ohm.—Various units of resistance, differing slightly from one another, have been adopted from time to time, and called the "*ohm*," but the value that was definitely recommended to the Board of Trade in 1892 by the Committee appointed to advise them, was defined thus: "*The resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice* 14·4521 *grammes in mass of a constant cross-sectional area, and of the length of* 106·3 *centimetres may be adopted as one ohm*"; and at the Electrical Congress held in Chicago in 1893, this value was unanimously accepted by the Chamber of Delegates, composed of members nominated by the Governments of the United States, Great Britain, France, Italy, Germany, Mexico, Austria, Switzerland, Sweden, and British North America

Finally, for the purpose of distinguishing this unit

of resistance from any other, it was decided to call it by the name of the "*international ohm.*"

A brief sketch of the history of the British system of electrical units, now the system of the world, is given in the Appendix, page 568. It should be read by all those who are interested in seeing how the interdependence of electrical theory and practice, each on the other, has led to the building-up of a complete system of electrical standards, now accepted by all nations as a common heritage.

Eventually the *international ohm* will be so generally used that no other unit of resistance will be met with, and probably the adjective *international* will then be dropped. For some years, however, the "*B.A. unit of resistance,*" the "*legal ohm*" (so called because it was legalised in France), and the "*international ohm*" must be carefully distinguished from one another. Their relative values are given in the following table :—

TABLE IV.

RATIOS OF THE PRACTICAL UNITS OF RESISTANCE.

1 international ohm	= 1·00235 legal ohm.
1 international ohm	= 1·01358 B.A. unit.
1 legal ohm	= 0·99765 international ohm.
1 legal ohm	= 1·01120 B.A. unit.
1 B.A. unit	= 0·98660 international ohm.
1 B.A. unit	= 0·98892 legal ohm.

46. **Volt.**—Since the ratio of the P.D. maintained between the terminals of a conductor to the current that flows in it is constant, it follows that the P.D. that must be maintained at the terminals of a resistance of *one international ohm* when a current of one ampere passes through it must have a perfectly definite value. This value is taken as the practical unit of P.D., and called the "*international volt.*"

If, instead of basing our unit of P.D. on the international ohm, we base it on the B.A. unit of resistance or on the legal ohm, then we obtain the "*B.A. volt*" and the "*legal volt.*" And the equations connecting the values of the three volts are exactly the same as those connecting the three ohms, viz. :—

TABLE V.

RATIOS OF THE PRACTICAL UNITS OF P.D.

1 international volt	= 1·00235 legal volt.
1 international volt	= 1·01358 B.A. volt.
1 legal volt	= 0·99765 international volt.
1 legal volt	= 1·01120 B.A. volt.
1 B.A. volt	= 0·98660 international volt.
1 B.A. volt	= 0·98892 legal volt.

There is, however, but one ampere, viz. that defined in § 6, page 23.

Example 22.—With a P.D. of 100 international volts maintained between the terminals of a glow lamp a current of 0·3 passes through it, what is the resistance of the lamp?

Answer.—333·3 international ohms.

Example 23.—If the P.D. be reduced to 98 international volts and the resistance of the filament remain as before, what current will pass through it?

Answer.—0·294 ampere.

Example 24.—By how much per cent. does the international volt exceed the B.A. volt?

Answer.—1·36 per cent.

Example 25.—A P.D. of 7 international volts is maintained between the terminals of a resistance of 2,475 legal ohms, what is the current that passes?

Answer.—·002835 ampere.

Example 26.—The standard P.D. employed by the London Electric Supply Corporation is 2,400 legal volts, what is that in international volts?

Answer.—2,394 international volts.

Example 27.—If a wire has 235 B.A. units of resistance, what is its resistance in international ohms?

Answer.—231·85 international ohms.

Example 28.—If a wire of uniform cross-section has a resistance of 54 B.A. units at a certain temperature, by how much per cent. must its length be reduced so that it may have a resistance of 50 international ohms at the same temperature?

Answer.—54 B.A. units equals $54 \times 0{\cdot}9866$, or 53·276, international ohms, therefore the length must be reduced by $\dfrac{3{\cdot}276}{53{\cdot}276}$, or by about 6·15 per cent., in order that the wire may have a resistance of 50 international ohms.

Example 29.—What resistances in legal ohms are respectively equal to 100, 200, 300, 400, and 500 international ohms?

Answer.—100·235, 200·47, 300·705, 400·94, 501·175 legal ohms.

47. Current Method of Comparing P.Ds.—From Ohm's law it follows that the current flowing through any conductor at constant temperature is directly proportional to the P.D. between its terminals. Such a conductor may be a coil of a galvanometer, or it may consist of a galvanometer G together with a wire W (Fig. 86) in series with it. And no matter how the shape of

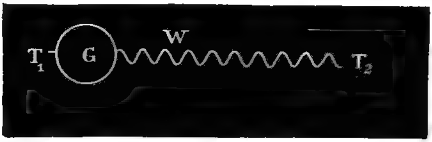


Fig. 86.—Galvanometer with Added Resistance for Measuring Potential Differences.

the circuit composed of G and W may be altered, provided that the joint resistance of G and W together is not changed, the current passing through the galvanometer will be directly proportional to the P.D. which is main-

tained between T_1 and T_2, the terminals of the arrangement. If then the galvanometer has been calibrated relatively for current, it is calibrated for the relative measurements of any P.D. which may be set up between T_1 and T_2 by connecting their terminals with any conductors between which a P.D. exists.

In place then of employing the zero electrometer (Fig. 83) we may use the combination of galvanometer and auxiliary resistance W to compare, for example, the P.D. between the points A and B (Fig. 87) with the P.D. between the points C and D in the conductor A B C D conveying a steady current. For the P.Ds. in question will be simply proportional to the two currents that flow through the galvanometer when the terminals T_1, T_2

Fig. 87.

(Fig. 86) are connected respectively first with the points A and B and then with the points C and D.

Further, since the resistance of a conductor is the name given to the ratio of the P.D. between its ends to the current that flows through it, and, since the current that flows through A B is necessarily the same as that flowing through C D, it follows that—

$$\frac{\textbf{resistance of A B}}{\textbf{resistance of C D}} = \frac{\textbf{potential difference between A and B}}{\textbf{potential difference between C and D}}$$

therefore

$$\frac{\textbf{resistance of A B}}{\textbf{resistance of C D}} =$$

$$\frac{\textbf{current when } T_1 \textbf{ and } T_2 \textbf{ are joined to A and B}}{\textbf{current when } T_1 \textbf{ and } T_2 \textbf{ are joined to C and D}},$$

the current in each case being the current through the galvanometric arrangement (Fig. 86).

Consequently, if the value of one of the resistances A B or C D be known in international ohms, the value of the other in international ohms can be at once found by the method of testing just described.

48. Reason for Using High Resistance Galvanometers for P.D. Measurements.—When using a galvanometer for the comparison of two P.Ds., or for the comparison of two resistances by the method described in § 47, it is not necessary that the galvanometer should be calibrated absolutely in amperes, for, as we have just seen, all that is required to be known is the ratio of the currents that produce different deflections, not the actual value of these currents in amperes. But there is one condition in connection with the galvanometric arrangement G W (Fig. 86) that it is most important to fulfil, and that is the condition that the application of the terminals T_1, T_2 to the points A and B or to the points C and D does not alter the distribution of potential that previously existed in the conductor A B C D. In fact, *the test must not alter the thing tested*, an all-important rule to remember in experimenting.

Whenever a galvanometer, properly constructed and calibrated, is introduced into any circuit the galvanometer measures the current flowing *after* the galvanometer has been inserted, but this is not necessarily the same as the current that flowed *before* the galvanometer was inserted. These two currents will only be the same in value when the resistance of the galvanometer is small compared with that of the rest of the circuit, and when the other conditions remain unchanged. It will, therefore, be only under these special circumstances that the deflection of a galvanometer will measure the current that passed through the circuit *before* the circuit was disturbed by the insertion of the galvanometer into it.

Similarly, whatever be the resistance, small or large, of a galvanometer or of a galvanometric arrangement G W (Fig. 86), provided that this resistance remains quite

constant, the relative P.Ds. between two pairs of points A and B, C and D, can be accurately compared by means of this galvanometric arrangement, only it must be carefully remembered that the P.Ds. that are thus compared are the values existing *after* the joining of the terminals T_1, T_2 to the points A and B, or to the points C and D, and not the values of these P.Ds. *before* the application of the measuring instrument. And it will be only when the resistance of G and W combined is very large compared with the resistance of the conductor A B and also with the resistance of the conductor C D that the application of the galvanometer will produce no disturbance in the distribution of potential along the conductor A B C D.

Therefore *for* P.D. *measurement it is desirable that the galvanometer* G *and the auxiliary conductor* W *should together have a high resistance, and that the required sensibility of the galvanometer should be attained by winding the galvanometer with a large number of convolutions of fine wire.*

49. Voltmeter.—A "*voltmeter*" is an instrument which enables the P.D. between its terminals to be read off directly in international volts. Whether the *voltmeter* be of the electrostatic type and its action depend on the attraction of electrified bodies, or whether it be of the galvanometer form and the P.D. be indirectly measured by the current it produces through a fixed resistance, it is obviously necessary that the sensibility of the instrument should not be affected by moving the instrument from place to place. In fact, a *voltmeter* must possess the constancy of an ammeter, with the addition that its resistance must be quite constant, and any ammeter of practically constant resistance when graduated to indicate the P.D. between its terminals in international volts, instead of the current passing through it in amperes, becomes a *voltmeter*.

The electrometer described and illustrated in § 42, page 164, gives the same reading for the same P.D.

between its terminals if the instrument be levelled each time after being moved. Its relative calibration is, of course, known since our fundamental definition of the relative value of P.Ds. is based on the use of this electrometer. If then we ascertain the P.D. in international volts (say V_1) that must be set up between the terminals T_1, T_2 of the instrument so as to bring the pointer p to the zero position when the index C has been turned through some particular angle, say a_1, the P.D. in international volts V_2 corresponding with *any* other angle a_2 through which the index C must be turned to bring p to zero is known from the equation—

$$V_2 = V_1 \sqrt{\frac{a_2}{a_1}}$$

or

$$V_2 = \frac{V_1}{\sqrt{a_1}} \sqrt{a_2},$$

$\frac{V_1}{\sqrt{a_1}}$ being a constant for the particular instrument.

A P.D. whose value is known in international volts can be applied to the terminals T_1, T_2 of the electrometer (and so the constant $\frac{V_1}{\sqrt{a_1}}$ can be experimentally found) by connecting T_1 and T_2 to the ends of a conductor, c (Fig. 89, page 185), whose resistance in international ohms, o, has been ascertained, and through which flows a current of A amperes, as measured by the ammeter A. For this P.D. is equal to $A \times o$ international volts.

This constant is about 2·37 for the zero electrometer illustrated in Fig. 83, page 164, that is to say, the index C has to be turned through about 360° to bring the pointer p to zero when a P.D. of 45 volts is maintained between the terminals of this instrument.

The dial at the top of the electrometer is initially graduated into degrees or other divisions of equal value.

But after the constant of the instrument has been experimentally determined, in the way just described, this degree scale may conveniently be replaced by one graduated in square roots with which the P.D. can be read off directly in international volts.

The electrometer then becomes a direct-reading "*electrostatic voltmeter*" of the zero type.

50. Ammeters used as Voltmeters.—If an ammeter with its scale graduated in volts instead of (or in addition to) its being graduated in amperes has a low resistance, it will be suitable for measuring any small P.D. that may exist between two points separated by a small resistance. For example, it may be used to measure the P.D. between two points close together in a thick copper electric-light main through which a current is flowing, or to measure the P.D. between the terminals of a galvanic cell of very low internal resistance. On the contrary, if the resistance of the instrument alone, or the resistance of the instrument and its auxiliary wire, W, combined (Fig. 86) be high, it may be used to test a larger P.D. between two points separated by a larger resistance; for example, the P.D. between the positive and negative electric-light main in a house.

Beginners sometimes feel mystified that the same instrument is sometimes employed to measure a current and at other times a P.D.; that in the former case, when it is called an ammeter, it may be "*short-circuited*" with impunity, but must not be disconnected, whereas when it is called a voltmeter it may be disconnected but on no account may it be *short-circuited.*

The difference arises not from any intrinsic dissimilarity between an ammeter and a current voltmeter, but from the different ways in which the two instruments are employed. An ammeter is put into the main circuit *in series* with the rest of the apparatus, as is the galvanometer G in Fig. 85, page 170, and the ammeter A in Fig. 89, page 185, whereas a voltmeter is placed as a

branch circuit *in parallel* with the part of the circuit, the P.D. between the terminals of which is to be measured; for example, the zero electrostatic voltmeter E in Fig. 85, and the voltmeter V in Fig. 89. If the voltmeter be of the current type, then both it and the ammeter simply measure a current directly, but the current that the instrument G in Fig. 85 and A in Fig. 89 measures is the current flowing through the main conductor, K in Fig. 85 and c in Fig. 89 respectively, whereas the current that the voltmeter measures is the current that the P.D. between the terminals K_1, K_2 of the main conductor K or the P.D. between the terminals of the main conductor c will send through a resistance which is quite external to the main circuit, viz. the resistance of the voltmeter itself.

If the resistance of an ammeter be but a small fraction of the resistance of the rest of the circuit in which it is placed, the only result of short-circuiting the ammeter by bridging its terminals with a short piece of thick wire is to electrically remove the instrument from the circuit, for the current remains unchanged in strength, and practically the whole of it now passes through the short circuit: whereas in short-circuiting a voltmeter we short-circuit all that part of the circuit with the terminals of which the voltmeter is connected, and thus cause a great, and possibly a dangerous, increase in the current in the remainder of the circuit. For example, the short-circuiting of an ammeter which is used to measure the electric-light current passing through a house will simply cut this particular ammeter out of circuit, whereas short-circuiting the voltmeter, which is placed across the house mains for measuring the P.D. supplied to the house, would momentarily extinguish all the lamps in the neighbourhood and compel the electric current-generating-station to produce an enormous current. Almost instantaneously either the piece of wire used to make the short circuit would itself be burnt up, or one of the "*fuses*," the name given to the pieces of easily-fusible metal placed in the circuit to diminish the

damage caused by such accidents, would itself be volatilised by the excessive current.

On the other hand, disconnecting one or both of the voltmeter wires from the main circuit stops, of course, the current through the voltmeter itself, but produces practically no effect on the main current, whereas disconnecting the ammeter stops the main current altogether, unless the ammeter has been short-circuited before being disconnected.

51. Moving Coil Voltmeter.—The moving coil ammeter, described in § 38, page 144, lends itself extremely well for use as a portable voltmeter in consequence of its freedom from outside magnetic disturbance, its quickness of action, its capability of being used in any position, and its great sensibility, so that the resistance of the coil and of the auxiliary wire W combined can be very high. Indeed, in a Weston voltmeter, intended to measure a maximum P.D. of about 140 volts, the resistance of the moving coil is about 100 ohms, and that of the auxiliary stationary wire about 16,000 ohms, which is a resistance far higher than that of any other type of voltmeter of the same range and quickness of action. The instrument, however, can only be employed to measure small currents, which is a disadvantage when it is desired to use it directly as an ammeter, but this becomes an advantage when the instrument is used as a voltmeter, since the smaller the current taken by a voltmeter, other things being equal, the better the voltmeter.

52. Calibrating a Deflectional Voltmeter.—If the law of the instrument be unknown as well as the P.D. in volts that produces any particular deflection, we can calibrate the instrument throughout the scale in volts in one or other of five distinct ways.*

1. Place the voltmeter V to be calibrated in parallel with a zero electrostatic voltmeter E and apply different P.Ds. between the common terminals of the two

* For methods in which a Clark's cell is employed *see* §§ 153 and 154, pages 507 to 513.

instruments. Measure each P.D. in international volts by means of the electrostatic voltmeter and observe the corresponding deflection on the deflectional voltmeter.

2. If the voltmeter to be calibrated has a very much longer, or a very much shorter, range than the voltmeter with which it is to be compared—for example, if the one reads from 0 to 500 international volts, while the other reads from 0 to 60 international volts—then we may proceed as follows :—

Place two conductors A B, C D (Fig. 88) in series, and, by using the method described in § 47, page 177, determine the resistance of the two conductors in series A D relatively to that of one of them A B. For example, let it be found that the resistance of A D is ten times that of A B. The actual resistance of the conductors need not be known, but we must make sure that the resistances of the voltmeters whose calibrations we desire to compare are large relatively to the resistances of A D and of A B.

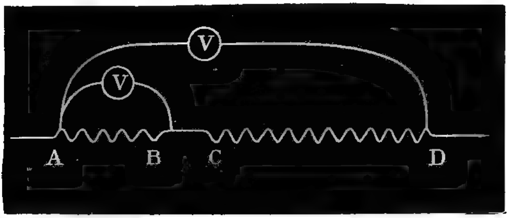


Fig. 88.—Comparing Two Voltmeters of Very Different Sensibilities.

Attach the terminals of the voltmeter of the shorter range to the points A and B respectively, and the terminals of the other voltmeter to the points A and D respectively. Send different currents of suitable, but not necessarily of known, values through the conductor A D. Observe the corresponding readings of the two voltmeters, and remember that the P.D. between the

points A and D is always ten times the corresponding P.D. between the points A and B.

3. Join the voltmeter V (Fig. 89) to be calibrated to the terminals of a conductor *c* whose resistance *o* is known in international ohms. Send different currents in succession through this conductor, and measure the currents with the ammeter A. Observe the deflections of the voltmeter which correspond with each of the currents A_1, A_2, A_3, &c., amperes, and note that they are produced by P.Ds. of A_1o, A_2o, A_3o, &c., international volts.

If the voltmeter V be an electrostatic one, so that no current whatever passes through it, the deflection of the

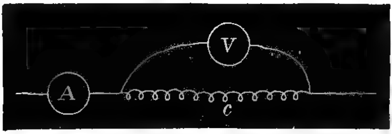


Fig. 89.—Calibrating a Voltmeter by using an Ammeter and One Known Resistance.

ammeter A will measure the true current passing through the conductor *c*. If, however, V be a voltmeter that takes a current, then it must not be forgotten that the current passing through the ammeter is the sum of the currents passing through the conductor *c* and through the voltmeter. The error introduced by assuming that the ammeter measures simply the current passing through *c* will be the smaller the less is the resistance of *c* compared with that of the voltmeter. It will be better, therefore, that *c* should have a comparatively small resistance, and that the necessary P.D. should be produced between its terminals by sending a strong current through it.

If, however, there be a risk that such a current will warm the conductor *c* and so change its resistance, then it is better to join up the apparatus as in Fig. 90.

In that case the resistance that must be used in calculating the P.Ds. set up between the terminals of the voltmeter V is $o + a$ international ohms where

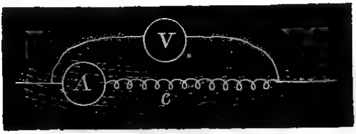


Fig. 90.—Calibrating a Voltmeter by using an Ammeter and One Known Resistance.

o, as before, is the resistance of the conductor c, and a is the resistance of the ammeter A. So that when the currents are A_1, A_2, A_3, &c., amperes respectively, the P.Ds. are $A_1\ (o + a)$, $A_2\ (o + a)$, $A_3\ (o + a)$, &c., international volts.

4. Let B_1, B_2, B_3, &c. (Fig. 91), be binding screws attached to different points of a conductor which may be composed all of one wire of uniform cross-section, or of different pieces of wire of any cross-sections joined up to one another in series. Compare the resistances of the

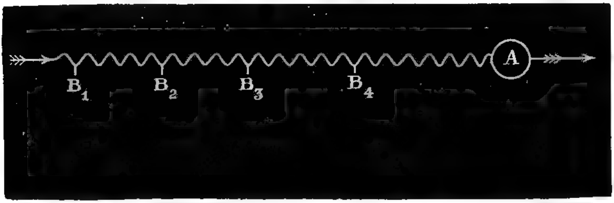


Fig. 91.—Calibrating a Voltmeter by using Several Known Resistances in Series, with One Known Current passing through them.

sections with one another by the method described in § 47, page 177, and compare the resistance of some one of the sections with a standard international ohm, or with some conductor whose resistance is known in

international ohms, then the resistance of each of the sections $B_1 B_2$, $B_2 B_3$, $B_3 B_4$, &c., will be known. Let these resistances be respectively o_1, o_2, o_3, &c., international ohms.

Send a current through the conductor $B_1 B_2 B_3$, and keep the current quite constant at some convenient number of amperes, as measured by the ammeter A. Then the P.D. between any pair of the binding screws attached to different points of the conductor is known in international volts; for example, the P.D. between binding screws B_2 and B_5 is A $(o_2+o_3+o_4)$ international volts. By connecting, therefore, the terminals of the voltmeter to be calibrated with each of the pairs of binding screws in succession a series of deflections is obtained, the P.D. corresponding with each of which is known in international volts.

5. If the voltmeter be a galvanometric one it may be calibrated by measuring its resistance g*, ascertaining the currents A_1, A_2, A_3, &c., in fractions of an ampere that produce the deflections d_1, d_2, d_3, &c. These deflections will then correspond with a P.D. of $A_1 g$, $A_2 g$, $A_3 g$, &c., volts maintained between the terminals of the voltmeter, or with a P.D. of A_1 $(g+w)$, A_2 $(g+w)$, A_3 $(g+w)$, &c., volts maintained between the terminals T_1 and T_2 (Fig. 86) where w is the resistance of the auxiliary wire W placed in series with the galvanometer.

Example 30.—An ammeter of 17 ohms' resistance has been graduated to read milliamperes (thousandths of an ampere) directly. What external resistance must be added to the instrument so that the same scale will measure P.Ds. directly in volts?

If a resistance of 1000–17 or 983 ohms be added to the galvanometer, a P.D. of x volts, maintained between the terminals of the ammeter and resistance combined will

* For the sake of brevity the word *international* will, throughout the remainder of this book, be omitted before the words volt and ohm, but it is to be understood that in all cases where no prefix is mentioned the word *international* is understood.

send x milliamperes through the arrangement, and will, therefore, produce a deflection of x on the scale.

Answer.—983 ohms.

Example 31.—A voltmeter having 2475 ohms' resistance has been calibrated to read off volts. It is desired that a deflection of n divisions shall correspond with a P.D. of $5n$ volts instead of n volts. What external resistance must be added to the voltmeter to obtain the result?

Answer.—4×2475 or 9900 ohms.

53. Voltmeters used as Ammeters.—Any voltmeter, whether electrostatic or of the current type in combination with a constant resistance, can be used and graduated as an ammeter. For, consider the arrangement No. 3, § 52, used for calibrating a voltmeter, and illustrated in Fig. 89. With every current which is measured in amperes with the ammeter A there is a certain deflection of the voltmeter. If, then, these deflections be marked not in volts but with the numbers of amperes as measured with the ammeter, the reading on the scale of V will at any time give the current in amperes passing through it and the conductor *c* together when the two are used in combination as shown. The graduation of the voltmeter scale in amperes will not, however, be correct if the voltmeter be used as a shunt to some other conductor having a different resistance from that of *c*.

The device just described enables a moving coil instrument, such as was described in § 51, through which only a small current can be passed, to indirectly measure any current no matter how large. In such a case, and generally when the voltmeter used as an ammeter is to be portable, the conductor *c* may be placed inside the case of the voltmeter.

It is to be noticed that the combination of voltmeter and conductor *c*, of fixed resistance, can be graduated and employed, as an ammeter, whatever the relative resistances of the voltmeter and the conductor may be. One important advantage, however, is gained by making the

resistance of c very high compared with the voltmeter, and that is the facility for altering the sensibility of the arrangement.

For, suppose that the conductor c of Fig. 89 takes the form of a short, wide strip (Fig. 92), having therefore a very low resistance, and that the voltmeter joined up as a shunt to it has a resistance of a ohms, large compared with that of the strip; further, suppose that a current of A amperes sent through the arrangement as measured by the ammeter A deflects the pointer of the voltmeter to the end of its scale.

Next, let a resistance of a ohms be put in series with the voltmeter (Fig. 92), then it will require twice the P.D.

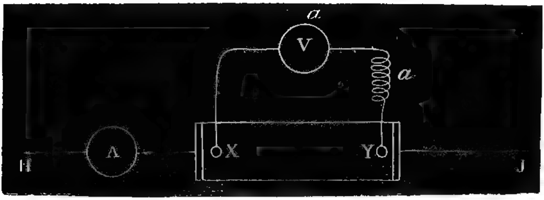


Fig. 92.

to be maintained between the points X and Y to produce the same deflection on the voltmeter as before. Therefore it will require twice the current to flow through the strip, and, since by hypothesis the resistance of the voltmeter is very high compared with that of the strip, the current passing through the voltmeter is inappreciable compared with that flowing through the strip. Therefore twice the current flowing through the strip means practically twice the current in the main circuit H J. In other words, by adding to the voltmeter branch a resistance of a ohms we have halved the sensibility of the arrangement which is used, as an ammeter, for measuring the current in the main circuit H J. And, generally, if a resistance of na ohms be added to the voltmeter branch the

current in H J that produces any particular deflection of the voltmeter will be $n + 1$ times the current required to produce the same deflection when the voltmeter terminals are joined direct to the points X and Y.

Example 32.—A strip of platinoid of resistance 0·017 ohm is shunted with a galvanometer of 305 ohms' resistance in series with a variable resistance. The galvanometer is of such sensibility that a P.D. of 0·5 volt causes a deflection of 270 scale divisions when the resistance in series with the galvanometer is 1,000 ohms. If the scale is a proportional one, what must be the resistance in series with the galvanometer in order that when 10 amperes pass through the strip the deflection shall be 100 scale divisions?

When 10 amperes pass through the strip the P.D. between its terminals is $10 \times 0{\cdot}017$, or 0·17 volt. Therefore the current that this P.D. produces through the galvanometer is $\frac{0{\cdot}17}{305 + r}$ where r is the resistance in ohms to be put in series with the galvanometer. But by hypothesis a current of $\frac{0{\cdot}5}{305 + 1{,}000}$, or 0·0003833, ampere produces a deflection of 270 scale divisions, and therefore, since the scale is a proportional one, a current of $\frac{100}{270} \times 0{\cdot}0003833$, or 0·000142, ampere will produce a deflection of 100 scale divisions. Hence

$$\frac{0{\cdot}17}{305 + r} = 0{\cdot}000142$$

or

$$r = 892 \text{ ohms.}$$

Answer.—892 ohms.

Example 33.—Calculate for the strip and galvanometer referred to in the previous question the resistances that must be placed in series with the galvanometer in

order that 20, 30, and 50 amperes through the strip may produce 100 divisions' deflection.

Answer.

Amperes through Strip.	Resistance in Series with Galvanometer to give 100 Divisions' Deflection.
10	892 ohms.
20	2,089 ,,
30	3,286 ,,
50	5,680 ,,

54. Gold-Leaf Electroscope.—If we desire to measure the P.D. between two insulated bodies which have been electrified by touching them, for example, one with a rubbed piece of ebonite, and the other with a rubbed piece of glass, it would be impossible to employ any form of current voltmeter. For no matter how fine or how long were the wire used in winding the galvanometer, or how large was the resistance of the added wire w (Fig. 86), the flow of electricity which enabled the P.D. to be indicated would at once destroy the very P.D. we desired to measure. An electrostatic voltmeter must, therefore, be employed in such a case, but as there is no difficulty in producing a P.D. of many hundreds of volts by means of rubbed ebonite or rubbed glass, the voltmeter may, for many purposes, be of a much rougher kind than the one already described.

When it is only required to know whether one potential is higher, or lower, than another, or whether the potential of a body is plus or minus, that is to say, whether a positive current would flow from the body to the ground, or from the ground to the body, if they were connected together by a wire, such a *qualitative* test can be conveniently made with a "*gold-leaf electroscope.*"

This instrument, as formerly constructed, had a

variety of faults, but the illustrated description that was given, in the earlier editions of this book, of the proper way to construct a *gold-leaf electroscope*, has induced some manufacturers, at any rate, to cease reproducing instruments possessing the glaring defects of the older types. In the present edition of the book it will be,

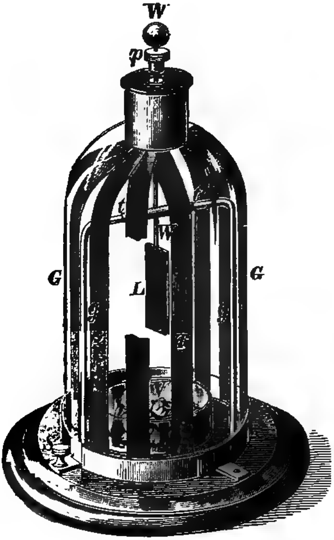


Fig. 93.—Ayrton and Perry's Gold-Leaf Electroscope.

therefore, sufficient to describe the way in which a gold-leaf electroscope may be satisfactorily constructed.

A glass shade G G (Fig. 93) rests on a wooden base, and is covered inside with the *conducting* varnish devised by Mr. Mather and the author (*see* § 58, page 199), or with strips of tin-foil T, placed only just so far apart as is necessary to enable the gold leaves to be easily seen. These strips of tin-foil are bent round the bottom of the

glass shade, and connected electrically with a brass ring, which encircles the outside of the bottom of the glass shade. To this ring three horizontal brass lugs are attached for enabling the shade to be screwed to the wooden base, and to one of them is fixed a binding screw, S, for holding any wire which we wish to electrically connect with the tin-foil coating. Inside the glass shade G G, thin rods of good insulating glass *g*, *g* are cemented into two short brass tubes, or collars, fixed to the base, and the glass rods are joined together at the top by being cemented into a little metallic tube *t t*, carrying the thick wire W W, and the gold leaves L. This wire W passes through the top of the instrument *without touching it*, and may carry at its top a little knob or a little binding-screw. V is a glass vessel containing lumps of pumice-stone soaked in strong sulphuric acid, which absorbs any water vapour in the interior of the electrometer, and thus keeps the glass rods *g*, *g* dry.

When the instrument is not in use the little metal plug or stopper *p* (which is made to slide a little stiffly on the wire W by the hole in the stopper being lined with cork) *should always be pushed down*, and the hole at the top of the instrument thus closed to keep out dust and damp. If this precaution be carefully attended to *on every occasion* that the electroscope is left unused *even for a short time*, and the surface of the glass rods *g*, *g* be initially carefully cleaned, the insulation of the instrument will remain so good, even for a year after the acid has been put on to the pumice-stone, that an electric charge given at any time to the gold leaves will remain practically undiminished by leakage during an hour even on a very damp day.

With a given gold-leaf electroscope *the divergence of the gold leaves depends simply on the* P.D. *between the gold leaves* L *and the tin-foil coating* T. For the gold leaves constitute a flexible needle corresponding with N in Fig. 83, page 164, and the tin-foil coating is the stationary inductor (called I in the same figure) to which the gold leaves are attracted with a force depending on the P.D.

between them and the tin-foil coating. This attraction causes the leaves to diverge, and to be, therefore, lifted; the angle of divergence for any particular P.D. being such that the attractive forces exactly balance the controlling forces introduced by the weight of the leaves which are slightly lifted from the vertical position. A gold-leaf electroscope is, therefore, a "*deflectional gravity-voltmeter*."

55. Sensibility of Gold-Leaf Electroscopes.—As already explained, gold-leaf electroscopes are frequently used merely as qualitative instruments, but employing method No. 2, § 52, page 184, a gold-leaf electroscope may be calibrated, if desired, by comparison with the zero electrostatic voltmeter (Fig. 83). The law connecting the divergence of the leaves with the P.D. set up between them and the case depends on three things (1) the length of the leaves, (2) the weight per square inch of the leaf, and (3) the size of the case. If the length of the leaves and the size of the case be fixed, it follows, from our original definition of what is meant by one P.D. being twice another, that the P.D. required to produce any particular divergence is simply proportional to the square root of the weight of the leaf per square inch.

Specimens of gold leaf from different gold-beaters appear to vary as much as 20 per cent. in the weight per square inch, but the lighter the leaf the lower will be the price, provided that it is not much below 40 shillings per book of 1,000 leaves, in which case cheapness may result from the impurity and not from the thinness of the gold. At 40 shillings per thousand sheets of 22 carat gold, the sheets being $3\frac{1}{4}$ inches square, the weight per square inch is about 0·013 grain. With leaves, each $2\frac{1}{4}$ inches long, cut from this quality of material and suspended in a conducting case $4\frac{3}{4}$ inches internal diameter, a divergence of about 56° is obtained for a P.D. of 1,000 volts, set up between the leaves and the case. Reducing the length of the leaves to $1\frac{1}{2}$ inch increases the divergence for the same P.D. to 60° and in addition it renders the various divergences between the leaves

in degrees more nearly directly proportional to the P.D. in volts.

The calibration curve can also be rendered much more nearly a straight line by increasing the diameter of the case, but this has the counterbalancing effect of diminishing the sensibility for the same leaves, as may be seen from the following table:—

LEAVES EACH $1\frac{1}{2}$ INCH LONG. P.D. OF 1,000 VOLTS MAINTAINED BETWEEN LEAVES AND CASE.

Internal Diameter of Case in Inches.	Divergence between Leaves in Degrees.
$4\frac{3}{4}$	60°
6	54°
8	48°
10	44·5°

Plotting a curve to represent the above four pairs of values and continuing the curve forwards, it is seen that the divergence rapidly approaches 40°, which means that however large may be the diameter of the conducting case the divergence will be about 40° when a P.D. of 1,000 volts is maintained between this case and a pair of leaves each $1\frac{1}{2}$ inch long cut from a 40-shilling book of 22 carat gold leaf.

With the leaves each $1\frac{1}{2}$ inch long the case can be made as narrow as $4\frac{3}{4}$ inches in diameter and still nearly direct proportionality of P.D. and divergence be obtained up to 70° whatever be the weight of the leaves. This is the size of leaf and case, therefore, that may be conveniently adopted, and the constant of instruments so constructed will vary from about 6° per 100 volts to 6° per 225 volts, as the material used in making the leaves costs 40 shillings per 1,000 sheets, or a few pence when the material is "Dutch metal."

For measuring P.Ds. of 2,000 volts, or higher, such as are now maintained between the underground mains with certain electric light systems, the leaves may be conveniently made out of lead foil instead of gold-leaf.

56. No Force Inside a Closed Conductor Produced by Exterior Electrostatic Action.—This fact may be experimentally illustrated with the cage composed of wire gauze c c (Fig. 94) mounted on an insulating stand, which will be found described in detail at the end of § 65, page 216. Inside the cage are suspended one pair of pith balls by means of silk fibres which are fairly insulating, and a second pair by pieces of cotton, which is relatively a conductor. Outside the cage are suspended one or more pairs of pith balls by silk fibres or by pieces of cotton. If now the cage be electrified, the pith balls outside the cage will diverge as in the figure, but those inside give no evidence whatever of any electric force even when the cage is so highly electrified that sparks can be drawn from it. If the openings in the cage be very large, it may be possible to produce some slight effect inside by powerful outside electrostatic action, but if the meshes of the wire-gauze be very small, or, better still, if the sides of the metal box be continuous throughout, then no matter

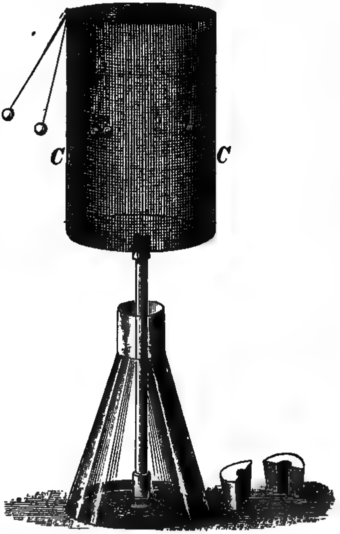


Fig. 94.—Apparatus for Proving that the Electrostatic Force inside a Closed Conductor is Nought.

how thin they may be, or how sensitive may be the apparatus for detecting electric force that is placed inside the box, no effect whatever can be produced on the apparatus by external electrification. *A continuous metal box then, even if its sides be made of the thinnest gold leaf, completely screens bodies inside it from external electrostatic action.*

If, however, a current be flowing along the surface of a long hollow conductor, then, whether or not the ends of the hollow tube be closed metallically so as to make it into a box, electric forces are exerted inside the tube.

57. Potential due to Exterior Electrostatic Action is Uniform at all Points inside a Closed Conductor.— This result follows from the conclusion arrived at in the last section, for, if it be impossible by means of external electrostatic action to produce a force on any measuring instrument placed inside a closed conductor, no voltmeter could give evidence of any P.D. inside the conductor produced by external electrostatic action. Hence *the potential at all points inside a conductor, subjected to outside electrostatic action, must be uniform, and have the same value that it possesses at a point on the surface of the conductor.*

If the conductor has a hole in its surface — as large, for example, as the opening at the top of a coffee-pot—the preceding will still be true, except for points in the space just inside the pot close to the opening, where the potential will differ somewhat from the uniform potential inside the pot. When, however, the surface of the conductor is as continuous as the wires of a meat-cover made of wire gauze, or even as continuous as the wires of a bird-cage, the potential is practically uniform at all points inside.

If then there be a number of electrical apparatus inside a metallic box, or inside a metallic room, some, or all, of the apparatus being connected with the box and some, or all, being insulated from it, *the internal distribution of potential will be quite independent of the potential of the box relatively to the earth, and will remain quite*

unaffected if the potential of the box be raised or lowered by outside action. Altering the potential of the box by outside action alters the potential of every object inside it by exactly the same amount, and, therefore, leaves the P.D. between any two objects inside the box unchanged.

The preceding only applies when the electric forces exerted on the box are produced by electricity at rest. For when a current is made to flow along a metal tube, the potential varies from point to point in the space inside the tube in the same way that it varies from point to point along the metal of the tube itself through which the current flows.

58. Voltmeters must be Enclosed in a Conducting Case.—In order that a voltmeter may correctly measure the P.D. between the needle and the inductors, it is necessary that no electric force shall be exerted directly on the moving system by bodies external to the voltmeter. The needle and the inductors of an electrostatic voltmeter must, therefore, be surrounded by a screen constructed of conducting material. Such a screen can be constructed of strips of metal foil stuck fairly close together *inside* the glass shade which covers up the instrument (Fig. 93, page 192), and the screening can be improved by connecting the vertical strips together by horizontal strips of metallic foil, as in Fig. 95. When such a coated glass shade is used with a gold-leaf electroscope, it acts both as inductors and as screen.

A good test of the power of any such coated glass shade to act as an electrostatic screen consists in joining the terminals of the voltmeter together by a piece of wire, and bringing a highly excited ebonite rod close to the voltmeter, when, if the screening is practically perfect, no deflection of the moving system will be possible.

Nor if the screen acts well will a deflection of the moving system be able to be produced by placing a voltmeter, whose terminals are connected together in the way just described, on an insulating stand, and electrifying the voltmeter as a whole so highly with an "*electrical*

machine," that sparks can be drawn from any part of the voltmeter on approaching the finger. For the instrument measures the P.D. between its terminals, and this P.D. must be zero when the terminals are joined by a wire, whether the instrument be electrified or not.

It will be found, however, that when the metallic foil is stuck on the glass shade, as indicated in Figs. 93 and 95, so that the moving system can be fairly well seen at a distance through the openings between the strips, the screening action, although considerable, is by no means complete, and that when the area of the metallic coating becomes sufficiently large compared with the area of the glass as to render the screening practically perfect, there is considerable difficulty in seeing the moving system sufficiently well to enable small changes in the deflection to be observed at a distance.

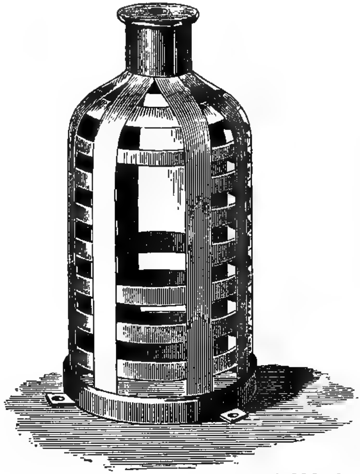


Fig. 95.—Metallic Screen for a Gold-leaf Electroscope.

Mr. Mather and the author, therefore, experimented on methods of coating the whole of the interior of the glass shade with a *transparent* varnish that should be sufficiently conducting to act perfectly as an electrostatic screen, and yet hard enough that the inside of the glass could be cleaned when desired without risk of the varnish being rubbed off. And this, they find, can be satisfactorily accomplished in either of the following ways :—

Method No. 1.—Dissolve $\frac{1}{4}$ ounce of transparent gelatine in 1 ounce of glacial acetic acid by heating them together in a water bath at 100°C. To this solution add half the volume of dilute sulphuric acid, which has been prepared by mixing 1 part of strong acid with 8 of distilled water by volume, and apply the mixture while still warm to the glass shade, which should be previously polished and warm. When this film has become very nearly hard apply over it a coating of Griffith's anti-sulphuric enamel, the chief ingredient of which is resin dissolved in fusel oil.

Method No. 2.—Thin the gelatine solution, prepared in the manner previously described, by the addition of acetic acid (say, 2 volumes of acid to 1 of solution), and after polishing the glass, float the thinned solution over the glass cold. Drive off the excess of acetic acid by warming, allow the glass to cool, and repeat the floating process, say, twice. Thin the anti-sulphuric enamel by the addition of ether, and float it over the gelatine layer applied as just described. Expel the ether by heating, and apply a second layer of this thinned anti-sulphuric enamel.

It is advisable to varnish the inside of the glass shades or glass fronts, not merely of electrostatic voltmeters, in one of the ways just described, but of current voltmeters, ammeters, or indeed of any instrument where the electrification of the glass produced by cleaning it on a dry day might cause a deflection of the pointer of the instrument—a cause of error that has been noticed with electrical measuring instruments placed in hot dry engine-rooms of electric-light stations.

59. The Potential of a Conductor.—"*The potential*" of a conductor is an abbreviation for the P.D. between the conductor and the earth. To measure this we must connect one terminal of the voltmeter by a wire to the conductor, and the other terminal to a system of gas-pipes, or better, to a system of water-pipes. If *the potential* of a body be high, it can be roughly

measured in a moist country like England by simply connecting the knob of a gold-leaf electroscope to the body without connecting the screw s (Fig. 93) metallically to the gas- or water-pipes of the building, since the film of moisture which condenses on the dusty wooden base of the electroscope will make a more or less good electric connection between the screw s and the ground unless special precautions be taken to insulate the wooden base from the ground.

When, however, *the potential* is smaller, and it is necessary to use a more sensitive electrostatic voltmeter, for example, that illustrated in Fig. 83, page 164, and generally in all cases when the measurement is made by means of a current voltmeter, it is absolutely necessary to make a good electric connection between one terminal of the voltmeter and the earth if we desire the instrument to accurately measure *the potential* of the other terminal (or *the potential* of a conductor connected with the other terminal) relatively to the earth.

If the voltmeter used to test *the potential* of a conductor be an electrostatic one, or a current voltmeter with a *soft iron* needle, the instrument will measure the value in volts, but it will not indicate whether the potential of the conductor is positive or negative (§ 41, page 162). Hence, if *the potentials* of two conductors, so measured, be 160 and 70 volts, the P.D. between the conductors (which can, of course, be measured by connecting the terminals of a voltmeter to them respectively) may be either 90 or 230 volts depending on whether the potentials of the two conductors are of the same or of different sign.

60. The Potential of a Body Depends Partly on its Position relatively to other Bodies.—The body whose potential is being tested may be the knob of a gold-leaf electroscope, then its potential is altered when an electrified body, such as a piece of rubbed ebonite, is brought near the knob, as evidenced by the change in the divergence of the gold leaves. Or the body under test may

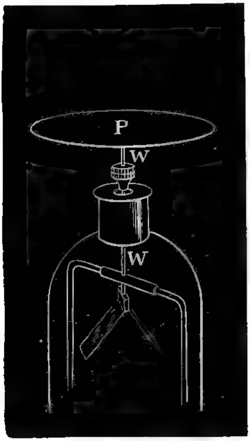


Fig. 96.

Fig. 97.

be a metal plate P, carried by the wire W W of the electroscope (Fig. 96). Electrify this plate by touching it with, say, an excited rod of ebonite, and remove the rod. Then the divergence of the leaves, which measures the potential of the plate, will be found to alter either when an electrified body is brought up to the plate, or even when an un-

Fig. 98.

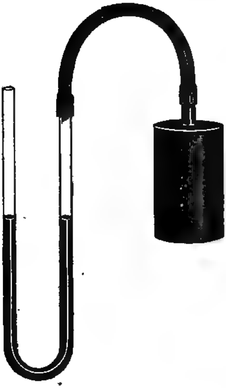


Fig. 99.

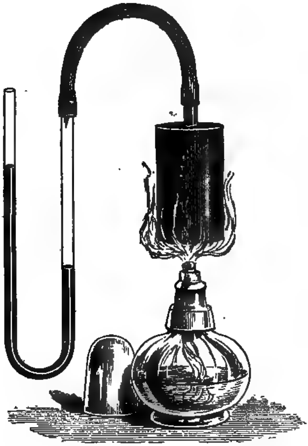


Fig. 100.

electrified body like the hand is approached (Fig. 97). If an insulated unelectrified metal plate, M (Fig. 98), be brought up by means of the insulating handle H, there will be a small diminution in the potential of P, and this diminution will be much increased if the plate M be connected with the earth. Indeed, if M, still connected with the earth, be brought very near to the plate P, and parallel to it, the potential of P will be reduced almost to zero, even if the plates be not made to touch one another.

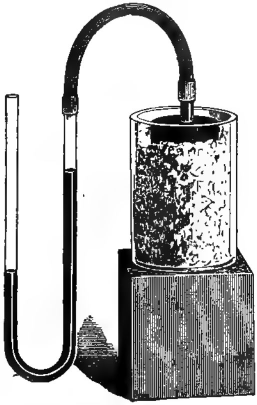


Fig. 101.

The change which is produced in the potential of a body by altering its position

relatively to other bodies is analogous with the change that occurs in the pressure of gas in a closed vessel when the temperature of the vessel and the gas it contains is altered, as illustrated by Figs. 99, 100, and 101. In Fig. 99 the pressure of the gas in the vessel is the same as the outside atmospheric pressure, and the liquid in the two limbs of the manometer is at the same level. In Fig. 100 the vessel is warmed and the internal gaseous pressure exceeds that of the external atmosphere, corresponding with the increase of the potential of an insulated body caused by bringing a positively charged body up to it. While in Fig. 101 the pressure is lowered because the vessel is cooled by being immersed in ice, corresponding with the diminution of the potential of an insulated body by bringing a negatively charged body up to it.

61. The Potential of a Body Depends Partly on its Size and Shape.—Supported by the insulating handle H (Fig. 102) is a little spring roller blind which can be pulled out by means of the silk thread T, the spring inside the roller R R causing the blind to roll up again when the thread T is released. Each side of the blind has tin-foil stuck on it to make the blind conducting. If now the blind be electrified when shut up, and its potential be measured, for example, by connecting the binding screw B at the end of the roller with the knob W of an electroscope, it will be found that the potential of the roller greatly diminishes as the blind is pulled out by means of the silk thread T. The original value of the potential will, however, be regained on allowing the blind to roll up, if the glass rod and the silk thread insulate well.

The change in the potential of a body produced by altering its size and shape is analogous with the change that occurs in the pressure of a gas in a closed vessel when the internal volume of the gasholder is altered without altering its temperature. Figs. 103, 104, and

105 represent a glass tube T permanently closed at the top and connected at the other end with a flexible indiarubber tube I containing mercury, the level of the top of which is seen in the open glass tube *t*. In Fig. 103 the internal pressure of the gas is the same as the

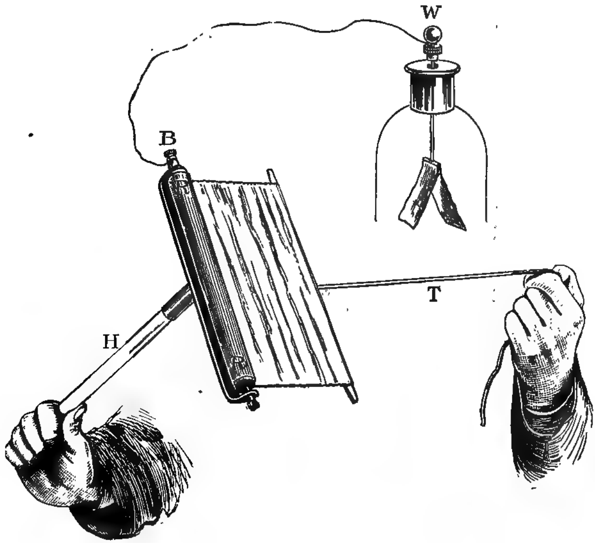


Fig. 102.—Apparatus for Proving that the Potential of a Body depends on its Size and Shape.

external atmospheric pressure, and the confined gas has a certain volume V. In Fig. 104 this volume has been diminished by raising the indiarubber tube and increasing the pressure on the confined gas, this pressure now exceeding the atmospheric pressure by an amount depending on the height that the mercury in the open glass tube *t* is above the level in the closed tube T. In Fig. 105 the volume of the confined gas is larger than V by the

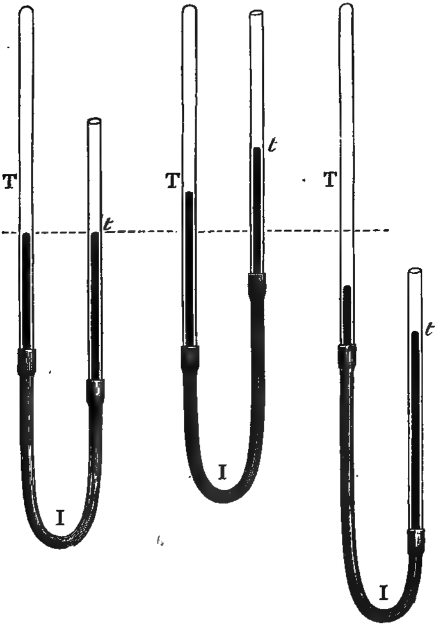


Fig. 103. Fig. 104. Fig. 105.

pressure on it having been made less than the atmospheric pressure by the lowering of the open glass tube *t*.

62. The Potential of a Body Depends Partly on a Third Condition: the Quantity of Electricity.—When the pressure of the gas in a vessel has been changed by varying the temperature of the gas, or by altering the internal volume of the vessel, the old values of the pressure can be regained by bringing the gas back to its original temperature and volume. Similarly when the potential of a body has been changed by altering the position of the body relatively to other bodies, or by

varying the shape and size of the body itself, the old value of the P.D. can be regained by replacing the bodies in their original positions relatively to one another, or by restoring the body to its original shape and size.

But there is a third way in which the pressure of the gas in a vessel can be varied without altering either

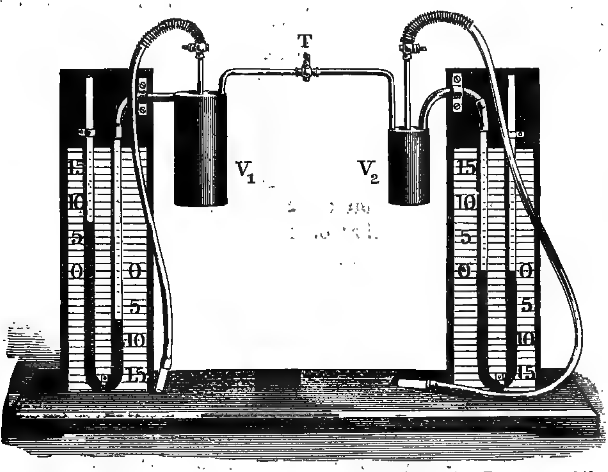


Fig. 106.—Apparatus for Illustrating the Analogy between the Pressures of the Gas in Two Vessels and the Electric Potentials of Two Conductors.

the temperature of the gas or the volume of the vessel, and that is by putting a greater or a less weight of gas into the vessel. For example, the gas in the vessel v_1 (Fig. 106) has a greater pressure than the atmosphere, while that in the vessel v_2 has the same pressure as the atmosphere. If now we open the tap T in the pipe connecting the two vessels, there will be a rush of gas from the vessel v_1 into the vessel v_2, the pressure in the former will fall, while that in the latter will rise. And we cannot, of course, bring things back again to

their original condition by simply closing the tap T, since some of the gas went out of the vessel V_1 when the tap was opened.

In the same sort of way if there be two insulated conductors, one A having a potential $+V_1$ volts, and the other B a potential nought, and if an electric connection be made between them by means of a wire, the potential of A will fall to some value $+V_2$ volts, less than V_1, while the potential of B will rise to the same value $+V_2$. And matters cannot be brought back again to their original condition by simply taking away the wire, since (using the figurative language which is derived from experiments with fluids, such as the one just referred to with the two vessels containing gas) a "*quantity of electricity*" has flowed through the wire from the conductor A to the conductor B, so that the "*electric charge*" on A has become smaller, and that on B larger than before.

Again, on opening the stop-cock at the top of either of the vessels V_1 or V_2 (Fig. 106), air can be blown into, or sucked out of, either of these vessels through the tube provided with a mouthpiece which is attached to each vessel, and the pressure of the gas in the vessel can be made to exceed, or to be less than, that of the atmosphere. So, experiment shows that a clean dry rod of ebonite can either be so highly electrified by a prolonged rubbing with a piece of clean dry flannel that the mere presence of the ebonite at the distance of a foot or two from the knob of a gold-leaf electroscope raises its potential to over 1000 volts; or, on the contrary, if the rod be only just touched at one spot with the flannel, or if no precaution be taken on a damp day to dry both the flannel and the rod, the rod after being rubbed may be held close to the knob of the electroscope or be actually made to touch it without the leaves being raised to a sufficiently high potential to more than just visibly diverge.

Hence just as the pressure of the compressed gas in a gas-bottle may vary from many pounds per square

inch, when the bottle has just been filled, to a fraction of a pound per square inch, when nearly all the gas has been used up, the potential of the same rod of ebonite may have very different values; and the analogy leads us to say that the potential of the ebonite is changed by more or less electricity being given to it, or by a change being produced in its *electric charge.*

It must, however, be carefully remembered that while we can conceive of the gas apart from the iron bottle containing it, just as we can think of the water filling a pint pot apart from the pot itself, we have no experience of the existence of electricity apart from the body that is said to be electrified. In fact, while we know the effects that can be produced by a so-called electrified body we have no certain knowledge of the physical change, if any, that is produced in a body by electrifying it.

It would, therefore, be more correct to speak of the "*amount of a body's electrification*" than of the *quantity of electricity* in, or on, the body; but, just as it is convenient to talk of an electric current as if it had an independent existence, apart from the wire through which it is said to be flowing, so it is convenient to speak of an *electric charge* or a *quantity of electricity* as if electricity had an independent existence.

We see then that just as the pressure of a gas (say oxygen) in a gasometer can be varied by—

1. *Altering the mass of the oxygen in the gasometer.*
2. *Altering the size of the gasometer without altering the mass of oxygen in it.*
3. *Altering the temperature,*

so the potential of a body can be varied by—

1. *Altering the quantity of electricity in it.*
2. *Altering the size, or shape, of the body without altering the quantity of electricity in it.*
3. *Altering its position relatively to other bodies.*

63. No Electricity at Rest Inside a Conductor.—This experiment can be easily tried by lowering any

insulated charged ball, by means of the silk thread or glass rod which supports it, into a somewhat deep pot like a coffee pot and touching the pot *inside near the bottom*, then whether the pot be insulated or not, previously charged or not, the ball will come away entirely discharged, so that if its potential be measured after it has been removed some distance away from the pot it will be found in all cases to be zero.

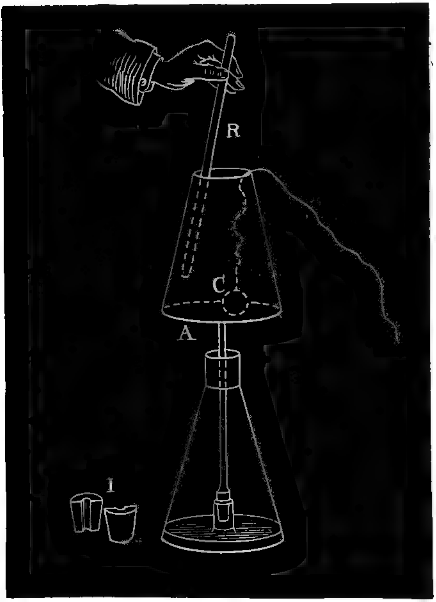


Fig. 107.—Apparatus for Proving that there is no Electricity at Rest inside a Conductor.

And the same conclusion is found to be practically true if the ball be touched against the *bottom of the inside* of a comparatively shallow metal teapot even when the lid is open, or against the inside of a box made of wire gauze even with fairly wide meshes.

If, however, the part of a charged insulated pot against which the insulated ball is touched, although inside, be very near the mouth of the pot, or be outside the pot, then the ball will *not* come away discharged.

Further, even if it be touched against the bottom of the inside of even a deep narrow pot, and there be also inside the pot a body whose potential differs from that of the pot, the ball will *not* come away discharged. For example, if the pot A be insulated and charged either positively or negatively, and a metal rod R (Fig. 107) be held in the hand with its end projecting some way into the pot, but not in contact with it, an insulated ball C, touched against any part of the inside of the pot will come away with an electric charge of the same sign as that initially given to the pot. In such a case, what is commonly called the inner side of the pot becomes electrically part of the outside as regards the rod; so that the middle of the upper side of a horizontal metal plate supported on a glass rod (Fig. 96, page 202) and far removed from other bodies, is electrically more the inner side of the plate than is the interior of the bottom of the pot (Fig. 107) with a very narrow mouth, but into which a metal rod R has been inserted.

If, however, we use the expression *inside* to refer to those parts of a conductor, whether on its surface or inside the material, which cannot, so to say, be seen by other bodies in the neighbourhood, we may conclude that *electricity at rest does not reside inside a conductor.* As far, then, as the electrostatic action of an electrified conductor on bodies outside it is concerned, it is quite immaterial whether the conductor be made of solid or hollow metal, or whether it be constructed of wood or of plaster, and be simply coated outside with tin foil or gold leaf.

64. Comparing Quantities of Electricity.—The next point to consider is what shall be meant by one quantity of electricity being twice another, three times another, &c. Analogy with a quantity of gas suggests that we should define the quantity of electricity on a conductor as being proportional to the potential of that conductor when it is fixed in size, shape, and in position relatively to other bodies. It is easy to fulfil the condition

that the conductor shall be fixed in size and shape, but if the conductor were, say, a ball, supported on a glass rod in the middle of a room it would be most difficult to satisfy the condition that the position of the conductor should be fixed relatively to all others unless it were surrounded by a closed conducting screen insulated from it but definitely fixed relatively to it.

Consequently we arrive at the following definition:—*When a given conductor* A *is entirely surrounded by another given conductor* B (Fig. 108), *the quantity of electricity on* A *is directly proportional to the* P.D. *between* A *and* B *as long as the position of* A *relatively to* B *is absolutely fixed.*

Fig. 108.

The fixity of the relative positions of A and B is as important as the fixity of the sizes of A and B, since the mere motion of A inside B would change the P.D. between A and B, although the charge on A remained quite constant.

If the weights of gas in two vessels of unknown size and of unknown temperature have to be compared, no information can be derived from a measurement of the difference of pressure between the gas in the two vessels, nor from separate measurements of the pressure of the gas in each vessel relatively to that of the atmosphere. So in the same way if the charges on two conductors C and D have to be compared, this comparison cannot be made by measuring the P.D. between C and D, nor by measuring the potential of C and D separately, whether C and D are inside B (Fig. 108) or not.

To compare the weights of the same kind of gas in different vessels at different temperatures by a simple pressure test, we must successively *entirely* empty the

gas in each vessel into a third vessel whose size is constant, and which, with the gas inside it, is brought in each case to exactly the same temperature: then the pressures of the gas in this third vessel in the two cases will be directly proportional to the required weights. This method would, however, be a very difficult one to employ on account of the time it takes, with any known form of air pump, to remove even the greater part of the gas from one vessel into another, not to mention the impossibility of completely performing the operation. With electricity, on the contrary, it is quite easy to completely discharge any conductor C into another conductor A, if C can be introduced *into* A and made to touch A on the *inside* at some distance from the opening, for, as explained in § 63, C will under these circumstances be entirely discharged.

If then we construct the conductor A and its conducting screen B in the form of two deep pots, as shown in Fig. 109, the inner one being insulated from the outer and the outer one insulated or not from the earth, the electric charges in any conductors C, D, &c., which can be introduced into A, may be compared. For all that has to be done is to put first one of the bodies C into A, by means of the silk thread or glass handle that supports C, touch A at the bottom, and measure the P.D. between A and B by means of an electrostatic voltmeter. Remove C and connect A and B metallically for a moment to bring them to the same potential; insulate A, next introduce another of the bodies D into A, touch A on the inside and again measure the P.D. between A and B. Then the values of the P.Ds. thus obtained will, as previously explained, be directly proportional to the charges in C and D.

Except for the purpose of obtaining more room inside A there is no necessity for removing C out of A before the insertion of D, since the moment that C has touched the inside of A it is entirely discharged, and produces no further electrical action.

65. Quantity of Electricity Produced by Rubbing Two Bodies Together.—When a charged body, for example C, is introduced into an insulated conductor such as A (Fig. 109), it is noticed that *after* C *has been lowered so far into* A *that it is well under the sides of* A, *when, in fact,* C *cannot be easily seen from outside* A, *no change in the potential of* A *is produced by further lowering* C, *or by moving* C *about inside* A, *or even by touching* C *against the inside of* A. In order then to measure the charge in C it is not necessary to discharge C into A, and all that need be done is to place C well inside A and observe the P.D. between A and B. The possibility of measuring the charge in C without having to discharge C is of great importance, as it enables the measurement to be carried out as easily whether C be an insulator or a conductor, whereas if C had to be thoroughly discharged into A the measurement would become impracticable when C were composed of any highly insulating material.

Consequently the apparatus seen in Fig. 109 may be conveniently employed for testing the amounts of positive and negative electricity that are simultaneously produced when two bodies are rubbed together. E and F (Fig. 109), are respectively discs of ebonite and wood, the latter being covered with cat's fur. The ebonite is a good insulator, while the cat's-fur and the wood are but poor insulators; by mounting, however, both discs on long thin insulating glass handles they may be rubbed together without any of the electricity produced by the rubbing being lost, provided first that the glass rods are clean and quite dry, second, that they be held at their extreme ends and are touched nowhere else with the hands.

When either of the discs, after they have been rubbed together, is held inside A the gold-leaf electroscope indicates a P.D. between A and B, with this difference, however, that when it is the ebonite disc that is inside A the divergence of the leaves can be increased by holding near the knob of the electroscope, or near the

wire connecting the knob with A, a piece of clean dry ebonite, that has been previously rubbed with flannel, whereas the divergence produced when the excited disc

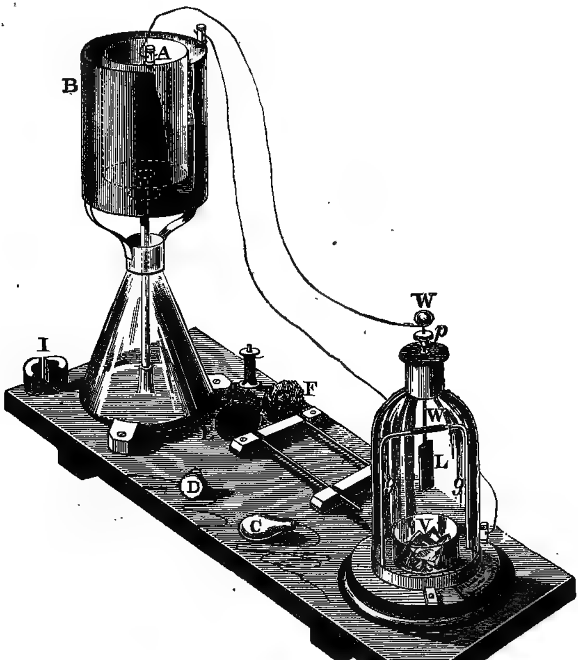


Fig. 109.—Apparatus for Comparing Quantities of Electricity.

of cat's-fur is inside A is diminished on the approach of the ebonite rod. The electrification of the ebonite and cat's-fur discs is therefore of opposite sign. But more than that, experiment shows that if, after the discs have been rubbed together, they be both inserted into A, then,

whether they are touching one another or not, or whether one or both be touching A, the P.D. between A and B remains absolutely the same as it was before the insertion of the discs, no matter how sensitive be the electrostatic voltmeter used to detect any change in the P.D. Hence we conclude that *the charges of electricity given to the ebonite and the cat's-fur in the operation of rubbing them together are not only opposite in kind, but are exactly equal in amount.*

As the quantities of positive and negative electricity that can be produced by rubbing *small* pieces of cat's-fur and ebonite together are not large, it may be necessary to remove the outer conducting pot B (Fig. 109) before making the experiment described above. The removal of this screen B does not lead to error in this case because we are not measuring the quantities of electricity on the cat's-fur and on the ebonite *separately*, but merely proving that the two quantities *together* neutralise each other's effect on the insulated pot A.

Before trying the preceding experiment it is well to make sure that there is no residual charge of electricity left on the ebonite disc from some previous rubbing. This can be ascertained by seeing whether the insertion of the disc into A sets up a P.D. between A and B. If it does, the ebonite disc should be *de-electrified* by being passed backwards and forwards through the flame of a spirit-lamp.

The insulating stand used in this, and in the preceding experiments, consists of a vessel, made of any kind of glass, with a tubulure, or collar, of glass fixed to its base in the centre. This tubulure is ground inside like the neck of a glass-stoppered bottle, and the ground end of a *clean* rod of a *highly insulating* glass fits into this tubulure in the same way as a glass stopper fits into a bottle. Various pieces of apparatus, for example, the wire cage, C C, in Fig. 94, page 196, the plate P in Fig. 96, page 202, the pot A in Figs. 107 and 109, &c., can be mounted on the top of this glass rod by means of a little bit of metal tube, which is soldered to the bottom of the

cage, plate, and pot, and which is of such a diameter that it slips somewhat tightly over the top of the rod.

Before the rod is inserted a small quantity of *strong* sulphuric acid is poured into the glass vessel; this, resting on the expanded bottom of the vessel, exposes a large surface for absorbing moisture from the air and keeps the rod *artificially dried.* Whenever this insulating stand is not required for use, *even for a short time*, the split india-rubber stopper I, seen resting on the base of the apparatus in Figs. 94, 107 and 109, should *always* be inserted in the neck of the glass vessel *to keep out dust and damp.*

The advantages of this insulating stand, which was devised by the author for experiments on bodies at high potentials, are:—

1. The rod can be easily taken out and cleaned. To clean such a rod hold it by the end, and wash it by means of a clean brush with soda and warm water to remove the dirt; then rub it with another brush while a stream of warm ordinary water flows over it, to remove the soda; and lastly, let a stream of distilled water flow over it to remove the trace of salt which is dissolved in ordinary water. The rod should be dried before a fire; or, better, by being hung up under a glass shade, or in some confined space free from dust, in which there is a vessel containing a little strong sulphuric acid. *On no account dry the glass rod by rubbing it with a cloth, nor touch it with the fingers except at the extreme end.*

2. The rod may be made of dense flint glass which insulates *well,* while the vessel may be made of any kind of glass that can be easily, and, therefore, cheaply blown, without reference to its insulating qualities.

3. As the rod is easily taken out, the sulphuric acid can be put into the vessel without splashing the rod; or the old acid, after it has become weak by absorbing water-vapour, may be emptied out, and fresh acid put in without fear of dirtying the rod. This it would be difficult to do, even with another opening in the vessel, if the rod were immovable.

66. Conduction and Induction.—A conductor can be electrified either by the *transfer* of electricity between it and another conductor, or merely by an *alteration in the distribution* of its electric charge, without any transfer of electricity between this conductor and any other body. In the former case the body is said to be electrified "*by conduction*" or "*conductively*," in the latter "*by induction*" or "*inductively*." Loading or unloading a ship would be analogous with electric *conduction*, while shifting some of the cargo from one part of the ship to the other would be analogous with *induction*.

When a body C charged with a quantity of electricity $+Q$ (measured in terms of a unit which will be described in Vol. II.) is introduced into a conductor A, there is induced on the inner side of A, as was first shown by Faraday, a charge of electricity equal to $-Q$, that is, exactly equal in amount to that on C, but opposite in sign. If A be insulated from the earth (Fig. 109), there is also *induced* on the outer surface of A a charge equal to $+Q$. If, further, A be entirely surrounded with a conductor B, there is induced on the inner surface of B a charge of $-Q$.

The P.D. between C and A depends on the charge on C, the sizes of C and A, and on their positions relatively to one another. The P.D. between A and B depends on the charge on the outer surface of A, the sizes of A and B, and on their positions relatively to one another. As long as C remains inside A these two P.Ds. are quite independent of one another, for by moving C about inside A the P.D. between C and A will be changed, but, as we have already seen, not that between A and B. If C be connected with A by means of a wire or by touching C against A, the charge of $+Q$ on C and of $-Q$ on the inner surface of A will neutralise one another; the P.D. between C and A will be destroyed, but the charge of $+Q$ on the outer surface of A and the charge of $-Q$ on the inner surface of B will exist as before, and, as already seen experimentally, the P.D. between A and B

will be unaffected. On the other hand, if A and B be connected with a wire instead of A and C, the charges on the outer side of A and on the inner side of B will neutralise one another; there will no longer be any P.D. between A and B, but the charges on C and on the inner surface of A will retain their former values, and so also will the P.D. between A and C.

A body charged positively, such as a dry clean rod of glass that has been rubbed with dry clean silk, gives, or tends to give, a positive potential to everything in the neighbourhood; that is to say, if the excited glass be brought near any previously uncharged conductor and that conductor be earthed, more or less positive electricity must flow from the conductor to the earth, or negative electricity from the earth into the conductor, in order to bring it to zero potential. For example, if the rod be brought near the insulated knob of a gold-leaf electroscope the outside tinfoil coating of which is uninsulated (Fig. 110), the gold leaves will diverge, indicating that the knob has a positive potential relatively to the coating. As a matter of fact, the knob of the electroscope is charged negatively, while the gold leaves are charged positively, but the potential of the knob and of the leaves relatively to the insulated coating is positive. In the Figs. 110, 111, 112, 114, 115, 116,

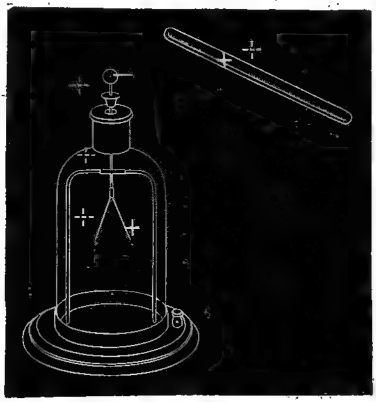


Fig. 110.—Excited Glass Rod brought near an Electroscope. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

117, and 118 the sign of the charge is indicated with *continuous* lines, while that of the potential is shown by *dotted* lines.

If now, without removing the glass rod, the knob be electrically connected with the tin-foil coating by means of a wire (Fig. 111), the potential of the knob and leaves will become zero; there will be no charge on the leaves, but a still larger negative charge will be induced on the knob than before. Removing the wire which connects the knob with the coating will produce no change provided that the position of the glass rod and its electrification both remain exactly as before, the gold leaves and the knob will still be at zero potential, the leaves will be uncharged, while the knob will possess a negative charge.

Fig. 111.—Interior of Electroscope Earthed. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

The knob and gold leaves may now have either a positive or a negative potential given to them by moving the glass rod nearer to or farther away from the knob, the latter case being shown in Fig. 112; and if the rod be replaced in its original position the potential of the knob will again become zero, unless some of the charge on the excited rod has leaked away. If the rod be removed altogether, the leaves and knob of the electroscope are left with a negative potential.

A negative charge and a negative potential would be given to the leaves if a rubbed ebonite rod had been brought simply into contact with the knob W of the electroscope; but, unless the ebonite were rather highly electrified, it would be found that, in consequence of ebonite being a good insulator, but little charge would be given up to the knob by mere contact with one spot of the rod. On the other hand, if the rod were highly electrified, its action, when close to the knob, would be so great as to certainly tear the gold leaves apart.

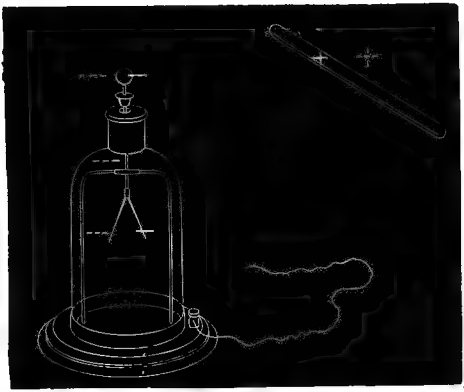


Fig. 112.—Earthing Wire Disconnected and then Excited Glass Rod Removed. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

If we desire to electrify the leaves of a gold-leaf electroscope positively, the most convenient method is to act inductively with a rubbed stick of ebonite in the following way:—(1) connect the knob of the electroscope with the tin-foil coating by means of a wire, or simply place one's finger on the knob; (2) bring the rubbed ebonite near to the knob; (3) withdraw the wire or finger from the knob; (4) lastly, take away the rubbed ebonite. The leaves are now positively charged, and have a positive potential relatively to the tin-foil coating.

67. Testing the Sign of the Electrification of a Body.—Use the body to give the leaves of a gold-leaf

electroscope a small inductive charge. Then bring up a rod of ebonite that has been highly electrified by rubbing it with cat's-skin or cloth—if it be highly electrified it will crackle while it is being rubbed; observe the change in the divergence of the gold leaves; an increase in the divergence means that the gold leaves are electrified negatively, and, therefore, that the body used to give them their induced charge was electrified positively, while a diminution in the divergence means the opposite.

It is important to carefully watch the gold leaves while bringing up the ebonite, especially if the divergence of the leaves be small, for it may be first reduced to zero, and then made to increase on bringing up the rod, and unless this first diminution in the divergence be noted, a wrong conclusion will be arrived at with reference to the sign of the body's electrification.

This method is better than that of starting by giving the electroscope a charge of electricity of known sign, for example, negative, since, although an increase in the divergence of the leaves, produced by bringing up the body under test, indicates quite correctly that the body is charged with electricity of the same sign as the leaves, a diminution in the divergence may mean either that the body is charged oppositely, that is positively, or not charged at all, or charged negatively, but so that its potential is much nearer zero than the potential of the knob and leaves of the electroscope. For example, although the knob and leaves may be first charged with a piece of well-excited ebonite, it will be found, on bringing up a large sheet of ebonite only slightly electrified, that the divergence of the leaves diminishes, in spite of the fact that the electrification of the ebonite sheet is of the same sign as the charge in the gold leaves.

68. Screening Outside Space from Inside Electrostatic Action.—In § 56, page 196, we saw that a closed box which may be made of even non-conducting material will completely screen the interior from outside electrostatic action if the surface of the box be entirely

covered with a continuous film of even the very thinnest gold leaf, and that the same result can be practically attained with the coating of transparent conducting varnish, prepared and applied in the manner stated in § 58, page 200. The converse of this, however, is by no means universally true, indeed the method of comparing electric quantities described in §§ 64 and 65 depends for its action on a charged body *inside* a conducting box being able to produce a potential and electric forces *outside* the box.

In § 66 it is seen that the charge induced on the outer surface of an insulated conducting box A, surrounding an electrified body, C, is always exactly equal to the charge on C, while the potential of A depends not only on this charge, but on the size of A and its position relatively to bodies outside it. Hence it follows that :—

1. If the box be insulated from the earth, and be not much larger than the electrified body inside it, the screening action will be extremely small.

2. If the box be insulated from the earth and the dimensions of its side separating the electrified body inside the box from the body acted upon outside be large compared with the distance between the two bodies, the screening action will be large.

3. If the box be earthed the screening action will be perfect whether the box be small or large.

In the cases just considered the body C is supposed to be electrified when *outside* the box A, and *before* it was introduced into A. If, however, the conducting box A be entirely closed up, and the body C electrified when it is *inside* A by some operation performed *inside* A, then it follows, from the experiment described in § 65, that this electrification of C inside A will produce no effect outside A, whether A be insulated or not, or charged or not.

In fact, a completely closed conducting box separates space into two parts which are entirely distinct from one another electrically, for no effect can be produced in

either by an electrostatic operation performed in the other.

69. Electric Density.—In addition to knowing the potential of a body and the charge on it, it is important to know the distribution of this charge; for while the quantity of electricity in a body as a whole may, for example, be nought, the body may be highly electrified positively at one part and negatively at another. To test the distribution over the surface of a conductor without altering it in the operation of testing we may use a "*proof-plane*" (Fig. 113), consisting of an insulating handle, H, carrying a thin disc of metal, M, so small

Fig. 113.—Proof-Plane.

in area that it practically coincides with any part of the surface of the conductor against which it may be laid flat. Then since, as proved in § 63, page 210, electricity at rest on a conductor resides only on its surface, it follows that if the *proof-plane* be pressed quite flat against the surface any quantity of electricity that was on the area now covered by the proof-plane will transfer itself to the surface of the *proof-plane*, and will be carried away with the proof-plane when it is withdrawn, provided it be moved *without tilting* along a line perpendicular to its surface.

The electricity on the plane can then be measured by the method described in §64, page 213, for comparing quantities of electricity, and so, by a series of contacts with different parts of the surface of the conductor and subsequent comparison of the various quantities of electricity removed by the proof-plane, the distribution of charge on the surface of the conductor can be mapped out.

If the investigation be made by means of one par-

ticular proof-plane the quantities of electricity on it at different times can be compared without placing the proof-plane inside the hollow conductor A (Fig. 109, page 215), which it was necessary to use when we desired to compare the electric charges on bodies of *different* sizes, and not merely the charges at different times on the same conductor. For the proof-plane being small in area it may be placed flat against the knob of the gold-leaf electroscope and so for the time become part of the surface of the knob. The charge on the proof-plane will then be transferred to a definite conductor, viz. the knob, gold leaves, and the stout wire supporting them, and as the conductor is practically in a fixed position relatively to other bodies the charge transferred to it will be approximately proportional to the potential of the system as measured by the divergence of the gold leaves.

And even if the knob of the electroscope be too small for even the diminutive proof-plane to coincide practically with its spherical surface, what has just been said will still remain true provided the plane be always held against the knob in the same position; for then the proof-plane, the knob, the supporting-wire and the gold leaves together will constitute a definite conductor which is practically in a definite position relatively to other bodies. Hence the charge on this conductor will be approximately proportional to its potential.

In making tests of the density at different parts of the surface of an insulated conductor it must not be forgotten that the charge on the conductor will be gradually removed by the successive applications and removals of the proof-plane, in addition to the slow loss of charge that will occur slowly or quickly from defective insulation. Hence unless there be some device similarly to that described in Vol. II. for keeping up the potential by a renewal of the portion of the charge lost, we must not conclude because a test made with a proof-plane at a point P on the surface of a conductor gives a larger result than that made *subsequently* at a point Q

that the density at P is really larger than at Q. What ought to be done is to make a test at the point P, then at the point Q, and lastly at the point P again, and the result obtained at Q should be compared with the mean of the two results obtained at P.

Fig. 114. — Insulated Ball. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

70. Examples showing the Difference between Potential, Quantity, and Density.—To familiarise the student with the difference between potential, quantity, and density, the following examples may be considered. A (Fig. 114) is an insulated conducting sphere charged positively, far removed from the inductive action of all other bodies, then its potential charge and the density on its two sides are given by 1 in the following table. Now let a large conductor B, in metallic connection with the earth (Fig. 115), be brought into the neighbourhood of A on its right side, then 2 will represent the electric state of A. Let B be brought nearer to A; A's state will now be given by 3. If, on the other hand, A and B be separated more and more, A's state will be more and more like that given in 1.

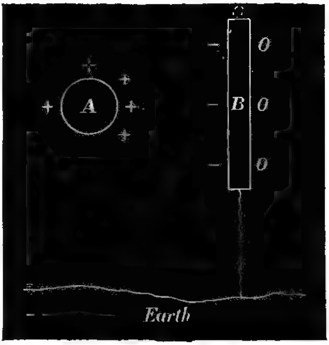


Fig. 115.—Earthed Conductor brought near an Insulated Positively Charged Ball. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

B being entirely removed, let a positively charged body C (Fig. 116) be brought into the neighbourhood of A on its left side, 4 will then represent A's state. Bring C nearer to A, but not so near that a spark or a brush

discharge can pass between A and C, A's state will now be changed to 5. Now while C is near A let A be earthed (Fig. 117), positive electricity will flow from A to the ground, and 6 will give the potential,

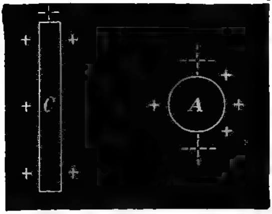


Fig. 116.—Positively Charged Body brought near an Insulated Positively Charged Ball. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

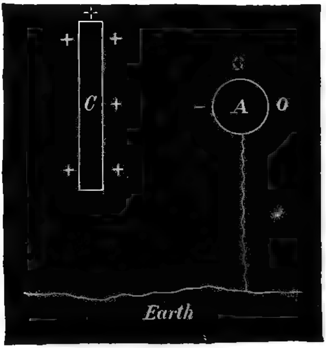


Fig. 117.—Positively Charged Body brought near an Earthed Ball. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

charge, and the density on the two sides of A. Lastly, let A be disconnected from the ground, and then let C be removed to a great distance from A, so that A is left alone, as in Fig. 114, then state 7 will be arrived at.

State of the Conductor A.

Number.	Figure.	Potential.	Charge.	Density.	
				Left Side.	Right Side.
1	114	Positive	+ Q say	Positive	+ and equal to density on left side
2	115	+ but less than in 1	+ Q as before	+ but less than in 1	+ but greater than in 1
3	115	+ but small	+ Q	+ but much less than in 1	+ and much greater than in 1

STATE OF THE CONDUCTOR A (*continued*).

Number.	Figure.	Potential.	Charge.	Density.	
				Left Side.	Right Side.
4	116	+ and greater than in 1	+ Q	+ but less than in 1	+ and greater than in 1
5	116	+ and much greater than in 1	+ Q	+ almost nought	+ and much greater than in 1
6	117	Nought	− q say	Negative	Nought
7		Negative	− q as in 6	− but less than in 6	Negative

Bringing the earthed conductor B near the right-hand side of A (Fig. 115) has exactly the same effect as cooling the right-hand end of a gas-bottle which has been placed horizontally, or cooling the vessel V_2 (Fig. 106, page 207) when the vessels V_1 and V_2 have been connected together by opening the tap T. The pressure of the gas at all parts of the bottle is uniformly reduced, while the weight of gas in the bottle remains unchanged, the density, or weight of a cubic inch of gas becomes less than before at the left-hand end of the bottle, or in the vessel V_1, and greater than before at the right-hand end, or in the vessel V_2.

Again, bringing a positively-charged body C near the left-hand side of A (Fig. 116) is like warming the left-hand end of a horizontal gas-bottle, or warming the vessel V_1. The pressure of the gas at all parts of the bottle is uniformly raised, while the weight of the contained gas remains unchanged, the density of the gas becomes less than before at the left-hand end of the bottle, or in the vessel V_1, and greater than before at the right-hand end, or in the vessel V_2.

Next let a negatively-charged plate M be brought near an insulated and uncharged plate P (Fig. 98, page

202), P for convenience being carried by the stout wire of a gold-leaf electroscope, whose outer coating is earthed, then the electric state of the plate P is given by 8 in the following table. The reason why P, as stated in the table, has a small positive charge is because by induction this charge comes into the plate from the gold leaves; the charge, however, in the plate and the leaves *together*, which was nought originally, must, of course, remain nought after M has been brought up. Without altering the positions of M and P, earth P, then its state will be given by 9 in the table. Next let P be insulated again, 9 will still give its state as long as the charge on M and its position be unchanged.

Now move M nearer to P, its state will become 10, whereas if, instead of moving M nearer to P, it be taken farther away from P, or taken away altogether, so that P and the electroscope are left alone (Fig. 96, page 202), P's state will be given by 11 in the following table.

STATE OF THE PLATE P.

Number.	Figure.	Potential.	Charge.	Density.	
				Upper Side.	Lower Side.
8	98	Negative	+ but small	Positive	Negative
9	98	Nought	+ and large	+ and larger than in 8	Nought
10	98	Negative	+ and larger than in 9	+ and larger than in 9	Negative
11	96	Positive	+ and smaller than in 9	+ and smaller than in 9	Positive

As regards the distribution of the density over the *upper* surface of P, this will be found to be practically uniform as long as the two plates are near together, parallel to one another, and with their centres in a line

perpendicular to both. Similarly, the density over the *lower* surface of M will also be uniform under the same conditions and numerically equal to the density on the upper surface of P, but opposite in sign. As regards the *upper* surface of M the density will throughout be found to be much greater near the edges of this plate than at the centre, so also will be the density over the *upper* surface of P and over the *lower* surface of M when the plates are separately a long distance from one another.

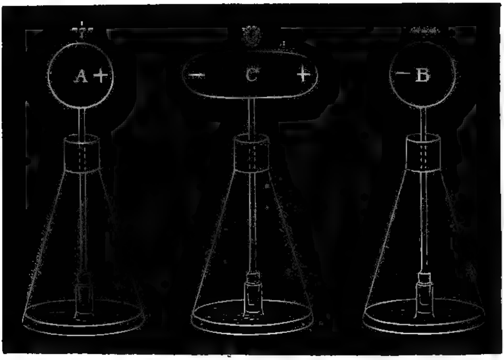


Fig. 118.—Insulated Uncharged Cylinder placed symmetrically between Two Insulated Balls Charged with Equal but Opposite Potentials. Sign of the Charge indicated by Continuous Lines; Sign of the Potential by Dotted Lines.

As a further illustration, let us take the case of an insulated uncharged cyclinder C, placed symmetrically between two charged spheres A and B, of equal size (Fig. 118). Suppose that the potentials of A and B are equal numerically, but of opposite sign, say, + 5,000 and −5,000 volts respectively. If A only were in the neighbourhood of C, the potential of the latter would be positive, but less than that of A, say, + 1,000 volts; if B only were near it would be −1,000, but if both are approached, as in Fig. 118, the potential of C will be

zero; and any point on the surface, whether at the centre or the extreme end, may be connected with the leaves of a delicate electroscope, whose case is earthed, without producing the slightest effect. So, in the same way, if the pressure of the gas in the two vessels V_1 and V_2 (Fig. 106, page 207) be first made equal to that of the atmosphere, by opening all the three taps, and then the taps at the tops of the two vessels be closed, but the tap T be left open, the temperature of the vessel V_1 may be raised very much and the temperature of the vessel V_2 lowered very much, without the pressure of the gas in the two vessels, and in the tube connecting them, altering from that of the atmosphere if the increase of pressure produced by the heating of the one vessel be made to exactly counterbalance the diminution of pressure produced by cooling the other.

The following table gives the condition of C when (12) A alone is near it, (13) B alone is near, (14) A and B both present, as in the figure :—

STATE OF CYLINDER C.

Number.	Potential.	Charge.	Density.		
			Left End.	Centre.	Right End.
12	+ but less than that of A	Nought	Negative	+ but small	Positive
13	− but less than that of B	Nought	Negative	− but small	Positive
14	Nought	Nought	Negative	Nought	Positive

CHAPTER IV.

RESISTANCE: ITS LAWS AND MEASUREMENT.

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71. Comparing Resistances.—By the method described in § 47, and illustrated in Fig. 87, page 177, two resistances can be compared if the relative calibration of a voltmeter only be known. Further, any of the methods described in § 52 for calibrating a voltmeter in volts, which depend on using a conductor whose resistance is known in ohms, can be used for measuring a resistance in ohms, if the voltmeter has been previously calibrated in volts. The one of these methods which is illustrated in Figs. 89 and 90, page 185, is particularly useful when we desire to know the resistance of a conductor which is much heated by the passage of a current through it—for

example, the resistance of the luminous carbon filament of a glow lamp, or the apparent resistance of the "*electric arc.*" The name "resistance" here means, as before, the ratio of the P.D. in volts to the current in amperes, but it is no longer a constant quantity and independent of the current passing, so that it is only by a sort of extension of the name "resistance" that it can be used at all in such a case. Indeed, had the early experience of currents passing through conductors been always with currents large enough to produce considerable warmth in the conductor, it is probable that we should never have acquired the conception we now possess of a conductor having a definite resistance as it has a definite length or a definite cross-section.

Frequently, when we are measuring the resistance of a conductor traversed by a strong current as, for example, the apparent resistance of an electric arc, we desire to know in addition the current which is flowing. In such a case the necessity of having to take simultaneous readings of an ammeter and a voltmeter in order to ascertain the resistance is no disadvantage, since two things have to be ascertained, and, therefore, two measurements must necessarily be made at the same time. But in other cases, when the resistance alone has to be ascertained, it may be a disadvantage to have to take readings of two distinct instruments simultaneously. Hence an instrument called an "*ohmmeter*" was devised by Professor Perry and the author to enable the resistance of any part of a circuit, through which a current is passing, to be measured by making a single observation.

72. Ohmmeter.—An *ohmmeter* contains a "*current coil*" C C (Fig. 119) and a P.D. or "*pressure coil*" *c c* placed at right angles to one another, and both acting on the same magnetic needle. The former coil has its terminals T, T connected with the circuit, the resistance of some portion of which it is desired to measure, so that C C is in series with the circuit, while *t*, *t*, the terminals

of the pressure coil, are joined with the points H and J, the ends of that bit of the circuit whose resistance, *o* ohms, is wanted, in the same way as a voltmeter, would be placed in parallel with H J.

The resistance of the *current coil* is made as low as possible, while the portion of the ohmmeter between the terminals *t* and *t* is made relatively very high, either by the *pressure coil c c* itself being wound with a very long fine wire, or by an auxiliary resistance being added to this coil and included in the instrument between the terminals *t*, *t*

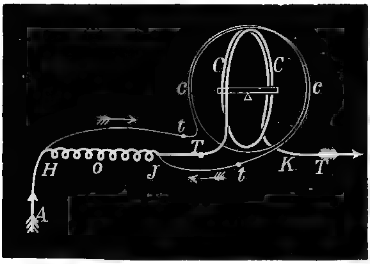


Fig. 119.—Diagram of Ohmmeter.

If the needle be short, the force due to the current passing round either of the two coils will be perpendicular to the plane of that coil (*see* § 20, Figs. 34, 38). Further, if the needle be made of hard steel so that its magnetism is not altered by the currents in the coils, these two forces will be directly proportional to the currents respectively. Hence the needle will be acted on by two forces at right angles to one another: one directly proportional to V, the P.D. in volts between the points H and J, the other directly proportional to A, the current in amperes passing through the conductor H J. Consequently, if matters be so arranged that no other magnetic forces than the two just mentioned act on the needle, it will place itself so that the tangent of the angle it makes with the plane of the pressure coil will be directly proportional to the ratio of V to A, that is, to *o* the resistance in ohms of the conductor H J (*see* § 22, page 83).

Further, if all extraneous magnetic action be avoided,

then, whether the needle be short or long, made of soft iron or of hard steel, it will place itself at right angles to the plane of the current coil, that is parallel to the plane of the pressure coil when *t*, *t* are both connected with the same point H, that is, when the resistance of the main circuit included between the two terminals *t* and *t* is nought. As the terminals *t*, *t* are separated so as to make contact with points of the main circuit farther apart, say, with H and K, the P.D. between the terminals of the pressure coil will increase, and the needle will deflect away from the plane of the pressure coil.

And, although the tangent of this deflection may not be directly proportional to the ratio that the P.D. between the points H and K bears to the current passing through the conductor H K J, the deflection will be quite constant as long as the terminals *t*, *t* are connected with the points H and K respectively, or with any two other points in the main circuit separated by the same resistance, whatever may be the current passing through the main circuit. For if the main current be doubled, the P.D. between the points H and K will be also doubled, therefore both the forces acting on the needle will be increased in the same ratio, and the deflection will remain as before. Hence, whatever the shape and sizes of the two coils and of the needle, the scale of the ohmmeter can be graduated to read off resistances directly in ohms, provided that the only forces acting on the needle be those due to the currents flowing round the pressure, and the current, coils *c c* and C C.

The principle of the ohmmeter has been employed by Mr. Evershed in constructing a commercial instrument that has been much used for measuring the resistance to leakage of electric-light wires and fittings.

73. Simple Substitution Method of Comparing Resistances.—If we merely wish to cut off a length of wire which shall have exactly the same resistance as that of some other conductor, for example if we desire to make a resistance exactly equal to that of a standard ohm, or a

standard ten-ohm coil, the following method may be adopted :—In circuit with the conductor whose resistance we wish to reproduce, place any convenient current-generator and a galvanoscope. Neither the resistance nor the relative calibration, much less the absolute calibration, of this galvanoscope need be known. Observe the deflection. Next remove this conductor, and put in its place a piece of the wire, out of which we desire to construct the resistance, of sufficient length that a smaller deflection of the galvanoscope is obtained with the same current-generator. Gradually diminish the length of this wire until the original deflection is obtained, then the resistance of this wire must be exactly equal to that of the conductor.

To detect any possible change in the sensibility of the galvanoscope, or in the strength of the current-generator during the test—a change in either of which would, of course, destroy the accuracy of the reproduction—it is well, after the wire has been shortened nearly sufficiently, to substitute the original conductor and see whether the deflection now obtained with it is exactly the same as it was at first. If it be found to be slightly different, then the final adjustment of the length of the wire must, of course, be made with the new deflection of the galvanoscope. Care must be taken not to accidentally shift the controlling magnet of the galvanoscope between the interchange of the conductor and the wire; further, the current-generator should not be allowed to send a current for so long a time through either the conductor or the wire that there is any evidence of a falling-off of its power.

In order to connect the galvanoscope and current-generator quickly, and conveniently, with either the known or the unknown resistance, a "*plug key, or switch*" (Fig. 120), may be conveniently employed. It consists of three pieces of brass, A, B, and C, fastened to a slab of ebonite, or hard wood, E E, and a brass plug, P, ground conically to fit tightly into either of the holes, H

or *h*, this plug being provided with an ebonite, or a wooden handle. If, therefore, the conical brass plug P is put into the hole *h*, the current will pass through the known resistance, while if the plug be put into the hole H, the current will pass instead through the unknown.

The current-generator is indicated symbolically by three thin lines, which stand for the copper plates of a battery, and by three shorter and thicker lines, which stand for the zinc plates or rods. The cells are under-

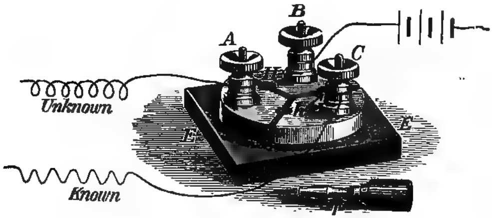


Fig. 120.—Plug Key.

stood to be coupled by the zinc plate, or rod, of the first cell being joined to the copper plate of the second, and the zinc plate of the second to the copper plate of the third, so that the six lines in Fig. 120 are a symbolical representation of the battery shown in Fig. 128, page 251. This symbolical representation, which is commonly used to stand for a battery, will be employed in the rest of this book, and will be found still further explained in § 161, page 537.

The preceding method of comparing the equality of two resistances is exactly analogous with Borda's method of double weighing, by means of which the weight of a body can be accurately compared with that of known standard weights, no matter how unequal be the lengths of the two portions of the beam of the balance, or how unequal be the weights of the scale pans.

The Differential Galvanometer.

74. Differential Galvanometer.—If any two conductors A and B (Fig. 121) be placed in parallel, and a current be sent through the arrangement, as indicated by the arrows, the resistances of A and B must be *inversely* proportional to the currents that flow in these conductors respectively. For the resistance of a conductor in ohms is the ratio of the P.D. maintained between its terminals in volts to the current that flows

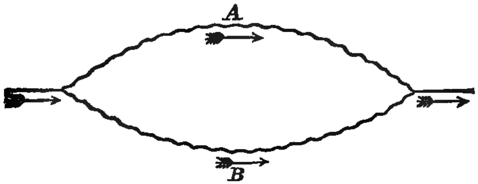


Fig. 121.

through it in amperes, therefore, since the P.D. between the terminals of each of the conductors A and B is the same, the resistances must be inversely as the currents.

To measure these currents, there might be inserted in A and B galvanometers that had been calibrated absolutely in amperes, or galvanometers the relative calibration of each of which was known as well as the relative calibration of each to the other. That is to say, not merely must we know the relative strengths of the currents producing any two deflections, say 40° and 20° in the same instrument, but we must know also what deflection on either corresponds with each particular deflection on the other. Such a joint relative calibration could very easily be carried out by placing the two galvanometers in series, and employing any of the methods described in Chapter I. for obtaining the relative calibration of a galvanometer.

There would, however, be an objection to inserting

galvanometers in A and B, arising from the fact that unless the resistances of the galvanometers were so small compared with the resistances of A and of B respectively, that they might be neglected, the resistances of the galvanometers must be taken into account in the comparison of the resistances of A and B. For let these resistances be a and b ohms respectively, the resistances of the galvanometers inserted in A and B be g_1 and g_2 ohms respectively, and let the currents be C_1 and C_2, flowing through A and B, C_1 and C_2 being measured rela-

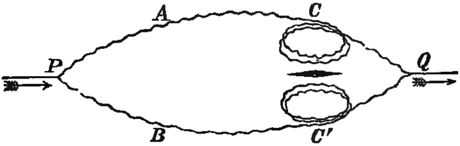


Fig. 122.—Diagram of Differential Galvanometer.

latively to the same unit of current which may, or may not, be the ampere, then

$$\frac{a+g_1}{b+g_2}=\frac{C_2}{C_1}=p \text{ say,}$$

or,

$$b=\frac{a+g_1-p\,g_2}{p}$$

an expression involving the resistance of each galvanometer.

If, however, we do not require to find out the ratio of any two resistances, but wish to make one resistance exactly equal to another resistance, or equal to some definite multiple of another resistance, say exactly ten times, then the arrangement can be simplified by combining the two galvanometers into one instrument, as indicated in Fig. 122, and so obtaining the arrangement known as a "*differential galvanometer.*"

For simplicity let us suppose that the *differential galvanometer* is to be used for testing whether one

resistance is equal to another, then the two coils C and C′ composing it (Fig. 122) are so wound and placed relatively to the needle that when equal currents pass round the two coils the magnetic effects exactly balance one another. The two coils C and C′ are also constructed in the way described further on, so that they not only have equal and opposite magnetic effects on the needle when traversed by equal currents, but also have equal resistances. If then a current be sent through the arrangement as indicated by the arrows, and the resistance of A or of B be altered until there is no deflection of the needle, it follows, since the P.D. between the points P and Q is the same for both the branches A and B, that the resistance of A plus that of the coil C is equal to the resistance of B plus that of the coil C′. Or since the resistance of C was made equal to that of C′ when the *differential galvanometer* was constructed, it follows that the resistances of the remainder of the two branches, that is the resistances of A and of B, are also equal.

The actual way in which the two conditions, *equality of magnetic effects and equality of resistance* of the wires of the two coils of the differential galvanometer, are fulfilled, is as follows :—Two reels of silk-covered copper wire are chosen so that the diameter of the wire on each is as nearly as possible the same, and the two wires are wound side by side on the galvanometer bobbin until it is nearly full ; the wires are then tested and cut, so that the resistance, but not, of course, necessarily the length, of each wire is the same. A current is now sent in *opposite* directions through the two coils *in series*, when it will be found that, although the wires have been wound on side by side, one of them will have a slightly greater magnetic effect than the other, partly perhaps because, being a trifle thicker, it has to be longer than the other, so as to have the same resistance, or partly because it is, on the whole, nearer the suspended needle than the other. To remedy this, a small portion of the wire having the greater magnetic effect is un-

wound, and without being cut, which would, of course, destroy the equality of the resistances of the two coils, the portion so unwound is coiled up out of the way in the base of the instrument. Thus, by unwinding more or less from the coil that was magnetically the more powerful, a very good balance can be obtained. In the use of differential galvanometers in which the needle is suspended by a silk fibre, a final and most delicate adjustment can be obtained by raising or lowering one of the levelling screws slightly, so as to tilt the needle nearer to or farther from one of the coils. And the spirit level attached to the instrument should then be permanently adjusted so that the bubble is in the centre of the glass cover of the level, after the instrument has been tilted in the manner just described.

If a differential galvanometer is to be used for ascertaining whether one resistance is a given multiple of another, for example, whether the resistance of the branch A is exactly ten times the resistance of the branch B, the coil C of the differential galvanometer must be arranged to have ten times the resistance of the coil C′ and the coil C must also produce the same magnetic effect on the needle as the coil C′ when the current flowing through A is one-tenth of that flowing through B. This result can be obtained by applying a *shunt* to the coil C′ (*see* § 104, page 311).

The Wheatstone Bridge.

75. Wheatstone's Bridge: its Principle.—The differential galvanometer is a very convenient apparatus for ascertaining whether one resistance is a certain definite multiple of another; but for accurately and rapidly comparing *any* two resistances, whether equal to one another or whatever may be their ratio, the "*Wheatstone's bridge*," or "*Wheatstone's balance*," as it is sometimes called, is more convenient.

As the late Sir Charles Wheatstone explained, when he first gave a public description of the balance method

of comparing resistances, the credit of its conception was due to Mr. Christie. The name of the better known man, however, has been universally attached to the arrangement which is shown symbolically in Fig. 123.

Two conducting branches, P S Q, P T Q, are joined in parallel, and a current sent through the arrangement, as indicated by the arrows, then in passing from P to Q, either along the conductor P S Q, or along the conductor

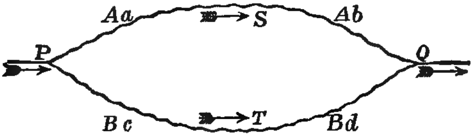


Fig. 123.

P T Q, there are points having all potentials between the potential of P and that of Q, therefore it follows that for every point in the conductor P S Q, there must be a point in the conductor P T Q having the same potential. Let S and T be two such points; then, if they were joined with the terminals of an electrostatic, or of a current voltmeter, or indeed with the terminals of any galvanometer, there would be no deflection. Given one point S, the corresponding point T can, therefore, be experimentally found by joining one terminal of an electrostatic voltmeter, or of any galvanometer, to S and touching the other conductor P T Q at different points with a wire attached to the other terminal of the electrostatic voltmeter, or of the galvanometer, until a point T is found for which there is no deflection. In practice a galvanometer is generally employed, since a galvanometer can be constructed so as to be a much more sensitive detector of a P.D. than an electrostatic voltmeter.

Let A be the current flowing along P S, then A must be the current flowing along S Q also, since no current passes through a galvanometer connecting the points S and T. Let B be the current flowing along P T Q, and let *a*, *b*, *c*, *d*

be the resistances respectively of PS, SQ, PT, TQ; then, since the potential difference between P and S is the same as the potential difference P and T,

$$A\,a = B\,c.$$

Similarly, since the potential difference between S and Q is the same as the potential difference between T and Q,

$$A\,b = B\,d.$$

Therefore, combining these two equations, we have

$$\frac{a}{b} = \frac{c}{d},$$

which is the law connecting together the resistances of the four "*arms*" of the Wheatstone's bridge.

This law may also be proved graphically, thus :—Let O, A, B, C (Fig. 124) be points in a conductor through which a *steady* current is flowing, and let O A, A B, B C be drawn so that the lengths of the lines represent, on some convenient scale, the resistances of the parts of the conductor between the points O and A, A and B, and B and C respectively, then if lines O P, A Q, B R, C S be drawn perpendicular to the straight line O A B C and of such lengths that they represent the potentials at the points O, A, B and C respectively, it follows from our fundamental definition of resistance that the points P, Q, R and S all lie in one straight line, and that the tangent of the angle this straight line makes with O A B C measures the current. This tangent will, however, only measure the current in amperes if the length of the horizontal line that represents an ohm is the same as the length of the vertical line that represents a volt.

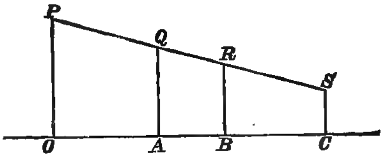


Fig. 124.

Suppose now that P P′ (Fig. 125) represents the P.D. between the points P and Q in Fig. 123, and suppose that P S represents the resistance a, S Q represents b, P T represents c, and T Q represents d, then, if the points P′ and Q in both the figures be joined by straight lines, and perpendiculars S S′, T T′ be erected, it follows that these perpendiculars represent the P.Ds. between the points S and Q and T and Q respectively of Fig. 123, on the same scale that P P′ represents the P.D. between the points P and Q. But the points S and T are by hypothesis selected such that no current flows through a galvanometer used to join them, therefore S S′ equals T T′.

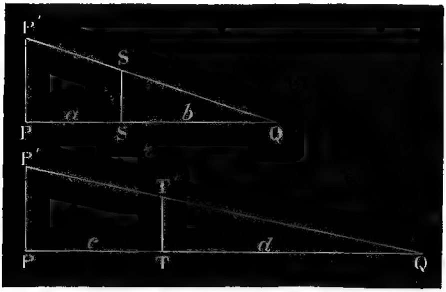


Fig. 125.

Further, from the properties of similar triangles, we know that

$$\frac{\text{S S}'}{\text{P P}'} = \frac{b}{a+b}$$

$$\frac{\text{T T}'}{\text{P P}'} = \frac{d}{c+d},$$

therefore, since S S′ equals T T′, we have

$$\frac{b}{a+b} = \frac{d}{c+d},$$

or

$$\frac{a}{b} = \frac{c}{d},$$

the same relationship as was previously arrived at as the law of the Wheatstone's bridge.

The last equation may be written in the form

$$\frac{a}{c} = \frac{b}{d},$$

and this is the equation that we should have obtained for no current through the galvanometer, had its terminals joined P and Q, and the current generator been placed between S and T. Hence, *when balance is obtained with a Wheatstone's bridge, the balance will not be disturbed by interchanging the galvanometer and battery.*

76. Wheatstone's Bridge: its Use and Simple Method of Constructing.—Any one of the four resistances a, b, c, d can be expressed in terms of one of the other resistances multiplied by the ratio of the two remaining resistances to one another. For example,

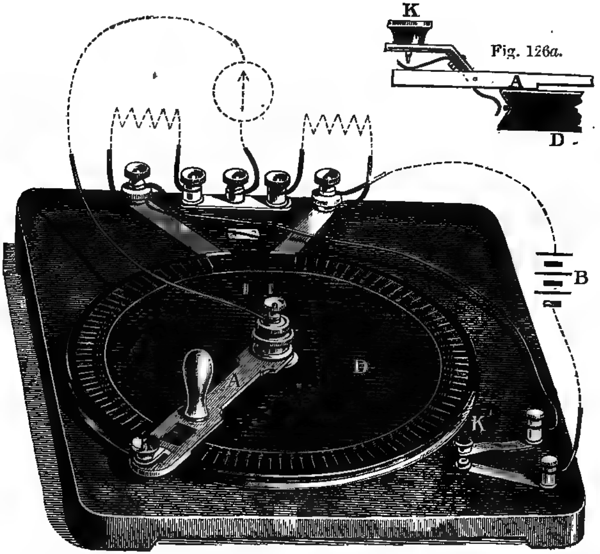
$$a = b \times \frac{c}{d},$$

$$a = c \times \frac{b}{d},$$

or $$d = c \times \frac{b}{a},$$

&c. If then the bridge be "*balanced,*" that is, if two points S and T have been found of the same potential, and we know the resistance of one of the arms, say b in ohms, and the ratio of the resistance of two of the other arms, say c to d, but not necessarily the values of either c or d in ohms, we can, from the first equation given above, find at once the value of the resistance of the fourth arm, a, in ohms. Similarly, if we know c in ohms and the ratio of b to a, but without necessarily knowing either b or a, we can at once find the value of d in ohms, from the third equation, &c. Hence, one mode of using the bridge to measure the resistance of a is to keep the ratio of c to d constant, and simply vary the resistance of b until no current passes through the galvanometer. Another method consists in keeping b constant, and

varying the ratio of c to d. For example, the resistances c and d may be the resistances of different lengths of the same kind of wire, in which case we know that c will be to d simply as the ratio of these lengths, whatever be the absolute resistance in ohms of the two parts (*see*



Figs. 126 and 126*a*.—Circular Metre Bridge.

§ 81, page 254). A form of Wheatstone's bridge, in which P T Q, of Fig. 123, was one piece of stretched wire, and the ratio of the "*proportional arms*" c to d varied by moving the connection of the wire leading to one terminal of the galvanometer, was originally employed by the Electrical Committee of the British Association, and is, for this reason, sometimes called the "*British Association bridge*"; at other times, the "*metre bridge*,"

from the stretched wire being often a metre long. The wire may be made of platinum, or better still, of platinum-iridium, which, being very hard, tends to prevent the wire from being worn at any part.

To protect the platinum-iridium wire from being accidentally knocked or damaged, it may conveniently be placed in a groove cut in the edge of an ebonite or slate disc, D (Fig. 126), and contact made with any point of it by means of the spring key K carried at the end of the movable radial arm A, and shown in detail in Fig. 126*a*. The small pin under the knob K is to prevent the knob being pressed down so much as to damage the platinum-iridium wire. The circuit of the battery B (Fig. 126) is closed by a separate key K′.

The scale round the edge of the disc in Fig. 126 is divided into centimetres and millimetres, but for rapid work it is more convenient to have this scale divided into ratios, as indicated for a few points in the following, where the top line of numbers gives the length of the platinum-iridium wire measured from the left hand, the second line of figures the ratio of the length on the left to the length on the right, and the third line the ratio of the length on the right to the length on the left:—

0	10	20	30	40	50	60	70	80	90	100
0	0·111	0·250	0·429	0·667	1	1·500	2·333	4	9	∞
∞	9	4	2·333	1·500	1	0·667	0·429	0·250	0·111	0

A form of metre bridge of greater range is shown in Fig. 127. It has three stretched wires *w w*, each a metre in length, and so arranged that either one of them alone, or two of them in series, or all three in series, can be made use of to form the two sides *c* and *d* of the Wheatstone's bridge (Fig. 123). When the plug E is, as in the figure, placed in the hole H, the current simply passes through the stretched wire which is nearest to the observer. If, on the other hand, the plug E be put in the hole *h*, then, since the brass plate P is permanently connected with the plate *p* by a thick copper strip under

the base of the instrument, the middle stretched wire is short-circuited, and the wire nearest to the observer is

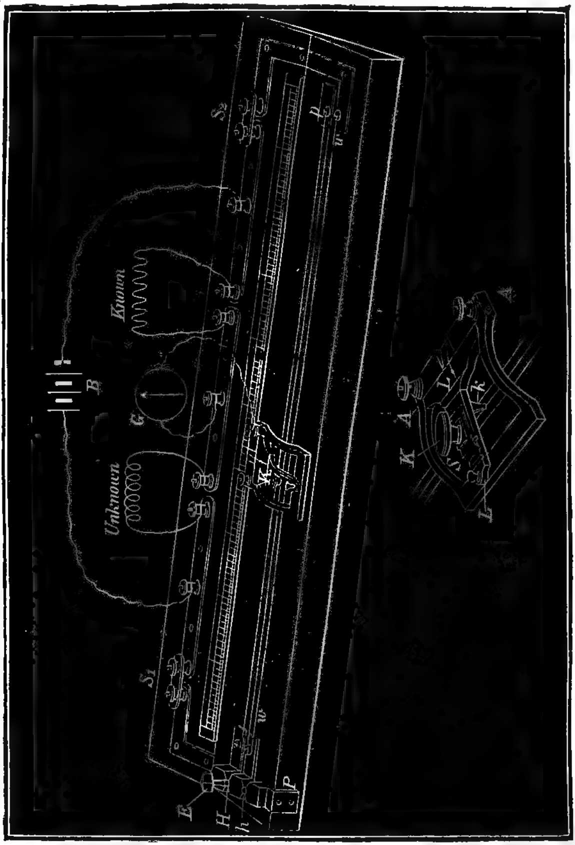


Fig. 127.—Three-Wire Bridge.

in series with the one farthest from him. Lastly, if the plug be removed altogether, the three wires are in series.

The object of thus lengthening the wire is to increase the sensibility of the test when desired, and a still further increase in the sensibility can be effected by removing the short-circuit pieces s_1, s_2, and inserting coils of known resistance in place of them. For example, suppose that the ratio of the unknown to the known resistance be $\frac{3}{2}$, then the slider K must be placed so as to divide the stretched wire into two parts having this ratio. Hence, if one of the three wires only be used, the lengths of the two parts which will give exact balance will be 60 and 40 centimetres, and an error of 1 centimetre in the position of the slider will correspond with an error in the determination in the ratio of

$$\frac{\frac{61}{39} - \frac{60}{40}}{1\cdot5} \times 100 \text{ per cent., or } 4\cdot3 \text{ per cent.}$$

If, on the other hand, the three wires in series be employed, then the lengths into which the three metres of wire must be divided to obtain exact balance will be 180 and 120 centimetres, and an error of one centimetre in the position of the slider will correspond with an error in the determination of the ratio of

$$\frac{\frac{181}{119} - \frac{180}{120}}{1\cdot5} \times 100 \text{ per cent., or } 1\cdot4 \text{ per cent.}$$

If now two coils, each having a resistance equal to, say, 500 centimetres of the stretched wire, be inserted in place of the short circuit pieces s_1 and s_2, an error of a centimetre in the position of the slider will only correspond with an error of

$$\frac{\frac{781}{519} - \frac{780}{520}}{1\cdot5} \times 100 \text{ per cent., or } 0\cdot32 \text{ per cent.}$$

Contact between the platinum-tipped knife-edge *k* and one or other of the stretched wires, is produced by

depressing the knob K, which causes the lever, L L, to which this knife-edge is attached to turn on an axis A A. On removing the pressure, the lever is pressed up by a spring underneath it; and the slider should never be moved with the knife-edge *k* depressed, as this would scrape the stretched wire and alter its diameter. In order to enable *k* to make contact with either the first, second, or third wire, the knob K is not fastened rigidly to the lever, but can slide along it in a slot, and be so placed that the near end of the spring S rests in either one of three grooves on the top of the lever, L L, corresponding with the three positions of *k* when it is in contact with the three stretched wires respectively.

77. Bridge Key.—In using a Wheatstone's bridge it is desirable to send the current through the four arms of the bridge *a*, *b*, *c*, *d*, before it is allowed to pass through the galvanometer, and this is especially important when testing the resistance of the copper conductor of a long submarine cable, since the current in such a case takes an appreciable time to reach its maximum value and become steady, due to the cable acting as a "*condenser*" (*see* Volume II.). Hence, if the galvanometer circuit were completed when the battery was attached to the bridge, an instantaneous swing of the galvanometer would be produced, even if *a* bore to *b* the ratio of *c* to *d*. And although, since the ratio of resistances having been effected, the deflection of the galvanometer would become nought as soon as the current in the four branches of the bridge became steady, great delay in the testing would be caused by this first swing of the needle. A similar difficulty would occur in measuring the resistance of an electromagnet or even of any coil without an iron core, if it were not specially wound; because whenever a coil is so wound that a current passing through it produces magnetic action, a short interval of time has to elapse, after putting on the battery, before the current reaches its maximum, or steady, value, arising from what is called the "*self-induction*" of the coil.

A key for sending the current through the four arms of the bridge before it is allowed to pass through the galvanometer is shown at K (Fig. 128), and is a modification of the one originally employed by the Electrical Committee of the British Association. On pressing down the button, contact is first made between the flexible piece of brass A and the flexible piece of brass P.

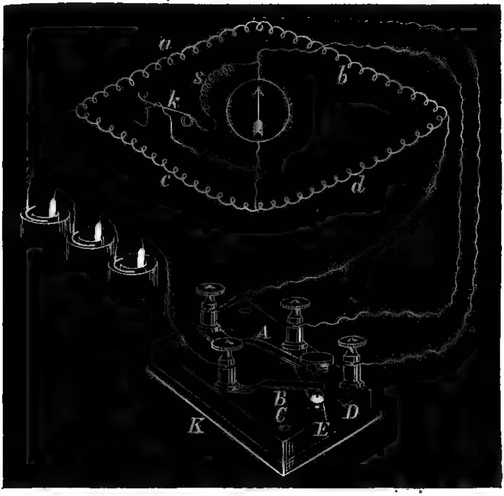


Fig. 128.—Bridge Key.

This completes the battery circuit, and causes the current to flow through the four arms of the bridge shown symbolically in Fig. 128 by the spiral lines. On the button being still further pressed down, B is brought into contact with a little knob of ebonite E on the top of the flexible piece of brass C. This does not complete any other electric current; but on the button being still further depressed, C is brought into contact with D, and the galvanometer circuit is completed.

This form of key is to be preferred to the ordinary bridge key, because all the connections are above the base of the key and in sight, whereas when the connections are made under the base, it occasionally happens that, without it being noticed, the pieces of guttapercha-covered wire used to make the connections are either badly insulated, or are loosely connected at their ends with the terminals of the key, and so introduce unnecessary resistance.

78. Use of a Shunt with the Bridge.—It is desirable to employ also another key *k* (Fig. 128), which may be quite simply made of a twisted bit of hard brass wire, bent so as to press up against a sort of bridge of hard brass wire, since the resistance at the contact is in this case of no consequence. When the key is not depressed, a portion of the current is shunted past the galvanometer through any convenient shunt *s*, the resistance of which need not be known as it does not enter into the calculations. The object of this shunt is merely to diminish the sensibility of the galvanometer when the first approximation is being made to the value of the unknown resistance. As soon as this has been done the key *k* should be depressed, and all the current in the galvanometer circuit arising from want of perfect balance allowed to pass through the galvanometer itself, and the resistances adjusted until perfect balance is obtained. Another device to expedite the testing, and also to prevent powerful currents being sent through the galvanometer, consists in not holding the key K down when the first rough approximation is being made, but merely giving it a tap, which has the effect, when the balance is far from perfect, of giving the needle of the galvanometer a slight impulse to one side or the other, according as the ratio of *a* to *b* is larger or smaller than that of *c* to *d*, instead of causing the needle to violently swing against the stops on one side or the other as it would do if the key K were held down before balance was arrived at.

79. Meaning of the Deflection on a Bridge Galvanometer.—A considerable amount of time will be saved in testing if the meaning of a deflection of the galvanometer needle, say to the right, be once for all definitely ascertained, and a note be made whether it means that the ratio of a to b is too large or too small. The simplest way of recording this, if we assume, for example, a to be the unknown resistance, is to put the words "*increase b*" and "*diminish b*" one on each side of the galvanometer, these being the directions to be followed according as the needle deflects towards one or other of them. The position of these two directions must, of course, be reversed if the terminals of the galvanometer, or of the testing battery, be reversed.

Example 34.—In measuring a resistance on the Wheatstone's bridge the resistances of the arms P T and T Q (Fig. 123) are 1,000 and 100 ohms respectively. The unknown resistance is placed in the arm P S, and the resistance in S Q is adjusted until balance is as nearly as possible obtained. It is found that when the variable resistance S Q is 546 ohms the galvanometer deflection is 15 divisions to the left, while if S Q is made 547 ohms the deflection is 27 divisions to the right. Find the value of the unknown resistance, assuming proportionality of deflection for small changes in the resistance S Q.

A change of 1 ohm in S Q produces a change of 42 divisions in the deflection, hence a change of $\frac{15}{42}$, or 0·36, ohm in S Q would cause a change of 15 divisions in the deflection. Consequently, if S Q were 546·36 ohms the galvanometer deflection would be zero, therefore the resistance tested is $\frac{1000}{100} \times 546{\cdot}36$, or 5463·6, ohms.

80. Conditions Affecting the Resistance of a Conductor.—The resistance of a conductor depends on four distinct conditions :—

(1) Its length.

(2) Its cross-section.

(3) The material of which it is composed, the purity of the material, and the hardness or density.

(4) The temperature.

It is therefore important that the student should ascertain by experiment how much change is produced in the resistance by varying each of these four conditions *separately*. And generally, in experimenting, it is to be remembered that *when it is possible to change several of the conditions under which the experiment can be made, it is of the utmost importance that only one of the conditions should be varied at one time.* The effect produced by the variation of one condition should be fully inquired into before any one of the other conditions is in any way altered, otherwise it will generally be quite impossible to gather from the results what portion of the variation in the effect was produced by a particular change in the conditions.

81. Variation of Resistance with Length.—In § 42, page 167, we saw that when a steady current passed through a uniform conductor the P.D. between any two points was proportional to the length of the conductor between the points. Combining this fact with the fundamental definition of resistance (§ 44, page 172), it follows at once that *the resistance of a uniform conductor is proportional to its length.*

This law may also be proved independently by using a high-resistance galvanometer as a voltmeter. For it is to be remembered that, although it would *not* be justifiable to prove that Ohm's law were true by using a *current*-voltmeter, seeing that the possibility of employing a galvanometer as a voltmeter depends on the fact that Ohm's law is true, galvanometers could be used as accurate voltmeters, when once Ohm's law has been proved to be true, even if the distribution of potential along a uniform wire conveying a steady current followed some law other than it actually does.

Fig. 129 shows a simple arrangement for testing the

distribution of potential in such a case. On pressing down the key a current is sent through a uniform wire of platinum—or better, of platinum-iridium—stretched along a graduated bar between the points W W′. A tangent galvanometer, whose coil has a high resistance compared with that of the straight wire W W′, has one of its terminals, B, connected with one end of this wire, W,

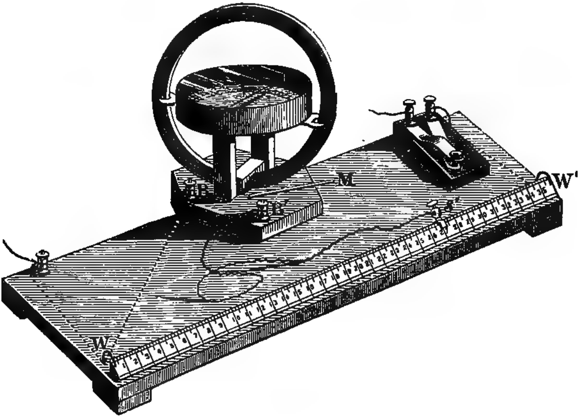


Fig. 129.—Apparatus for Testing the Distribution of Potential along a Wire Conveying a Current.

while its other terminal, B′, can be connected with any point of the stretched wire by means of the loose flexible wire and the binding screw *s*′.

Then experiment shows, if the sensibility of the galvanometer is kept unchanged by the adjusting magnet not being moved during the experiment, and if the current flowing through the wire W W′ be kept quite constant, that the tangent of the deflection of the galvanometer, and therefore the P.D. between its terminals, is directly proportional to the length of the wire W *s*′.

Instead of the binding screw s' (Fig. 129) we may conveniently use a movable spring key; but whatever method be employed of making connection with different points of the stretched wire, the contact must be loosened before the connecting device is moved along, otherwise the wire will be scraped, and its cross-section will no longer remain perfectly uniform.

If it be desired to try this experiment with a longer wire than can be conveniently used in a straight form, we may employ the frame (Fig. 130), consisting of six or more wooden cylinders having a screw groove cut on each. Lengths of say 5, 10, 20, 30, 40 and 50 feet of wire of the *same material* and having exactly the *same thickness* throughout, say 0·01 of an inch, may be wound in the grooves on the respective cylinders, and by connecting the binding screws together in pairs the whole of the wire may be joined up in series. If a current be sent through the whole of the wire joined up in series from left to right through the wire on the first cylinder, right to left through that on the second, &c., and if the current be maintained constant, it will be found that the P.D. between the terminals at the ends of any one of the cylinders is proportional to the length of wire on that cylinder.

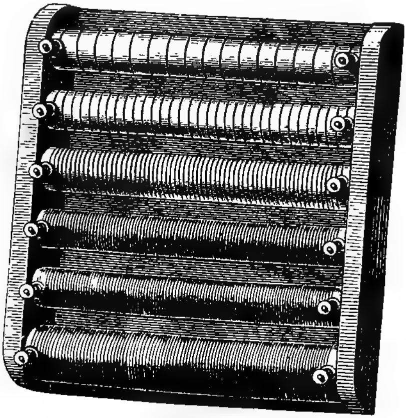


Fig. 130.

82. Variation of Resistance with Cross-Section.—For ascertaining the law of variation of the resistance of a conductor with its cross-section the spiral grooves in another set of six cylinders (Fig. 130) have wound in them wires all composed of the *same material* and of exactly the *same length* (say, twenty-one feet), but having diameters respectively of, say, 0·008, 0·0095, 0·01, 0·0136, 0·0158, 0·019 of an inch. The resistances of these wires may be tested by any of the methods described in the §§ 71 to 76, in terms of some one resistance taken as a standard; and when this is done, it is found that *the resistances of the different conductors of the same material are inversely as the squares of their diameters*—that is, *inversely as their sectional areas.*

83. Variation of Resistance with Material.—The cylinders in this case have wound on them wires of exactly the *same length* (say, twenty-one feet) and having exactly the *same diameter* (say, 0·01 of an inch), but made of the following materials respectively—copper, brass, platinum, iron, lead, and German silver; and when the resistances are tested by any of the methods described in §§ 71 to 76, it is found that the metals, as given in this list, are arranged in increasing order of resistance, and that the resistances are, roughly, as the numbers 1, 4, $5\frac{3}{4}$, 6, 12, 13.

84. Resistance of Metals and Alloys per Centimetre Cube and per Inch Cube.—The "*specific resistance*" of a material is usually expressed as the resistance in "*microhms*" or millionths of an ohm, at 0°C. of a centimetre cube, or of an inch cube—that is, the resistance from one face to the opposite face across the cube. It has been customary hitherto in books to give a table of the specific resistances of a number of pure materials and alloys expressed to four significant figures as determined by Dr. Matthiessen nearly thirty years ago; and such a table will be found in the earlier editions of "Practical Electricity." But during the last few years a

number of investigations have been carried out on the resistance of copper—the material generally employed for an electric conductor—and it has been found that a diminution of from 3 to 4 per cent. can be produced in the resistance of copper by compressing it, without any change being made in its chemical composition. The resistance of different samples of copper chemically of the same quality diminishes more rapidly than the density increases; so that even if we compare the resistances of different copper wires of the same *length* and *weight* (Table No. VI.) instead of the same *length* and *cross-section*, it will still be found that the denser, and therefore the thinner, wire has the less resistance. The difference between the resistances of copper wires of the same length and weight is, however, less of course than between the resistances of wires of the same length and cross-section.

TABLE VI.

Variation of Specific Resistance with Density, for High Conductivity Copper.

Density.	Resistance in International Microhms per Centimetre Cube, at 0°C.		Resistance in International Ohms of a wire 1 metre long, weighing 1 gramme.	
	Hard Drawn.	Annealed.	Hard Drawn.	Annealed.
8·86	—	1·6076	—	0·1424
8·88	—	1·605	—	0·1425
8·89	1·635	1·587	0·1454	0·1411
8·90	1·629	—	0·1450	—
8·92	1·615	—	0·1440	—
8·94	1·597	1·557	0·1428	0·1392

This table is extracted from an article by Mr. T. C. Fitzpatrick "On the Specific Resistance of Copper," in the *Electrician* for October 3rd, 1890. (*See* the note on page 312.)

Similar investigations have not yet been made with other materials, so that the specific resistances given in

the following table (No. VII.) must be regarded as being only approximately correct, and they are, therefore, stated only to three significant figures. The substances are arranged in order of increasing specific resistance, and the unit employed is the international microhm.

TABLE VII.

PURIFIED SUBSTANCES ARRANGED IN ORDER OF INCREASING RESISTANCE FOR THE SAME LENGTH AND SECTIONAL AREA.

Name of Metal.	Resistance in International Microhms at 0°Centigrade.		Relative Resistance.
	Centimetre Cube.	Inch Cube.	
Silver, annealed	1·48*	0·583	1
Copper, annealed ... from	1·55	0·610	1·04
to	1·61	0·633	1·09
,, ,, (Matthiessen)	1·594	0·6277	1·077
Silver, hard drawn ...	1·58	0·622	1·07
Copper, hard drawn from	1·59	0·626	1·07
to	1·64	0·646	1·11
,, ,, (Matthiessen)	1·630	0·6418	1·10
Gold, annealed	2·05	0·807	1·38
Gold, hard drawn... ...	2·089	0·822	1·41
Aluminium, annealed ...	2·90	1·14	1·96
Zinc, pressed	5·61	2·21	3·79
Phosphor bronze (about)	7·8	3·07	5·27
Platinum, annealed ...	9·04	3·55	6·09
Iron, annealed	9·69	3·82	6·56
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard or annealed	10·8	4·27	7·33
Nickel, annealed	12·4	4·90	8·41
Tin, pressed	13·2	5·19	8·91
Lead, pressed	19·6	7·71	13·2
German silver ... from	19·0	7·48	12·8
to	30·0	11·8	20·2
Platinum-silver alloy (1 oz. platinum, 2 oz. silver), hard or annealed ...	24·3	9·58	16·4

* Profs. Dewar and Fleming give 1·468, and Mr. Fitzpatrick 1·481.

TABLE VII. (*continued*).

Name of Metal.	Resistance in International Microhms at 0°Centigrade.		Relative Resistance.
	Centimetre Cube.	Inch Cube.	
Platinoid ... (about)	34	13·4	23
Antimony, pressed ...	35·4	13·9	23·8
Manganin ... (about)	42	16·7	28·7
Mercury	94·08	37·04	63·6
Bismuth, pressed	108	42·5	73

From the preceding table we see that of the various pure metals *annealed silver* is the one having the least, and bismuth the one having the *greatest*, resistance for a given length and sectional area.

The numbers given in the foregoing table can be used to ascertain the resistance of a wire or rod of any length and of any cross-section composed of any one of the materials at 0°C. For example, if s be the specific resistance per cubic centimetre, l the length, and d the diameter of the wire in centimetres, the resistance is

$$\frac{4\,l\,s}{\pi\,d^2} \text{ microhms.}$$

85. Resistance of Metals and Alloys for a Given Length and Weight.—It is frequently convenient to know, not the resistance of a given volume of a material, but the resistance of a given *length* having a given *weight*. In the following table (No. VIII.) will be found the resistances in international microhms at 0°C. of wires one foot long weighing one grain, and one metre long weighing one gramme; the substances being arranged in increasing order of resistance for a given *length* and *weight*, this order being different from that employed in Table No. VII., where the substances were arranged in increasing order for the same *length* and *cross-section.*

TABLE VIII.

PURIFIED SUBSTANCES ARRANGED IN ORDER OF INCREASING RESISTANCE FOR THE SAME LENGTH AND WEIGHT.

Name of Metal.	Resistance in International Ohms at 0°Centigrade of a wire.		Relative Resistance.
	1 foot long weighing 1 grn.	1 metre long weighing 1 grn.	
Aluminium, annealed ...	0·1071	0·0747	1
Copper, annealed from	0·199	0·139	1·86
Copper, annealed to	0·209	0·143	1·91
" " (Matthiessen)	0·2037	0·1421	1·90
Copper, hard drawn from	0·208	0·142	1·90
Copper, hard drawn to	0·218	0·146	1·95
" " (Matthiessen)	0·2078	0·1449	1·94
Silver, annealed	0·218	0·152	2·04
Silver, hard drawn ...	0·238	0·166	2·22
Zinc, pressed	0·575	0·401	5·37
Gold, annealed	0·577	0·402	5·37
Gold, hard drawn ...	0·587	0·409	5·47
Phosphor bronze (about)	1·0	0·70	9·3
Iron, annealed	1·082	0·755	10·1
Tin, pressed	1·38	0·960	12·8
Nickel, annealed	1·51	1·06	14·1
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard or annealed	2·36	1·65	22·1
German silver ... from	2·37	1·66	22·2
German silver ... to	2·87	2·01	26·9
Platinum, annealed ...	2·74	1·93	25·8
Lead, pressed	3·19	2·22	29·7
Antimony, pressed ...	3·41	2·38	31·8
Platinum-silver (1 oz. platinum, 2 oz. silver), hard or annealed	4·19	2·92	39·1
Platinoid	4·40	3·03	40·6
Manganin ... (about)	5·12	3·57	47·8
Bismuth, pressed	15·2	10·6	142
Mercury	18·36	12·80	171

From Table No. VIII. we see that of the metals *aluminium* has the least resistance for a given *length*

and *weight;* whereas we saw from Table No. VII., page 259, that for a given *length* and *cross-section* it was *annealed silver* that had the *least* resistance.

86. Variation of Resistance with Temperature.—To ascertain the way in which the resistance of metals and alloys varies with the temperature, small coils of silk-covered wire composed of the different materials may be conveniently wound on paper cylinders and inserted in narrow glass test-tubes, G_1, G_2, G_3, and G_4 (Fig. 131), the test-tubes being supported from a wooden disc. One end of each of the coils may be soldered to a common terminal, T, while the other ends of the coils are soldered to the terminals T_1, T_2, T_3 and T_4. The test-tubes are inserted in the water-bath W (Fig. 131*a*), which can be warmed, with the Bunsen burner B, standing on a sheet of asbestos, A A, to a temperature which is indicated by the thermometer *t t*, enclosed in a brass tube to prevent mechanical injury; and the resistances of the different coils of wire can be measured with a Wheatstone's bridge, differential galvanometer, or other suitable arrangement, the measuring apparatus being protected from the heat of the burner by means of the double wooden screen S.

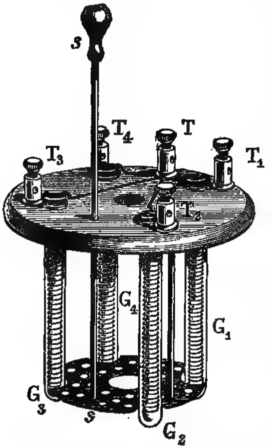


Fig. 131.—Coils of Wire used in the Apparatus for Measuring the Variation of Resistance with Temperature.

In carrying out experiments of this kind, it must be borne in mind that, as the glass bulb of a thermometer is very thin, and as mercury is a substance having a very small "*specific heat*,"* a thermometer rapidly acquires the

* The *specific heat* of a substance is the ratio of the amounts of heat required to raise equal masses of the substance and of water through 1°.

temperature of the liquid in contact with it; whereas a mass of metal inserted in the same liquid may have a very different temperature from the liquid which immediately surrounds it, especially if the temperature be

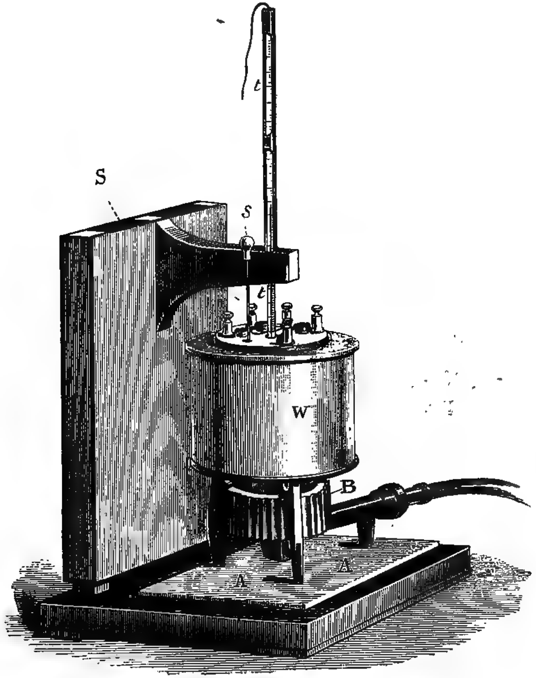


Fig. 131*a*.—Calorimeter for Measuring the Coils of Wire shown in Fig. 131.

rapidly rising or falling. Further, a liquid, being a very bad conductor of heat, the temperature in different parts of it will be different, unless it be kept constantly in motion; therefore a stirrer, *s s*, is provided with the "*calorimeter*" seen in Figs. 131 and 131*a*. Lastly, the

water-bath W is made in two separate parts in order that the current of hot water which rises by "*convection*" from the heated bottom of the water-bath may not come directly into contact with the glass tubes.

Before taking a measurement of the resistances of the coils of wire at any particular temperature, it is well to adjust the flame of the Bunsen burner so as to maintain the temperature of the thermometer constant for some minutes, the water being constantly agitated with the stirrer *ss* during the time. For the longer the time during which the temperature of the water in the water-bath is all kept at a uniform and constant temperature, the greater is the probability that the coils of wire have acquired the temperature indicated by the thermometer *t*.

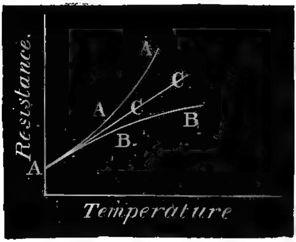


Fig. 132.

The exact law connecting the variation of the resistance of a metal with the temperature depends not only on its chemical constitution, but on its molecular condition, such as its hardness, and therefore, although a large number of experiments have been carried out on this subject by various people, the results are conflicting. According to Dr. Matthiessen and M. Benoit, if r_t be the resistance of a conductor of pure metal at any temperature t, and r_0 be its resistance at the freezing temperature,

$$r_t = r_o\,(1 + a\,t + b\,t^2),$$

where a and b are positive coefficients, and such a law gives a curve connecting resistance and temperature something like A A A (Fig. 132), indicating that the resistance of a pure metal increases more rapidly than the temperature. The late Sir W. Siemens, however, who was the first to experiment on the variation of resistance

at high temperature, showed that this formula was only correct between the limits of 0° and 100° C., and he suggested the formula

$$r_{\text{T}} = \frac{r_o}{273}\left\{ a\sqrt{\text{T}} + \beta\,\text{T} - 1 \right\},$$

where T is the temperature measured from the absolute zero, which is 273°C. below the freezing-point. As a and β are positive coefficients, this second equation would give a curve something like A B B (Fig. 132), which is quite different in shape from A A A. In fact, a curve of the shape A B B indicates that the resistance of a metal does not increase as rapidly as the temperature. Prof. Callendar, on the other hand, who has devoted much time during the last few years to a study of the temperature variation of resistance, especially of platinum at high temperatures, concludes that the first formula generally represents the truth more correctly than the second, but he finds that b equals $-0{\cdot}000{,}000{,}533$, for platinum a being 0·003,448. A long series of experiments made by Mr. Kilgour and the author on platinum, between 0° and 340° C., shows that for this metal a is about 0·0036, and that b varies from $-0{\cdot}000{,}000{,}109$ to $-0{\cdot}000{,}000{,}645$. Hence the curve for platinum is of the shape A B B (Fig. 132), similar to that found by the late Sir W. Siemens. Lastly, the measurements carried out in 1893 by Mr. Kennelly in America appear to show that the resistance of *commercial* copper follows a straight-line law—that is, the equation connecting resistance and temperature is of the form

$$r_t = r_o\,(1 + a\,t),$$

without any term containing t^2, and gives therefore a straight line something like A C C (Fig. 132).

When t is measured in degrees Centigrade the value of a for *commercial* copper is 0·00406—according to Messrs. Kennelly and R. A. Fessenden—a result not differing much from that published in 1894 by Messrs.

J*

J. W. Swan and J. Rhodin for pure electrolytically deposited copper when hard drawn, the temperature coefficient of which they find to be 0·00408. When, however, this pure copper is *annealed*, the coefficient rises to as high a value as 0·00416, according to these experimenters, or 0·00428 according to Profs. Dewar and Fleming. Dr. Matthiessen's experiments on copper, on the other hand, led to a value of about 0·00387 for a.

In spite, then, of all the work that has been done on this subject, we cannot say that we are sure of the shape of the curve; but we are, however, quite sure that the coefficient a has a value something like 0·004 for pure metals, excluding iron and nickel, for which it is about 0·005 and 0·007 respectively. Further, we are also sure that a has only about one-tenth of this value for certain alloys. So far, then, the results of the very important research carried out by Dr. Matthiessen thirty years ago to discover a conductor of a permanent character with a low temperature coefficient have been fully confirmed. The following table (No. IX.) contains a few of his results, the substances being arranged in decreasing order of variation of resistance with temperature :—

TABLE IX.

Approximate Percentage Variation in Resistance per 1°C. at about 20°C.

Iron . . . . . about	0·5
Antimony	0·389
Copper, annealed	0·388
Lead, pressed	0·387
Silver, annealed	0·377
Gold, annealed Zinc, pressed Tin, pressed . . .	0·365
Mercury	0·072
Gold-silver alloy (2 oz. gold, 1 oz. silver), hard or annealed	0·065
German silver, hard or annealed . .	0·044
Platinum-silver alloy (1 oz. platinum, 2 oz. silver), hard or annealed . . .	0·031

From this we see that, whereas (of the substances experimented on by Dr. Matthiessen) an *alloy of platinum-silver*, hard or annealed, is the one of which the resistance changes *least* by temperature, *German silver*, which is a very much cheaper alloy, being composed of copper, nickel, and zinc, is nearly as good in this respect.

The resistance of different specimens of German silver wire may, however, vary from about 0·027 per cent. to 0·044 per cent. per 1°C.; for the name "*German silver*" is applied indifferently to alloys the composition of which varies from 100 parts of copper, 79 parts of nickel, and 53 of zinc to 100 parts of copper, 20 parts of nickel, and 30 of zinc.

The coefficient b of the term t^2 in the formula

$$r_t = r_o\,(1 + a\,t + b\,t^2)$$

is said to be zero for the platinum-silver alloy, to have a small positive value for German silver, a small negative value for the gold-silver alloy, for mercury and for platinum (*see* page 265), and a comparatively large positive value for iron.

87. Conductors of Large Specific Resistance have Small Temperature Coefficients.—On comparing Table No. IX. with Table No. VII. (page 259) it will be observed that, if the metals and alloys be arranged in increasing order of specific resistance, they are arranged roughly in decreasing order of temperature variation, or, in other words, *the poorer the conductor the smaller its temperature variation of resistance.* And not only does the temperature variation become less and less as the specific resistance of the material increases, but it passes to the other side of zero and is *negative* in the case of a bad conductor like carbon, which, in the form used in electric arc-lamps, has a specific resistance of about 0·01 *ohm* per centimetre cube—a value, roughly, 6,250 times as great as that of copper. That is to say, the resistance of carbon

diminishes with *increase* of temperature; for example, the resistance of the carbon filament of a glow-lamp, when glowing at its normal brilliancy, is only about three-quarters of the resistance it possesses when cold. This property of carbon has been utilised by making a resistance of a metallic wire in series with a carbon filament, so arranged that the increase of the resistance of the wire caused by rise of temperature was practically balanced by the simultaneous diminution in the resistance of the carbon filament.

When we come to still poorer conductors, such as gutta-percha or indiarubber, which are, therefore, usually termed insulators, the temperature coefficient is not only *negative*, but is numerically much larger than it is for any metal. For example, the gutta-percha which is usually employed in the manufacture of submarine cables has a specific resistance of about 350×10^{12} *ohms* per cubic centimetre at 24°C., or about 200 million million million times the specific resistance of the copper conductor; but this high resistance is *diminished* to *one-ninth* by an increase of temperature of only 15°C.

This connection between high specific resistance of a metallic alloy and low temperature coefficient has led people to seek for metallic alloys of higher and higher specific resistances. About eight years ago it was found that adding a trace of tungsten to German silver raised its specific resistance from about 20 microhms to 34 microhms per cubic centimetre, and lowered its temperature coefficient from about 0·044 to 0·02 per cent. per 1°C. The substance thus produced is called "*platinoid*," and has been much used in the construction of resistances.

At the present time, *thin* platinoid wire can be purchased having a specific resistance as high as 35·9 microhms per cubic centimetre at 0°C, and a temperature coefficient as low as 0·0178 per cent. per 1°C.

Going still farther, Mr. Weston, by adding manganese to copper instead of, or in addition to, nickel,

has succeeded in preparing alloys whose resistance does not vary at all for ordinary changes of temperature, or, like the resistance of carbon, actually *diminishes* with rise of temperature.

These manganese alloys have been very fully investigated at the Physikalische Technische Reichsanstalt, the German Government physical laboratory at Berlin, and these alloys, with as much as 30 per cent. of manganese and having a specific resistance over 100 times that of copper, have been prepared. The particular alloy, however, which the work of this Institute has shown to be the best for ordinary purposes is one containing 85 per cent. of copper, 12 per cent. of manganese, and 3 per cent. of nickel by weight, and is called "*manganin.*" *Manganin*, which has a specific resistance of about 42 microhms per cubic centimetre, or about 28 times that of copper, is now manufactured commercially in Germany, and, excepting when the most minute accuracy is desired, the variation of the resistance of commercial manganin may be regarded as zero for ordinary changes of temperature. (*See* the note on page 312).

Iron wire or iron ribbon, on account of its low price, is sometimes used for resistances to carry large currents. There is not, however, much to recommend its use, since the temperature coefficient of iron is high, and its specific resistance is only about six times as large as that of copper. Mercury, on the other hand, placed in long shallow troughs cut in a wooden or ebonite board (Fig. 133), is an excellent material to employ for a resistance to carry a large current, when it is desired to alter this resistance between wide limits quite gradually. For mercury has about 63 times the volumetric resistance of copper and a low temperature coefficient, the resistance r_t of a column of mercury of given length and cross-section at any temperature t°C. being given, it is said, by the formula

$$r_t = r_o \ (1 + 0{\cdot}000{,}748{,}5 \ t - 0{\cdot}000{,}000{,}398 \ t^2).$$

where r is the resistance at 0°C. Further, it is clear that by moving the copper bridge pieces B, B (Fig. 133) the resistance between the terminals T, T can be altered between wide limits and altered perfectly gradually.

With the bridge pieces B, B placed as shown in Fig. 133, the current passes from right to left along the whole length of the front mercury trough, from left to right along the whole length of the second, from right to

Fig. 133.—Set of Mercury Troughs used as an Adjustable Resistance for Large Currents.

left along about two-thirds of the length of the third mercury trough, and from left to right along about two-thirds of the length of the fourth. When the current is very large, it may be sent from right to left through two or more of the mercury troughs *in parallel*, and back again to the other terminal through two or more of the other mercury troughs joined *in parallel*.

The following table (No. X.) gives a list of the *approximate* specific resistances and temperature coefficients of materials commonly employed in the construction of resistances :—

TABLE X.

SPECIFIC RESISTANCE AND PERCENTAGE TEMPERATURE VARIATION OF MATERIALS USED IN CONSTRUCTING RESISTANCE COILS.

Name of Material.	Approximate.	
	Resistance in International Microhms per Centimetre Cube at 0°C.	Percentage Variation of Resistance per 1°C.
Iron	9·69	0·5
Phosphor bronze	7·8	0·08
German silver	19 to 30	0·04 to 0·028
Platinum-silver (1 oz. platinum, 2 oz. silver)	24·3	0 03
Platinoid	34 to 35·9	0·02 to 0·0178
Manganin (85 oz. copper, 12 oz. manganese, 3 oz. nickel)	42	0
"Ja Ja" wire	51·5	—0·0076
Carbon	4,500 to 10,000	—0·03

Example 35.—To find the resistance of a wire 52 metres long, 1 square millimetre in section at 22°C., made of pure copper, hard drawn, density 8·90.

Resistance required in ohms.

$$= \frac{1{\cdot}629}{10^6} \times \frac{52 \times 100}{\frac{1}{100}} \times (1 + 0{\cdot}00407 \times 22).$$

Answer.—0·923 ohm.

Example 36.—To find the resistance of a wire 110 feet long, $\frac{1}{20}$th of an inch in diameter, at 46°C., made of pure annealed platinum.

Taking as a mean correcting factor for platinum $(1 + 0{\cdot}0036t - 0{\cdot}000{,}000{,}38t^2)$, the resistance in ohms

$$\text{equals } \frac{3{\cdot}55}{10^6} \times \frac{110 \times 12}{\frac{\pi}{4} \times \frac{1}{20^2}} \times (1 + 0{\cdot}0036 \times 46 - 0{\cdot}000{,}000{,}38 \times 46^2).$$

Answer.—2·78 ohms.

Example 37.—At what temperature will a wire $3\frac{1}{2}$ miles long, $\frac{1}{12}$th of a square inch in section, made of German silver, have a resistance of 22·23 ohms?

Taking as a mean value 8·27 microhms per inch cube. (*See* Table VII.)

$$22 \cdot 23 = \frac{8 \cdot 27}{10^6} \times \frac{3 \cdot 5 \times 5280 \times 12}{\frac{1}{12}} \times (1 + 0 \cdot 00044 \times t).$$

Answer.—22°·73 C.

Example 38.—Which has the greater resistance, a copper wire 20 feet long, 0·015 inch in diameter, or a platinum-silver wire 10 feet long, 0·037 inch in diameter, at 0°C.?

The resistance of the copper wire will be to that of the platinum-silver as $\frac{20 \times 1 \cdot 56}{0 \cdot 015^2}$ is to $\frac{10 \times 24 \cdot 33}{0 \cdot 037^2}$, or as 0·792 to 1.

Hence, the copper wire has rather more than three-quarters of the resistance of the platinum-silver wire.

Example 39.—What will be the resistance at 40°C. of a copper strip 1 mile long, section $\frac{1}{2}$ inch × 1 inch, having 95 per cent. of the maximum conductivity of hard-drawn copper?

The resistance per inch cube $= \frac{1 \times 0 \cdot 626}{0 \cdot 95 \times 10^6}$.

∴ the resistance of the strip

$$= \frac{5280 \times 12 \times 0 \cdot 626}{0 \cdot 95 \times 10^6 \times \cdot 5} (1 + 0 \cdot 0040 \times 40)$$

$$= 0 \cdot 0969 \text{ ohm.}$$

Answer.—0·0969 ohm.

Example 40.—What will be the weight of an iron wire 100 yards long, having a resistance of 1 ohm at 0°C.?

An iron wire 1 foot long, weighing 1 grain, has 1·082 ohm resistance at 0°C. Hence, an iron wire x feet long, weighing x grains, has $x \times 1 \cdot 082$ ohms at

0°C. If the weight is y grains, the resistance is $\frac{x}{y} \times x \times 1{\cdot}082$. x is here 300, and the resistance is 1 ohm. Therefore, $\frac{300^2}{y} \times 1{\cdot}082 = 1$, or $y = 13{\cdot}92$ lb.

Answer.—13·92 lb.

Example 41.—At what temperature, approximately, would a German silver coil, which had 1 B.A. unit of resistance at 16°C., have the resistance of 1 international ohm?

One international ohm equals 1·01358 B.A. unit, therefore the temperature must be raised sufficiently to increase the resistance of the coil by 1·36 per cent.; German silver increases in resistance by about 0·044 per cent. per 1°C. (Table IX.), therefore if t be the increase of temperature above 16°,

$$0{\cdot}044 \times t = 1{\cdot}36$$
$$\text{or } t = 30^\circ{\cdot}9 \text{ C. approximately.}$$

Answer.—The B.A. coil will have a resistance of 1 international ohm at about 46°·9 C.

Example 42.—At what temperature would a metre of mercury 1 square millimetre in section have 1 international ohm resistance?

Answer.—87°·5 C.

Example 43.—A set of resistance coils made of platinum-silver are correct at 14°C. Between what limits of temperature approximately may they be used without correcting the results, if the temperature error is not to exceed $\frac{1}{4}$ per cent.?

The resistance of platinum-silver increases about 0·03 per cent. per 1°C., as stated in the last table; therefore, if t be the number of degrees above or below 14°C., within which the coils may be used without the error exceeding $\frac{1}{4}$ per cent.,

$$0{\cdot}03 \times t = 0{\cdot}25,$$
$$\therefore t = 8^\circ.$$

Answer.—The limits of temperature are, therefore, approximately 6° and 22°C.

Example 44.—If the greatest change of temperature at some particular place between summer and winter is from $-8°$ to 25°C. in the shade, what is the greatest percentage variation in the resistance of a set of German silver coils?

Answer.—1·45 per cent. approximately.

88. Conductivity.—"*Conductivity*" is the reciprocal of resistance, and the name "*mho*" has been suggested by Lord Kelvin as the name for the unit of conductivity; thus the specific conductivity of annealed silver is $\frac{1}{\cdot000{,}001{,}501}$, or 666,500 *mhos.*

It is common to speak of specimens of copper as possessing various percentage conductivities, such as 95, 98, or 101 per cent. What is meant in such cases is that the specific resistance of such copper bears to Matthiessen's standard the ratio of 100 to the number mentioned. Thus hard-drawn copper of 98·5 per cent. conductivity has a resistance per cubic centimetre at 0°C. of $\frac{100}{98\cdot5} \times 1\cdot630$ microhm, or 1·655 microhm.

Example 45.—If the resistance of a sample of commercial metal is 97·5 ohms, whereas the resistance of the same piece of metal, if quite pure, would be 94·3 ohms at the same temperature, what is its percentage conductivity in terms of that of the pure metal?

The conductivity of the sample of commercial metal $= \frac{1}{97\cdot5}$.

The conductivity of the same if pure would $= \frac{1}{94\cdot3}$;

∴ if x be the percentage conductivity,

$$\frac{1}{97\cdot5} = \frac{x}{100} \times \frac{1}{94\cdot3};$$

$$\therefore x = 96\cdot72.$$

Answer.—96·72 per cent. conductivity.

Example 46.—What will be the resistance at 37°C.

of a copper wire 20 metres long, weighing 12 grammes, and having 92 per cent. of the conductivity of pure annealed copper according to Matthiessen's standard?

Answer.—5·04 ohms.

89. Comparison of Electric and Heat Conductivities.—The reciprocals of the numbers given in column 4 of Table No. VII., page 259, will express the relative electric conductivities of the metals for the same length and sectional area. These numbers are given in column 2 of Table No. XI., the electric conductivity of silver being called 100. On comparing these with the conductivities of the metals for heat for the same length and sectional area as given in column 3 of Table No. XI., which are the numbers obtained by Wiedemann and Franz, we observe that the metals arrange themselves *approximately*, but not absolutely, in the same order for the two conductivities.

TABLE XI.

APPROXIMATE RELATIVE CONDUCTIVITIES PER CUBIC UNIT.

Name of Metal.	Electric.	Heat.
Silver, annealed	100	100
Copper, "	94·1	74·8
Gold, "	73	54·8
Platinum	16·6	9·4
Iron	15·5	10·1
Tin, pressed	11·4	15·4
Lead	7·6	7·9
Bismuth	1·1	1·8

If, however, we experiment with worse and worse conductors, we find that the electric conductivity diminishes much more rapidly than the heat conductivity. For example, the electric conductivity of copper is about 10^{20} times the conductivity of vulcanised indiarubber,

whereas the heat conductivity of copper is only about 10^4 times that of vulcanised indiarubber. *Hence, while we can obtain insulators for electricity, or bodies which relatively to the metals do not practically conduct electricity at all, insulators for heat are unknown.*

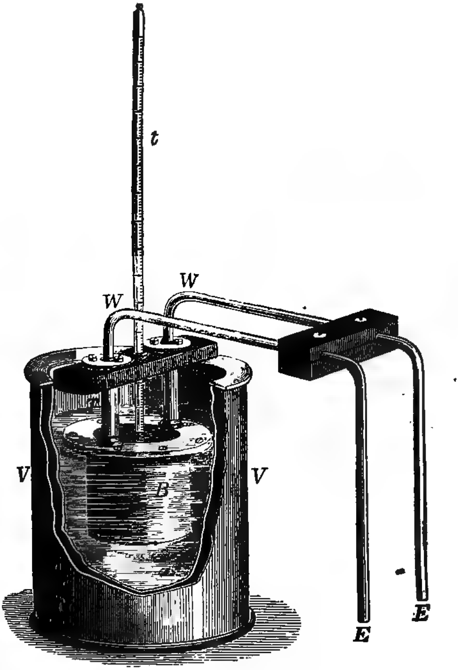


Fig. 134.—Standard Resistance Coil.

90. Standard Resistance Coil.—A resistance coil, when used as an *accurate standard*, is wound inside a brass box, B (Fig. 134), which is inserted in a vessel of water, V V, and the temperature of the water is accurately

noted by means of the thermometer *t*. The hollow cylindrical brass box B, which holds the coil, is made of large diameter outside and inside, so as to expose as much surface as possible to the water in order that the coil inside may acquire the temperature of the water as quickly as possible. It is desirable to provide a stirrer for agitating the water and bringing it all to one temperature; and if water hotter or colder than the room is employed, the height of the brass box may well be made much less than in the figure, and the bulb of the thermometer supported exactly at the same level as the box to avoid the risk of different horizontal layers of the water being at different temperatures, and the mean temperature of the coil not being that of the layer round the bulb of the thermometer. The vessel V V may with advantage have double sides, with an air-space between them, as seen in the figure, to prevent transference of heat between the water and the outside space.

The tubes T, T are to prevent the coil being short-circuited by water getting into the holes through which the rods W, W, attached to the ends of the coil, are brought out. These tubes are made of brass, but they are lined with tubes of ebonite to prevent electric contact between these brass tubes and the rods W, W. Electric connection with these rods is made by dipping their ends E, E into little cups containing clean mercury.

91. Construction of Plug Resistance Boxes.—Although it is theoretically possible, with a wire bridge such as is illustrated in Figs. 126 and 127, pages 246 and 248, to compare any resistance, however large or small, with a standard ohm, there is a practical limit to the range of resistances that can be compared with a single standard coil by using a Wheatstone's bridge. For if the ratio of the proportional arms *c* and *d* (Fig. 123, page 242) be very far from unity, tests made with the bridge become very unsensitive—that is to say, a considerable difference between the values of the ratios of *a* to *b* and of *c* to *d* will cause but a small current to pass through the

galvanometer, unless the battery be very large. In finding the resistance a from the equation $a = b \times \frac{c}{d}$, it is better, therefore, to make the ratio of c to d not greater than about 100 or less than about $\frac{1}{100}$th, and to obtain balance by varying the value of b.

For this purpose it is convenient to use for b a "*resistance box*" such as is seen in perspective in Fig. 140, page 288, and in section in Fig. 135. And when measuring a resistance by means of the simple substitution method (§ 73, page 235), and when using a differential galvanometer (§74, page 238) to measure a variety of unknown resistances, a set of coils of known resistances is indispensable.

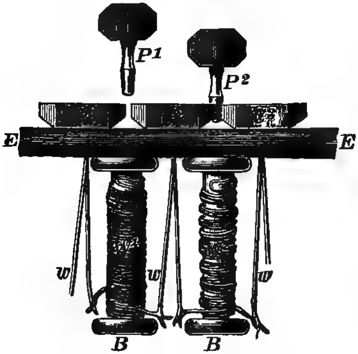


Fig. 135.—Interior of a Resistance Box.

A *resistance box* contains coils of wire, w^1 (Fig. 135), &c., wound on wooden, or ebonite, bobbins, B, &c. The ends of these coils are soldered to stiff wires, w, which again are fastened to the brass pieces c^1, c^2, c^3, &c., the latter being screwed to the wooden or ebonite top, E E, of the resistance box. When a plug P^2 is inserted tightly between the contact pieces c^2 and c^3, the current flows along the short path c^2 P^2 c^3 across the metal plug, and practically none of it passes through the wire wound on the bobbin w^2. If, however, a plug P^1 be withdrawn, then all the current passes through the coil w^1, and none across the space separating c^1 and c^2. Hence by taking out one or more plugs the resistance offered to the flow

of current may be increased at will, and the strength of the current decreased. The brass pieces c^1, c^2, c^3, are *undercut*, as seen in the figure, so that a strip of clean wash-leather can be inserted between them and the ebonite cleaned.

If the ebonite between the brass pieces were left dirty, there would be leakage of the electricity across the film of dirt when the plug was removed, and the resistance between two of the brass pieces would be a little less than that of the coil of wire connecting them.

The value of each coil is clearly marked on the outside of the box, so that the number adjacent to each plug indicates the resistance which is put *in* the circuit when the plug is taken *out*. The brass plugs and the holes into which they fit are made *conical*, and the plugs should be well *ground into the holes* during manufacture. To prevent a resistance being introduced between the plug and the two pieces of brass on each side of it, a good contact is necessary; and to ensure this, a plug, when put into the hole, should receive a *slight downward screwing motion*, when it will be found, with well-made plugs, that, although there is no screw-thread on the plug, the plug can, as it were, be screwed into the hole, so that even the whole resistance box may be easily lifted up by taking hold of one plug after it has properly been put in. Such closeness of contact it would be extremely difficult to secure by simply pressing down the plug, unless a large downward pressure were employed and a corresponding tugging when taking it out, which would soon wrench off the ebonite head. The ebonite heads are usually screwed on to the tops of the brass plugs; but to prevent the head unscrewing in use, a pin should always be driven through the ebonite top and the head of the brass plug after they have been fitted together.

The holes in the figure, seen in the brass pieces themselves (Fig. 137, page 284), may be used for holding the plugs when they are not placed between the pieces of brass to short-circuit the intervening coil; but this use of

the holes in the brass pieces cannot be recommended, since, when the resistances corresponding with the holes that are unplugged are being rapidly counted, a plug stuck in one of the pieces of brass is liable to be mistaken for a plug between two pieces of brass, and hence coils which are actually in circuit are liable to be missed out in the counting up. Further, unless the pieces of brass are very large, the ebonite head of a plug stuck into one of them prevents the next plug being properly inserted or easily removed, when the resistance of the next coil is to be subtracted from, or added to, the resistance in circuit.

The temperature of the coils of a resistance box is liable to change—first, from variation of the temperature of the room in which the box is placed; secondly, from the resistance box being used in rooms of different temperature; and, thirdly, on account of the slight heating of a coil of wire which is produced even by a weak current passing through it. Further, it is very difficult to measure the exact temperature of the coils by putting a thermometer into the box: for, although the thermometer may correctly measure the temperature of the air inside the box, it is quite possible to have a considerable difference between the temperature of a mass of tightly wound silk-covered wire and the air surrounding it, since the temperature of the former varies very slowly.

In order, therefore, that the resistances of the coils may be known, it is necessary that the wire used in winding them should be composed of a metal whose variation with temperature is very small. Resistance coils are, therefore, always constructed of either German silver, platinum-silver, platinoid, or manganin wire.

92. Mode of Winding Resistance Coils, and Gauge of Wire Employed.—Not only must a special metal be employed in making resistance coils, but the wire must not be wound on the bobbin in the ordinary way. If it were wound as cotton is on a reel, then each bobbin in a resistance box would act as a magnet when a current

passed through it, and a box full of electromagnets would be a most inconvenient thing to have near a delicate galvanometer used in testing resistances, since the tester would be constantly in doubt as to whether the deflection observed when putting on the current was due to want of adjustment in the resistance or to the temporary magnetisation of the adjacent resistance box. Hence the silk-covered wire of a resistance coil is wound back on itself as shown in Fig. 135, so that the current, in passing through the wire, first goes several times round the bobbin in one direction and then an equal number of times back again in the opposite direction, so that the two magnetic effects neutralise one another.

The disturbing magnetic effect that might otherwise have arisen when using resistance coils is overcome by this double mode of winding; but the magnetic action of a current passing round an ordinary reel of wire, or a coil wound for a galvanometer or for an electromagnet, &c., which cannot, of course, be doubly wound, must be carefully taken into consideration when anything of this form has to be tested for resistance. As such coils have to be tested after being wound, they must, when it is desired to test them, be placed so far away from the galvanometer, or in such a position relatively to the galvanometer, that the mere passage of the current round the coil produces by itself no direct action on the galvanometer needle when no current is allowed to pass through the galvanometer itself.

When a resistance coil has to carry a large current, the heat produced by the current can be more easily dissipated, and an excessive rise of temperature avoided, by using coils of uncovered wire hanging freely in the air. To use coils doubly wound like those seen in Fig. 135 would be very difficult with bare wires hanging in the air, for there would be great danger of the convolutions of bare wire which constitute one half of the coil touching those of the other half. If this occurred, the resistance of the coil would be, of course,

altered, and, in addition, since the potentials of the adjacent parts of the two halves where the current enters and leaves the coil would differ considerably if the current were strong, there would be considerable risk of sparking if a contact occurred.

To overcome this difficulty, and still to obtain a magnetic balance, the following arrangement, due to Mr. Mather and the author, and seen in Fig. 136, may be employed. Each bobbin consists of two spirals, of the same resistance and containing the same number of convolutions, joined up *in parallel*, but one coil is wound right-handed fashion and the other left-handed. The current, therefore, divides into two equal parts, which, circling round the two coils in opposite directions, balance one another's magnetic effects. With this device the points of the two coils which are adjacent have practically the same potential; therefore no serious change will be caused in the parallel resistance, nor will injurious sparking occur if by chance the two coils swing into contact.

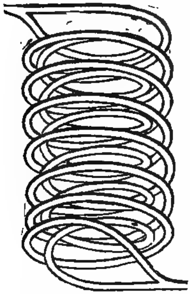


Fig. 136.—Ayrton and Mather's Non-inductive Resistance Coil for Large Currents.

The selection of the gauge of wire to be used in winding a coil which is to have a particular resistance depends on the current the coil has to carry without heating, the accuracy with which the resistance has to be adjusted, and the price to be charged for the coil. That a coil having a particular resistance will be the more expensive the larger the diameter of the wire, is clear, for, since the resistance of a wire is proportional to its length and inversely as the square of its diameter, the greater the cross-section of the wire the greater must be its length, and hence the weight of metal required for a given resistance will be proportional to the cube of

the diameter of the wire employed. More silk will also be employed in insulating a long thick wire than a short thin one. If, however, the surface of the wire be exposed to the cooling action of the air, as in Fig. 136, the rise of temperature of the long thick wire traversed by a given current will be less than that of a short thin wire of the same resistance. And, lastly, as the final adjustment of the resistances of the coils, seen in Fig. 135, is effected by sliding the bare end of the wire of a coil over the end of the stiff wire *w* while the solder connecting the two is hot and melted, and since a given alteration in the length of a long thick wire produces a less change in the resistance than the same change in the length if the wire be thin, coils wound with thick wire can be more exactly adjusted than if wound with a thin wire so as to have the same resistance.

93. Values of Coils for Resistance Boxes and for Commercial Wheatstone's Bridges.—Two things have to be considered in deciding on the values that should be given to the coils respectively of a resistance box—the first, what is the maximum resistance it is required that the box should possess when all the plugs are withdrawn? secondly, how many coils is the box to contain? If, for example, a total resistance of 100 ohms be desired and the box is to contain twelve coils, it will be found convenient to wind the coils so that they have the following resistances respectively:—0·1, 0·2, 0·2, 0·5, 1, 1, 2, 5, 10, 10, 20, and 50 ohms, since with these twelve coils any resistance between 0·1 ohm and 100 ohms can be obtained with an accuracy of 0·1 ohm. For example, 37·6 ohms would be made up by unplugging the 1st, 4th, 7th, 8th, 10th, and 11th coils, or by unplugging the 1st, 4th, 5th, 6th, 8th, 9th, and 11th coils.

The addition of four more coils of 100, 100, 200, and 500 ohms respectively to the preceding twelve will enable any resistance between 0·1 and 1,000 ohms to be built up with an accuracy of 0·1 ohm. Similarly the

addition of four more coils of 1,000, 1,000, 2,000, and 5,000, making twenty in all, extends the range from 0·1 ohm to 10,000 ohms.

If, however, the box is to be used with a Wheatstone's bridge, a resistance much smaller than the smallest, or much larger than the largest, resistance in the box can be measured by making the ratio of the proportional

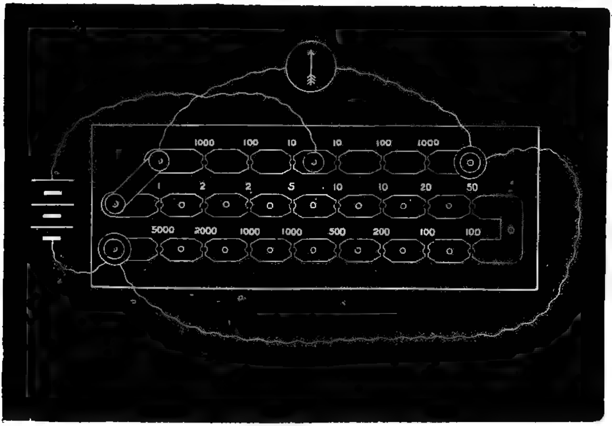


Fig. 137.—Top of a Commercial Wheatstone's Bridge.

arms unequal. Hence resistances of a small fraction of an ohm, or of hundreds of thousands of ohms, can be measured with more or less accuracy by comparing them with a resistance box which does not contain coils either of a very small or of a very large resistance. But as already explained in § 91, page 277, there is a practical limit to the inequality that can be given to the arms of a bridge if the measurements are to be accurate. Hence the values of the resistance coils in commercial bridges are frequently those shown in Fig. 137, the proportional arms consisting of two 10-ohm, two 100-ohm, and two 1,000-ohm coils,

while the comparison coils have resistances of 1, 2, 2, 5, 10, 10, 20, 50, 100, 100, 200, 500, 1,000 1,000, 2,000, and 5,000 ohms respectively. With this arrangement resistances between $\frac{1}{111}$ of an ohm and 1,110,000 ohms can be measured with a galvanometer of sufficient sensibility.

For the sake of ease in calculation, it is usual to employ only one of the coils in each half of the proportional arms at a time, so that in practice the ratios used are $\frac{10}{10}$, $\frac{100}{100}$, $\frac{1000}{1000}$, $\frac{10}{100}$, $\frac{10}{1000}$, $\frac{100}{10}$, $\frac{100}{1000}$, $\frac{1000}{10}$, $\frac{1000}{100}$, and the range of resistance that can be tested, lies between 0·01 ohm and 1,000,000 ohms. Having decided on the particular ratio to be employed, the particular pair of proportional coils that should be selected to produce this ratio and produce the greatest sensibility depends on the approximate value of the unknown resistance. For example, if equal arms are to be used and the unknown resistance is about 130 ohms, use the two 100-ohm, and not the two 10- nor the two 1,000-ohm proportional coils. Again, if the unknown resistance is of the order 400 ohms and the true resistance is desired to the first decimal place, a ratio of $\frac{1}{10}$th must of course be employed, and this ratio should be produced by means of the 100- and the 1,000-ohm proportional coils rather than by means of the 10- and the 100-ohm pair. Or generally *the greatest sensibility will be obtained when the four arms of the bridge are as nearly equal as is consistent with the ratio adopted.*

94. Portable Forms of Wheatstone's Bridge.—When a Wheatstone's bridge has to be carried about, it is convenient to have the battery and the galvanometer keys permanently attached to the box containing the comparison and the proportional coils. Such a combination is seen in Fig. 138, which represents the arrangement called a "*Post Office Bridge.*"

Occasionally the battery and galvanometer are in

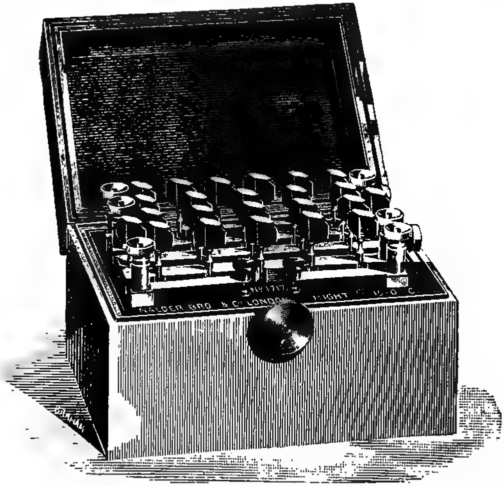


Fig. 138.—Post Office Wheatstone's Bridge.

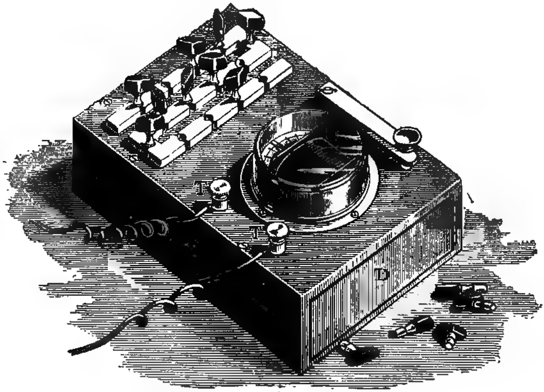


Fig. 139.—Portable Wheatstone's Bridge.

addition attached to the resistance box so that the comparison coils, proportional coils, keys, battery, and galvanometer, all form one piece of apparatus (Fig. 139). Hence, the only connection that has to be made when a resistance has to be measured is joining the two ends of the unknown resistance to the terminals T, T.

The portable bridge seen in Fig. 139 is intended for the measurement of a small range of low resistances. There are, therefore, only two proportional coils of equal value—viz., 10 ohms, contained inside the box, and the comparison coils have the values 0·01, 0·02, 0·02, 0·05, 0·1, 0·1, 0·2, 0·5, 1, 1, 2, 5, 10.

When the battery is exhausted the cells can be withdrawn, by opening the door, D, refilled and used again.

95. Calibrating a Galvanometer by using Known Resistances and a Constant P.D.—From Ohm's law it follows that if a constant P.D. be maintained between the ends of any circuit, the current passing through the circuit is inversely proportional to its resistance. In § 159, page 522, it will be seen that if the ends of the circuit be joined to the terminals of a galvanic cell of very low internal resistance compared with that of the circuit itself, a constant P.D. will be automatically maintained. Consequently, if the terminals of the apparatus shown in Fig. 140 (one terminal T only being actually seen in the figure) be connected with a cell of very low internal resistance, the current passing through the galvanometer or detector D will be inversely proportional to the sum of the resistances of the key K, the detector D, and the resistance box R.

Now the resistance of the key K will be extremely small when the spring is depressed, if the platinum contact points be clean. These can be best cleaned by inserting a piece of ordinary paper between them, pressing down the spring, and pulling away the paper while the points are pressed together. Neither

emery nor glass-paper should be used, as it rubs away the platinum, and still less should the contacts be scraped with a knife or a file. The resistance of the galvanometer D is a constant, say g ohms, and the resistance of the box R may be made to have different values.

First unplug such a resistance r_1 ohms in the box that the deflection of the galvanometer is $d_1°$, which may

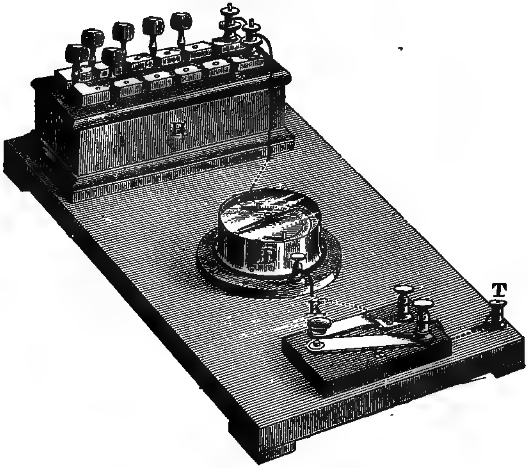


Fig. 140.—Calibrating a Galvanometer by using Known Resistances.

conveniently be made about 10°. Next unplug such a resistance r_2 ohms that

$$r_2 + g = \tfrac{1}{2}(r_1 + g),$$
or
$$r_2 = \tfrac{1}{2}(r_1 - g),$$

and let the deflection now be $d_2°$. Then, since the resistance in circuit has been halved and the P.D. has remained constant, the deflection $d_2°$ corresponds with

twice the current that produced the deflection $d_1°$. Similarly, if we unplug a resistance r_3 ohms such that

$$r_3 = \tfrac{1}{3}(r_1 - g)$$

and obtain a deflection $d_3°$, this deflection will correspond with *three* times the current that produced the first deflection $d_1°$, &c. In this way a series of deflections can be obtained corresponding with currents proportional to 1, 2, 3, 4, &c., and a relative calibration curve can be drawn: and then, if desired, a scale may be constructed for the galvanometer, in accordance with the method described in § 15, page 58, by means of which the relative strengths of currents can be read off at once.

Shunts.

96. Shunts.—We have already seen (§ 29, page 110), when calibrating a galvanometer by comparing it with a standard galvanometer, and again when using a Wheatstone's bridge (Fig. 128, page 251), that it is sometimes convenient to employ a by-path, or shunt, to convey a portion of the current, so that the current passing through the galvanometer is less than the current in the main wires connected with it.

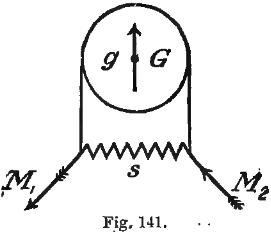


Fig. 141.

We will now consider what must be the relative resistances of the shunt and galvanometer to allow any particular fraction of the whole current to pass through the galvanometer.

Let g be the resistance of a galvanometer, s that of the wire shunting it, and let V be the P.D. between the terminals of the shunted galvanometer which is joined to the mains M_1 and M_2 (Fig. 141). Then if G and S be

the currents that pass respectively through the galvanometer and shunt,

$$G = \frac{V}{g}$$

$$S = \frac{V}{s}$$

$$\therefore \frac{G}{S} = \frac{s}{g},$$

or *the currents in the galvanometer and shunt bear to one another the inverse ratio of the resistances.*

Also, by a well-known rule of proportion, it follows that

$$\frac{G}{G + S} = \frac{s}{g + s},$$

$$\text{and} \quad \frac{S}{G + S} = \frac{g}{g + s},$$

but $G + S$, the sum of the currents flowing through galvanometer and the shunt respectively, is equal to the current M in the mains M_1 or M_2, hence

$$\frac{G}{M} = \frac{s}{g + s},$$

$$\text{and} \quad \frac{S}{M} = \frac{g}{g + s},$$

or the *current in either branch bears to the whole current the ratio of the resistance of the other branch to the sum of the resistances of the two branches.*

97. Multiplying Power of a Shunt.—Since

$$M = \frac{g + s}{s} \times G,$$

the fraction $\frac{g + s}{s}$ is frequently called the "*multiplying power of the shunt*"—that is, the quantity that the current flowing through the galvanometer must be multiplied by to obtain the total current, or current in the main.

As an example of the last equation, let us suppose that we desire that G shall be one-tenth of M; then

$$\frac{s}{g+s} = \frac{1}{10},$$

$$\text{or } s = \frac{1}{9}g,$$

or generally, if we desire that $\frac{1}{n}$th of the whole current shall pass through the galvanometer,

$$\frac{s}{g+s} = \frac{1}{n},$$

$$\text{or } s = \frac{1}{n-1}g.$$

Example 47.—A galvanometer of 2,572 ohms' resistance is shunted with a resistance of 285·8 ohms. What fraction of the main current passes through the galvanometer?

Answer.—$\frac{G}{M} = \frac{s}{g+s} = \frac{285{\cdot}8}{2857{\cdot}8} = \frac{1}{10}.$

Example 48.—A galvanometer has 5,461 ohms' resistance. What must be the resistance of the shunt in order that $\frac{1}{100}$th of the main current may pass through the galvanometer?

Answer.—$\frac{s}{5461+s} = \frac{1}{100}$, therefore $s = 55{\cdot}16$ ohms.

Example 49.—A galvanometer and its shunt are both wound with copper wire. The multiplying power of the shunt is 100 when the temperatures of the galvanometer and of the shunt coils are the same. What is the multiplying power when the temperature of the galvanometer coils is 5°C. above that of the shunt?

Answer.—101·98.

98. Combined or Parallel Resistance.—It would be, of course, possible to substitute for the two resistances g and s, which are in parallel (Fig. 141), a single wire of

resistance x such that *for the same potential difference*, V, *between its terminals*, the current flowing through it should be equal to the sum of the currents flowing through the two parallel circuits.

Such a resistance x is called the "*combined resistance*" or the "*parallel resistance*" of the circuits.

To find x we have:—

the current that would flow through $x = \frac{V}{x}$,

the current flowing through $g = \frac{V}{g}$,

the current flowing through $s = \frac{V}{s}$.

And since, by hypothesis, the first current is equal to the sum of the two latter—

$$\frac{V}{x} = \frac{V}{g} + \frac{V}{s}$$

$$\text{or } \frac{1}{x} = \frac{1}{g} + \frac{1}{s},$$

therefore *the conductivity* (see § 88, page 274) *of the equivalent conductor is equal to the sum of the conductivities of the two branch circuits.*

From the last equation it follows that

$$x = \frac{g\,s}{g + s},$$

or the combined resistance of two conductors in parallel is equal to the product of the resistances divided by their sum.

Hence, for example, when s is made equal to $\frac{1}{9}\,g$, so that G, the current in the galvanometer, is one-tenth of M, the current in the mains—

$$\frac{g\,s}{g + s} = \tfrac{1}{10}g,$$

or the parallel resistance of the galvanometer and shunt

is, in this case, equal to one-tenth of that of the galvanometer alone.

In the same way, if there be any number of resistances, a, b, c, d, &c., in parallel (Fig. 142), and x be a single resistance, such that *with the same potential difference* at its terminals the current that will flow through x

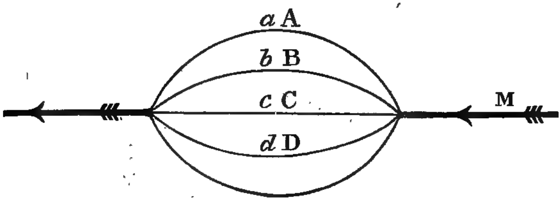


Fig. 142.—Branch Circuits in Parallel.

is equal to the sum of the currents that flow through all the resistances a, b, c, d, &c., it follows that

$$\frac{1}{x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \text{\&c.},$$

or *the conductivity of the equivalent conductor is equal to the sum of the conductivities of all the branch circuits;*

$$\text{and } x = \frac{1}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \text{\&c.}}\,.$$

For experiments on the combined resistance of branch circuits in parallel, four, or more, coils of wire may be conveniently wound in grooves turned on a wooden bobbin, as seen in Fig. 143, one end of each coil being connected with the binding screw S_1, and the other ends respectively with the small brass mercury cups C_1, C_2, C_3, and C_4, as indicated symbolically by the zigzag lines marked on the face of the bobbin. The brass piece R R, to which the binding-screw S_2 is attached, has small holes drilled in it, and into each of these holes a drop of mercury is poured, so that, by inserting one or more of the wire bridge pieces B_1, B_2, B_3, B_4, into the pair

of mercury cups, any one of the coils can be inserted alone between the binding screws S_1 and S_2, or any two or more coils can be joined up in parallel. Then, by measuring the resistance of each of the separate coils, and of the coils joined up in various ways in parallel, the formulæ given above can be experimentally verified.

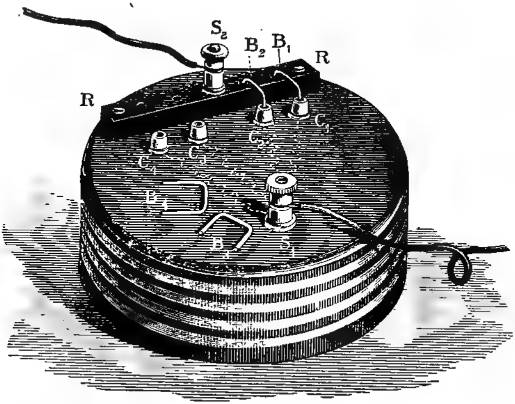


Fig. 143.—Set of Four Coils used for Testing the Resistance of Branch Circuits in Parallel.

Example 50.—Resistances of 25, 32, 17, and 40 ohms are put in parallel with one another. What is the combined resistance?

$$\frac{1}{x} = \frac{1}{25} + \frac{1}{32} + \frac{1}{17} + \frac{1}{40} \quad \therefore x = 6{\cdot}4 \text{ ohms.}$$

Answer.—6·4 ohms.

Example 51.—A coil of wire has 1,125 ohms' resistance. What resistance placed in parallel with it will make the combined resistance 1,000 ohms?

Answer.—9,000 ohms.

Example 52.—The wire of a resistance coil has 10,000 ohms' resistance, but the surface of the ebonite between the terminals, having been imperfectly cleaned,

has a resistance of only 870,000 ohms. What is the combined parallel resistance between the terminals?

Answer.—9,886 ohms.

99. Currents in Parallel Conductors.—If A, B, C, D, &c., be the currents in the various branch circuits (Fig. 142), and M be the current in the main, we have

$$\frac{A}{B} = \frac{\frac{1}{a}}{\frac{1}{b}}$$

$$\frac{A}{C} = \frac{\frac{1}{a}}{\frac{1}{c}}$$

$$\frac{A}{D} = \frac{\frac{1}{a}}{\frac{1}{d}}.$$

&c.;

$$\therefore \frac{A}{A + B + C + D + \&c.} = \frac{\frac{1}{a}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \&c.},$$

$$\text{or} \quad \frac{A}{M} = \frac{\frac{1}{a}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \&c.}$$

$$\text{Similarly,} \quad \frac{B}{M} = \frac{\frac{1}{b}}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \&c.}$$

&c.

Example 53.—Resistances of 12, 7, 2, and 30 ohms are placed in parallel with one another, and a current of 10 amperes, as measured by an ammeter in the main circuit, passes through the combination. What are the currents in the respective branches?

Answer.—1·097, 1·881, 6·583, and 0·439 amperes respectively.

100. Usual Method of Constructing a Shunt Box.—The three coils, having respectively the $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of the resistance of the galvanometer, are usually inserted in a small box *b* (Fig. 144), which accompanies the galvanometer. The terminals of the galvanometer, as well as the two wires which connect the galvanometer

with the rest of the circuit, are joined to the binding screws S, S on the shunt box, and each of the three shunt coils has one of its ends connected with the brass piece C, while the other ends are connected respectively with the brass pieces D, E, and F, as indicated symbolically in Fig. 145. If, then, the brass plug P′ be inserted in the hole between the brass bar A B and the brass piece C, all the current will pass from A B to C, through the plug, and practically none will pass through the galvanometer, since the resistance of the path from A B to C through the plug is extremely small compared with that through the galvanometer.

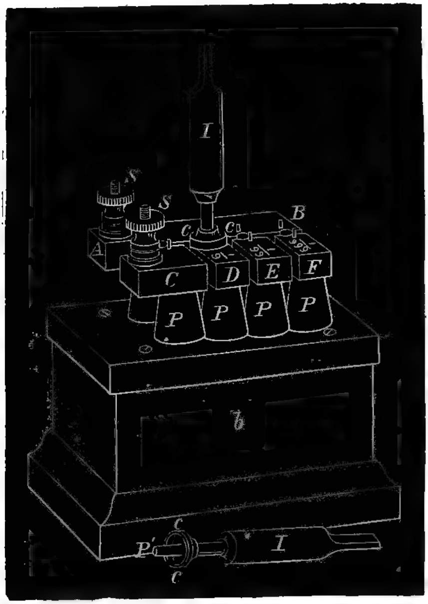


Fig. 144.—High Insulation Shunt Box.

If, on the other hand, the plug be inserted in the hole between A B and D, as in Fig. 144, the current will pass from A B to D through the plug, and from D to C through the coil in the shunt box, which connects with C. And as this coil has $\frac{1}{9}$th of the resistance of the galvanometer, $\frac{1}{10}$th of the total current will pass through the galvanometer. Similarly, if the plug

be inserted in the hole between A B and E or in the hole between A B and F, $\frac{1}{100}$th or $\frac{1}{1000}$th of the whole current will pass through the galvanometer.

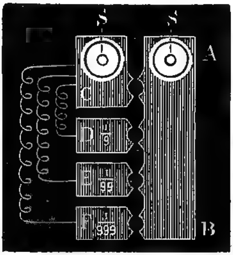


Fig. 145.—Top of Shunt Box, showing Parallel Arrangement of Shunts.

Instead of employing three coils whose resistances respectively are $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of that of the galvanometer, and joining one end of each of these coils to the brass piece C, the coils may be joined up in series between the brass pieces C and D, D and E, E and F respectively, like the coils of an ordinary resistance box (Fig. 146). In this case the coils must have resistances $\frac{1}{999}g$, $(\frac{1}{99} - \frac{1}{999})g$, and $(\frac{1}{9} - \frac{1}{99})g$, and the block marked F will correspond with the $\frac{1}{9}$th shunt, while that marked D will correspond with the $\frac{1}{999}$th shunt, as indicated symbolically in Fig. 146.

In order to obtain very good "*surface insulation*" (*see* Vol. II.), the brass pieces A B, C, D, E, and F are, in the particular shunt box shown in Fig. 144, mounted on ebonite pillars P, P, P, P; and, to avoid the insertion of the plug into one or other of the holes pushing these pillars outwards and so preventing the plug making firm contact with the pieces of brass on each side of it, there is a spring cap *c c*, sliding on the plug, which passes over the two vertical pins on each side of the hole, and so holds the brass pieces together against the wedging action which tends to force them asunder when the plug is pressed in. The plug has

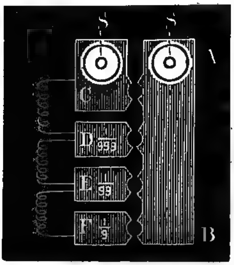


Fig. 146.—Top of Shunt Box, showing Series Arrangement of Shunts.

K*

a long ebonite handle I, *which should be held by the flat part at the end* to prevent leakage taking place along the surface of the handle and through the body of the experimenter to the ground.

101. Increase of the Main Current Produced by Applying a Shunt.—Although the current passing through an unshunted galvanometer is the same as the current in the main, and although the current passing through a shunted galvanometer is always $\frac{s}{g+s}$ times the current in the main, it must *not* be assumed that the application of a shunt to a galvanometer diminishes the current passing through it in

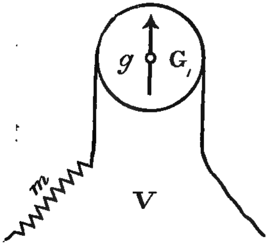


Fig. 147.

the ratio of unity to $\frac{s}{g+s}$. For the application of the shunt diminishes the resistance in the circuit by the difference between g and $\frac{g\,s}{g+s}$, and this diminution of the resistance of the circuit increases the current in the main, more or less, depending on the arrangement of the circuit; so that the current in the main *after* the application of the shunt is greater than the current in the main *before* the shunt was applied by an amount that may be very small, or may be very large.

Let the circuit consist of a resistance m in series, with a galvanometer of resistance g, and let a fixed P.D. of V volts be maintained between the terminals of this circuit (Fig. 147), then G_1, the current passing through the main or through the galvanometer, equals

$$\frac{V}{m+g}.$$

Next let the galvanometer be shunted with a shunt of resistance s (Fig. 148), and let the P.D. of V volts be still maintained between the terminals of the complete circuit shown in Fig. 148, then the current now passing along the main equals

$$\frac{V}{m + \frac{g\,s}{g + s}},$$

and G_2, the current now flowing through the galvanometer equals

$$\frac{s}{g + s} \times \frac{V}{m + \frac{g\,s}{g + s}},$$

$$\text{or} \quad G_2 = \frac{s\,V}{m\,(g + s) + g\,s};$$

$$\therefore \quad \frac{G_2}{G_1} = \frac{s\,(m + g)}{m\,(g + s) + g\,s}.$$

Now the value of this ratio depends on the size of m; for example, if m be very large compared with g,

$$\frac{G_2}{G_1} = \frac{s}{g + s};$$

whereas if m be very small compared with g,

$$\frac{G_2}{G_1} = \text{unity}.$$

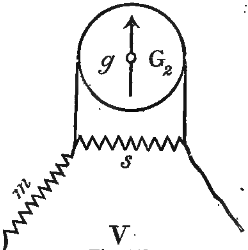


Fig. 148.

That is to say, *if the resistance external to the galvanometer be very large, the galvanometer current* after *the application of the shunt bears to the galvanometer current* before *its application the ratio of s to g + s; while, on the other hand, if the resistance external to the galvanometer be very small, shunting the galvanometer produces very little effect on the current passing through it.* And

this arises from the fact that on applying the shunt in the first case the main current is not changed, while in the second it is increased by an amount almost exactly equal to the current that is shunted past the galvanometer.

For example, let g be 1,000 ohms and s the $\frac{1}{9}$th of g—
(1) let m be 100,000 ohms, then the true ratio of G_2 to G_1 is

$$\frac{111\cdot\dot{1} \times 101{,}000}{100{,}000 \times 1{,}111\cdot\dot{1} + 111{,}111}, \text{ or } 0\cdot1009 \text{ about,}$$

whereas the value of $\frac{s}{s+g}$ is 0·1, which differs by about 1 per cent. from the true ratio, so that the current through the galvanometer is reduced practically to one-tenth of its previous value;
(2) let m be 10 ohms, then the true ratio of G_2 to G_1 is

$$\frac{111\cdot\dot{1} \times 1{,}010}{10 \times 1{,}111\cdot\dot{1} + 111{,}111}, \text{ or } 0\cdot92 \text{ about,}$$

whereas the approximate value of the ratio is unity, which differs by about 8 per cent. from the true ratio, so that the current through the galvanometer remains practically unchanged by the application of the shunt.

An important example of this independence of currents in parallel circuits that can be produced by making the value of m in Fig. 148 very small occurs in the wiring of a house for electric lighting. The glow lamps are all connected *in parallel* with the house mains as indicated in Fig. 149, which represents a portion of the plan of the ground floor of a house, and shows the way in which the electric lighting mains and branch mains are run. At the place where the house mains, H, are connected with the street mains, S, a constant, or nearly constant, P.D. is maintained by the Electric Supply Company, the value of this nearly constant P.D. being frequently 100 volts. Each lamp, L, or each group of lamps, is provided with a switch so

that the current can be turned on to, or off from, each lamp, or group of lamps, independently; and it is obviously important that the turning on, or off, of a switch in one part of a house shall not sensibly affect the light given by the glow lamps in some other part of

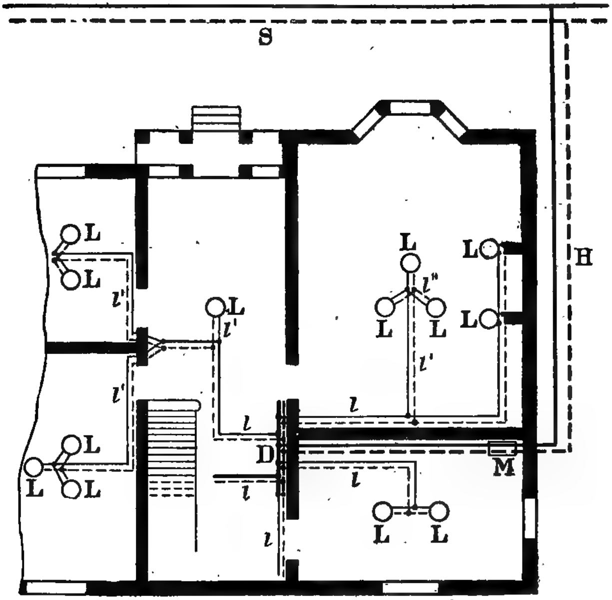


Fig. 149.—Part of the Plan of an Electrically Lighted House.
S, Street Mains; H, Mains to House; M, Supply Meter; D, Distribution Board; l, Leads to the Rooms; l', Branch Leads; L, Glow Lamps.

the house. Now a glow lamp is a very sensitive indicator of any variation of the current passing through it, for the light given out by a new glow lamp, when glowing at about its normal brilliancy, varies about *six per cent.* for each *one per cent.* variation of the current passing through it. Hence it is extremely important to

arrange matters so that the current passing through each lamp is nearly independent of the current passing through any other lamp, and this result is attained by making the resistance of the house wires H, l, l' small compared with the resistance of the carbon filaments of the lamps, in accordance with the principle proved in this section for a galvanometer and shunt.

Example 54.—A galvanometer of 8,100 ohms' resistance is in a circuit having 500,000 ohms' resistance external to the galvanometer. What is the percentage change in the main current made by shunting the galvanometer with a $\frac{1}{9}$th shunt?

Answer.—1·46 per cent.

Example 55.—If a galvanometer have 1,980 ohms' resistance, and a shunt be attached so that the current passing through the galvanometer is only $\frac{1}{100}$th of the total current, what will be the resistance of the shunt, and by how many ohms will the resistance of the circuit be diminished by employing the shunt?

Answer.—Resistance of shunt = 20 ohms.
Diminution of resistance = 1960·2 ohms.

Example 56.—A pair of "*leads*" or branch conductors runs from the street mains, where a P.D. of 100 volts is maintained, to a hall where 150 glow lamps are in use. Each of the lamps would take 0·5 ampere at 100 volts. What must be the resistance of the leads in order that, when all the lamps are burning in parallel, the P.D. between their terminals is 98 volts?

Answer.—The resistance of each lamp is $\frac{100}{0{\cdot}5} = 200$ ohms. Hence the current taken by each lamp at 98 volts is $\frac{98}{200}$, or 0·49 ampere, and the total current through 150 lamps in parallel is $150 \times {\cdot}49$, or 73·5 amperes. The resistance of the leads must be such that there is a "*drop*" of pressure of 2 volts when the current is 73·5 amperes; hence the resistance is $\frac{2}{73{\cdot}5}$, or 0·0272 ohm.

102. Principle of Universal Shunts.—When using a shunt to compare the *relative* strengths of two currents, it is unnecessary to know what is the exact fraction of the main current that passes through the galvanometer, for all that has to be known is the way in which this fraction is varied when the shunt is altered. Carrying out this idea, Mr. Mather and the author have devised a method of applying shunts to a galvanometer in which the resistances of the coils of the shunt box need have

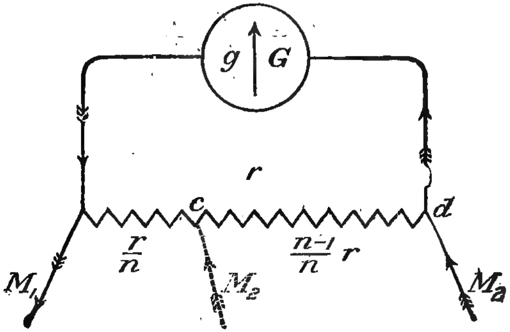


Fig. 150.—Principle of Ayrton and Mather's Universal Shunt.

no special connection with the resistance of the galvanometer itself. Hence *the same shunt box can be used with any galvanometer*.

For example, let a galvanometer of any resistance g ohms be *permanently* shunted with any resistance r ohms (Fig. 150), and when a current of A amperes comes along the main M_2 and leaves by the main M_1 let the deflection of the shunted galvanometer be a. Next, let the main M_2 be moved from the point d to the dotted position at the point c, the fraction of r between the points d and c being $\frac{n-1}{n}$. Now when a stronger current of A′ amperes comes along the dotted main M_2 at

the point c and leaves by the main M_1, let the deflection be a'; then, if the deflections of the galvanometer are directly proportional to the currents passing through it,

$$\frac{A'}{A} = n\frac{a'}{a}$$

whatever be the values of g and of r.

For let the galvanometer current in the first case be G, and in the second G′, then

$$G = \frac{r}{g + r}A$$

$$\text{and} \quad G' = \frac{\frac{r}{n}}{g + \frac{r}{n} + \frac{n-1}{n}r}A';$$

$$\therefore \frac{G'}{G} = \frac{1}{n} \cdot \frac{A'}{A}.$$

$$\text{But} \quad \frac{a'}{a} = \frac{G'}{G},$$

$$\therefore \frac{A'}{A} = n \cdot \frac{a'}{a}.$$

For example, if n be 10 or 100 the ratio of the currents, A′ to A will be exactly 10 times or 100 times the ratio which the deflection a' bears to a, *independently of the values of g and of r.*

103. Method of Constructing a Universal Shunt Box, and its Advantages.—A "*universal shunt box*" constructed on this principle is seen in Fig. 151. The terminals A and B of the shunt box are *permanently* connected respectively with the terminals of the galvanometer, while the terminals B and C of the box are connected with the two main wires which lead the current up to and away from the galvanometer and shunt. The ends of a coil of *any* resistance r ohms are

permanently connected as shown, and at points in this coil corresponding with $\frac{r}{10}$, $\frac{r}{100}$, $\frac{r}{1,000}$ ohms permanent connections are made with the various blocks of the shunt box as illustrated.

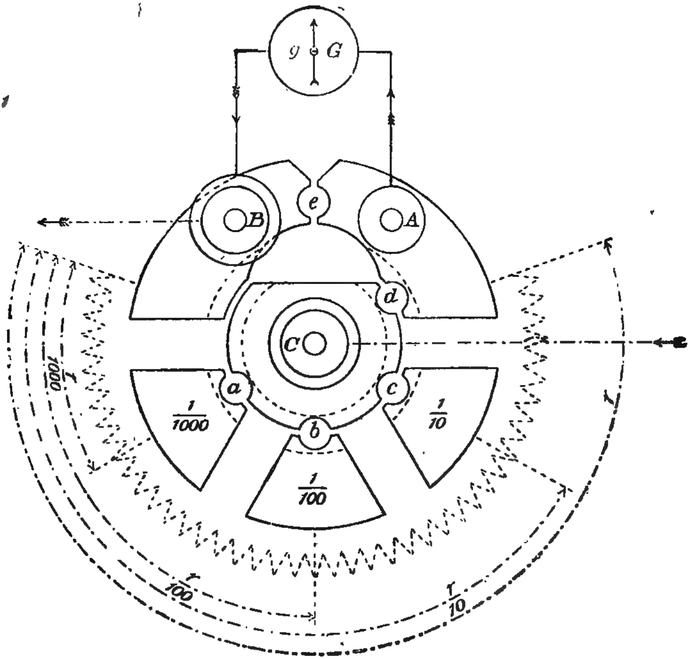


Fig. 151.—Plan of Ayrton and Mather's Universal Shunt Box.

Then, whatever be the resistance of this coil r compared with the resistance of the galvanometer g (either, or both, of which may, therefore, be unknown), it is easy to show that if G amperes be the current flowing

round the galvanometer when a plug is placed into the hole marked *d*, it will be $\frac{G}{10}$, $\frac{G}{100}$, $\frac{G}{1,000}$ amperes respectively when the plug is put instead into the holes marked *c*, *b*, and *a* respectively, if there be the same current in the main circuit.

This method of altering the shunting of a galvanometer by using a fixed resistance *r* and varying the position of the mains, instead of keeping the mains fixed and varying the resistance of the shunt as in Fig. 148, has several important advantages, viz. :—

(1) The *same* shunt box can be used with any galvanometer, &c.

(2) Variations of the temperature of the room produce no error, for we are only concerned with the accurate subdivision of the resistance *r*, which may be made of German silver, platinoid, manganin, or any wire of low temperature coefficient; whereas with the ordinary method of constructing a shunt box (Figs. 145, 146), unless the coils of the shunt box be constructed of the same metal as the coils of the galvanometer, and unless both the shunt box and the galvanometer have *exactly* the same temperature, the multiplying powers of the shunts will not be exactly those marked on the box, even although great care may have been taken by the maker in the original adjustments of the shunt coils. But since the shunts are not inside the galvanometer case, it is very difficult to ensure, from the readings of thermometers, that there is not a difference of some two or three degrees between the mean temperatures of the shunt box and of the galvanometer; so that, although both the shunt coils and the galvanometer coils be wound with copper wire, the resistances of the shunts may easily be 1 per cent. wrong relatively to that of the galvanometer.

In fact, except in the rare case when the galvanometer as well as the shunts is wound with German silver, manganin, or other wire of low temperature coefficient, it is useless spending time making accurate adjustments of the resistances of the shunt coils as ordinarily constructed.

(3) The coils of the *universal shunt box* can be more easily adjusted than those of an ordinary shunt box, for fractions of an ohm need not be used. Indeed, the smallest coil of a *universal shunt box* may have a resistance of several ohms if desired; whereas with a shunt box as ordinarily constructed the resistances of the $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th shunt must be exactly $\frac{1}{9}$th, $\frac{1}{99}$th, and $\frac{1}{999}$th of the resistance of the galvanometer. Therefore, if the resistance of the galvanometer be 1,000 ohms, for example, the shunts must have 111·1, 10·10, and 1·001 ohms' resistance; hence we must construct the $\frac{1}{1000}$th shunt accurately to the one-thousandth of an ohm if we wish it to be correct to even $\frac{1}{10}$th per cent.

Hence it is cheaper to construct a *universal shunt box* that can be used with any galvanometer than an ordinary shunt box which is only intended to be employed with one particular galvanometer.

(4) Lastly, whatever be the values of g, n, and r (Fig. 150), provided that r is less than $g\ (n + \sqrt{n^2 + n})$, the use of the *universal shunt box* produces less change in the resistance of the circuit from its original value than the employment of the ordinary shunt box.

The truth of this last statement may be proved as follows:—With the ordinary method of keeping the main leads M_1, M_2 (Fig. 141, page 289) permanently connected with the galvanometer G, and of varying the current through the galvanometer by altering the resistance s of the shunt, the change that is produced in the resistance of

the circuit on applying this shunt is, as well known, from g to $\frac{g\,s}{g + s}$; that is, the resistance is diminished by $\frac{g^2}{g + s}$. If, then, the shunt be such as to allow $\frac{1}{n}$th of the current to pass through the galvanometer, s equals $\frac{g}{n - 1}$, and the resistance in circuit is diminished by $\frac{n - 1}{n}\,g$ when this shunt is applied. For example, if n be 10, the resistance of the circuit will be diminished by $\frac{9}{10}$ths of g.

Whereas on moving the main M_2 from the point d to the point c, when using the universal shunt box (Fig. 150, page 303), the resistance of the circuit is altered from $\frac{g\,r}{g + r}$ to $\frac{(g + \frac{n-1}{n}\,r)\frac{r}{n}}{g + r}$; that is, the resistance of the circuit is diminished by

$$\frac{1}{g + r}\left\{\frac{n - 1}{n}\,g\,r - \frac{n - 1}{n^2}\,r^2\right\},$$

or increased by

$$\frac{1}{g + r}\left\{\frac{n - 1}{n^2}\,r^2 - \frac{n - 1}{n}\,g\,r\right\},$$

for this expression can be positive or negative, depending on the value of r relatively to that of $n\,g$. Therefore the change in the resistance of the circuit when using the universal shunt box bears to the change when employing the ordinary shunt box the ratio of

$$\frac{1}{g + r}\left\{\frac{n - 1}{n^2}\,r^2 - \frac{n - 1}{n}\,g\,r\right\} \text{ to } \frac{n - 1}{n}\,g,$$

and this ratio, which simplifies to

$$\frac{1}{g + r}\left(\frac{r^2}{n\,g} - r\right)$$

on dividing by $\frac{n - 1}{n}\,g$, will be less than unity as long as r does not exceed $g\,(n + \sqrt{n^2 + n})$.

From this it follows that, if r be not more than 20·488 times the resistance of the galvanometer, the change in the resistance of the circuit, from its original value, produced by using the $\frac{1}{10}$th, $\frac{1}{100}$th, and $\frac{1}{1000}$th shunts will be *less* with a universal shunt box than with an ordinary shunt box. So that if the resistance of no one of a set of galvanometers be less than 1,000 ohms, the universal shunt box may have a total resistance of 20,488 ohms, and still this additional advantage will be gained with *each* of the galvanometers. Or, if the resistance of the universal shunt box be 10,000 ohms, this additional advantage will be gained with any galvanometer of high or low resistance, provided that the galvanometer resistance be not less than 488 ohms.

When the movable main M_2 (Fig. 150) has to be connected with only two points of the fixed resistance r which permanently shunts the galvanometer—that is, when n has only to have one value—it is well to select r so that it is about equal to $n\,g$. For in that case the expression

$$\frac{1}{g+r}\left\{\frac{n-1}{n^2}\,r^2-\frac{n-1}{n}\,g\,r\right\}$$

is practically zero, and, as this expression has been shown to be equal to the change produced in the resistance of the circuit when the main M_2 is moved from d to c, it follows that when r is about equal to $n\,g$ there is practically no change made in the resistance of the circuit by shifting the main. Consequently, whether the resistance of the circuit external to the galvanometer be small or large, the shifting of the main from d to c so as to vary the fraction of the main current which passes through the galvanometer produces practically no change whatever on the main current itself.

For example, if the galvanometer has 1,000 ohms' resistance, and we select r so that its resistance is about 10,000 ohms, then, whatever be the resistance of the rest of the circuit, the main current will be practically

the same whether the main M_2 be joined to the point d or c; whereas the current passing through the galvanometer will be exactly ten times as great with the first position of the main as with the second, whatever be the materials the resistance r and the galvanometer coils are respectively made of, and whatever be the temperatures of the shunt box and of the galvanometer.

With the ordinary method, on the other hand, of making the tenth shunt equal to $\frac{1}{9}$th of the resistance of the galvanometer, it is only when the shunt coil and the galvanometer coils are constructed of the same material and are at exactly the same temperature that the galvanometer current is exactly $\frac{1}{10}$th of the current in the main. And, further, if the resistance external to the galvanometer be comparatively small, the application of the shunt so diminishes the resistance of the circuit, and therefore produces such a large increase in the main current, that the application of the shunt is of little value in diminishing the current passing through the galvanometer.

Example 57.—A *Universal Shunt*, 7,000 ohms in resistance, is employed with a galvanometer having a resistance of 1,270 ohms. What fractions of the main current pass through the galvanometer if the part of the shunt included between the mains is 10 ohms, 70 ohms, 700 ohms, and 7,000 ohms?

The ratio of the galvanometer current to the main current is

$$\frac{s}{g+s} = \frac{10}{8270}, \frac{70}{8270}, \frac{700}{8270} \text{ and } \frac{7000}{8270}$$

respectively, or the fractions are in the ratio $\frac{1}{700}$, $\frac{1}{100}$, $\frac{1}{10}$ to 1.

Example 58.—Taking the galvanometer and shunt referred to in the preceding question, find the percentage difference in maximum sensibility between the galvanometer used with the *universal shunt* and used in the ordinary way.

If the universal shunt is used, the maximum sensibility is obtained when the mains are across the galvanometer terminals, and the galvanometer takes $\frac{7000}{8270} = \cdot 846$ of the main current. If the ordinary method is employed, the galvanometer takes the whole of the main current for maximum sensibility. Hence, the universal shunt gives 15·4 per cent. less maximum sensibility.

Example 59.—If a galvanometer of 1,270 ohms' resistance be employed, and if the resistance of the circuit external to the galvanometer be 200,000 ohms, calculate the percentage variation that will be made in the main current when the sensibility of the galvanometer is diminished from its maximum to one-hundredth of the maximum, first by using a shunt specially constructed for the particular galvanometer, secondly a *universal shunt* of 7,000 ohms in resistance.

Answer.—The percentage change in the main current will be 0·62 when using the ordinary shunt, and 0·50 when the *universal shunt* is employed.

104.—Use of Shunts with a Differential Galvanometer.—We have seen (§ 74, page 241) that if the two coils C and C′ (Fig. 122) of the differential galvanometer have equal resistances, and if, in addition, they be so adjusted relatively to the needle that no deflection is produced when equal currents flow round the coils, no deflection will be produced when A and B have equal resistances, and a P.D. is set up between the points P and Q by any convenient current generator. If, now, one of the coils, say C, be shunted with a shunt, having, say, $\frac{1}{9}$th of the resistance of C, then the parallel resistance of C and its shunt will be $\frac{1}{10}$th of the resistance of C alone (§ 98, page 292). Therefore if the resistance of A be also diminished to $\frac{1}{10}$th of what it was before, the total resistance of the branch PACQ will become $\frac{1}{10}$th of what it previously was, hence ten times as much current will pass through A as through B, but of this

larger current only $\frac{1}{10}$th part will pass round the coil C, and, consequently, there will still be no deflection of the needle. We can generally conclude that if one coil, C, having a resistance g ohms, of a differential galvanometer be shunted with a shunt of s ohms, no deflection will be produced when

$$\frac{\text{resistance of A}}{\text{resistance of B}} = \frac{s}{s + g}.$$

If, therefore, B consist of a box of resistance coils, the resistance of which can be varied from, say, 1 to 10,000 ohms, we can, by the addition of a tenth shunt to one or other of the coils C or C′, of a differential galvanometer, measure resistances varying between 0·1 and 100,000 ohms.

Note to page 258.—Copper, having a specific resistance 1 or even 2 per cent. smaller than the purest specimens which Dr. Matthiessen could prepare chemically, can now be bought commercially in large quantities. But experiments carried out at the Central Technical College by Mr. J. H. Reeves (*Phil. Mag.*, May, 1896) do not support the view that this extra high conductivity is produced by high density, seeing that when the 9 specimens of Messrs. Bolton and Sons' soft copper wire, which Mr. Reeves tested, are arranged in order of diminishing specific resistance, they are not arranged in order of increasing density. Indeed, the best specimen, which had a resistance per cubic centimetre at 0°C. of 1·566 microhm, had a density of only 8·888, which is considerably less than the density usually assigned to copper.

Note to page 269.—A wire called "*Ja Ja*" is now (1896) manufactured at the Westphalian Rolling Mills which has a resistance of about 51·5 microhms per cubic centimetre, and a negative temperature coefficient of —0·0076 per cent. per 1°C., according to the results of tests communicated by Mr. S. Evershed to the author. This material is very valuable for the added resistance in a voltmeter (W of Fig. 86, page 176), since if the coil of the voltmeter be made of copper, having a temperature coefficient of, say, 4 per cent. per 1°C., while the added coil is wound with "*Ja Ja*" wire and has a resistance about 52·6 times that of the copper coil, the resistance of the voltmeter as a whole will be unaffected by changes of temperature.

CHAPTER V.

ELECTRIC ENERGY AND POWER.

105. Work Done by a Current.—Whenever an electric current flows through a circuit work is done, just as whenever a water current flows through a pipe or along a river bed the flowing water does work on the obstacles that obstruct its passage. When a water stream of Q cubic feet per minute falls down a height of f feet, the work done by the water in m minutes equals

$$62{\cdot}43\ \mathrm{Q} f m \text{ foot pounds very approximately,}$$

62·43 being approximately the weight of a cubic foot of water in pounds. So when an electric current of A amperes flows from a point a to a point b through *any* circuit, the potential at b being V volts lower than the potential at a, the work done on the circuit $a\,b$ by the electric current in m minutes equals

$$44{\cdot}23\ \mathrm{AV}m \text{ foot pounds very approximately.}$$

Neither the current of water nor the current of electricity is changed, but the current of water in falling from one level to a lower level, and the current of electricity in falling from one potential to a lower potential, gives up energy, and the amount of the energy lost by the current in *m* minutes is given in all cases by the preceding expressions, provided that there is no apparatus in the circuit in question which gives energy to the current instead of receiving energy from it.

When the stream of water is a steady one, and when it flows through a uniform tube such as *t t* (Fig. 77, page 154), all the energy lost by the water between any two points P_1 and P_3 is converted *directly* into heat, and is employed in slightly warming the water and the tube; so in the same way, when a steady electric current flows through a wire, the wire and the surrounding bodies being at rest relatively to one another, the energy lost by the current is turned *directly* into heat and the wire is warmed. If, however, the obstruction to the passage of the water be produced not merely by objects at rest but by the paddles of a water-wheel which can be moved by the falling water, then a portion of the energy lost by the water appears as mechanical energy given to the water-wheel; so in the same way, when there is a magnet or a piece of iron near the wire conveying the steady electric current, and when the relative positions of the wire and the magnet or iron can be changed by electromagnetic attraction, then a portion of the energy given up by the current is employed in doing work on the movable system. For example, when a current is sent through a galvanometer with a pivoted needle, or through a coil of wire suspended in a magnetic field, or through the coil of an electromagnet with a movable armature, or generally through any "*electromotor*," the current not only does work in heating the wire through which it flows, but it also does work in producing mechanical motion against the controlling force. As soon as the galvanometer needle or the suspended coil

has been deflected to such a position that the force due to the current is balanced by the controlling force, or when the armature of the electromagnet has been pulled down against some stop, or the *electromotor* has been brought to rest by some opposing force becoming greater than the electromotor can overcome, no more mechanical work is done by the current, and all the energy it subsequently loses is directly turned into heat and goes to warm the wire through which the current flows.

The expression 44·23 AVm foot pounds, then, represents in all cases the total amount of work done in m minutes by a current of A amperes flowing through a circuit under a P.D. of V volts, provided that there is no apparatus in the circuit which gives energy to the current instead of receiving energy from it; and the expression may be divided into two parts, one part representing the energy which is lost by the current and turned *directly* into heat, and the other the energy lost by the current which is converted into some form of energy other than heat. If an electromotor be driven by the current and be employed to grind corn or to turn a grindstone, this second portion of the energy will also be turned into heat; but this heat will not be produced by a direct conversion of electric energy into heat, but by a conversion first of electric energy into mechanical energy, and secondly of mechanical energy into heat. If, on the other hand, the electromotor be used to raise blocks of stone to the top of a scaffolding for building purposes, then this second part of the energy will not be turned into heat at all.

If the circuit through which the current flows contains an electrolytic cell, then, although no mechanical work will be done by the current in this cell, chemical change will be effected, and when, as a consequence, chemical energy is added to the electrolytic cell, the work done by the current in producing this chemical energy is analogous with the work done in producing mechanical energy, and must be added to the work done by the

current in directly heating the conductor to obtain the expression 44·23 AVm foot pounds.

If, on the contrary, chemical energy disappears from the cell on the passage of the current, this energy is transformed into electric energy, and the electrolytic cell, therefore, acts as a current generator and introduces electric energy into the circuit. In that case the *total* amount of work done by the current in m minutes in the portion of the circuit which contains this electrolytic cell is *not* equal to 44·23 AVm foot pounds, since electric energy is introduced by the cell in question into the portion of the circuit under consideration, as well as there being a transference of electric energy between this portion and the remainder of the circuit. If, however, we confine ourselves to this transference of energy between two portions of a circuit, we may say that *if a P.D. of V volts be maintained between any two points L and M in a circuit, the amount of electric energy transferred in m minutes between the portion of the circuit LM and the rest of the circuit by a current of A amperes equals in all cases* 44·23 *A V m foot pounds.*

In certain exceptional cases the electrolytic cell may act simply as a resistance and be merely warmed by the passage of the current, but for that to be the case the work done in producing chemical action at one plate of the cell must be exactly balanced by the work given out in the same time by the chemical action at the other plate. This will be approximately the case when the cell consists, for example, of two copper plates immersed in a solution of sulphate of copper, and when the liquid is kept vigorously stirred so that, although copper is thrown into solution at the anode and taken out of solution at the kathode, the density of the liquid is not allowed to diminish round the anode plate nor to increase round the kathode plate, which would happen if the liquid were left at rest while the current was passing.

Example 60.—An arc lamp takes 12 amperes at

50 volts pressure. How many foot pounds of energy does it receive per minute?

Answer.—26,538 foot pounds per minute.

Example 61.—A resistance coil of 1,500 ohms has a P.D. of 12 volts maintained between its terminals. How many foot pounds of energy does it receive per minute?

Answer.—4·25 foot pounds per minute.

Example 62.—What current at 100 volts' pressure will supply 1,000 foot pounds per second to a given circuit?

$$\text{Foot pounds per second} = \frac{44 \cdot 23}{60} \times \text{AV},$$

$$\text{therefore A} = \frac{1000 \times 60}{44 \cdot 23 \times 100} \text{ amperes.}$$

Answer.—13·56 amperes.

106. Joule.—To avoid the employment of a numerical coefficient in the equation connecting work with current, P.D., and time, *the work done per second when a current of one ampere flows through a circuit between the terminals of which a P.D. of one volt is maintained, is called a "joule;"* therefore J, the work in joules done in s seconds by a current of A amperes flowing through a circuit between the terminals of which a P.D. of V volts is maintained, is given by

$$J = AVs.$$

Consequently,

1 joule = 0·7372 foot pounds very approximately,
or **1 joule = 10^7 "*ergs*"* exactly.**

Example 63.—A pressure of 110 volts is maintained between the electric-light mains of a house, and twenty glow lamps in parallel, each taking a current of 0·3 ampere, are turned on for five hours nightly for thirty nights. How much energy in joules does the house receive?

* An *erg* is the work done when a force of one "*dyne*" is exerted through one centimetre; and a *dyne* is the force which, exerted on a mass of one gramme, generates each second a velocity of one centimetre per second.

Answer.—$20 \times 0{\cdot}3 \times 110 \times 5 \times 3600 \times 30$, or 356·4 million joules.

107. Heat Produced by a Current.—When a circuit acts simply like a resistance, so that the *whole* of the energy given up by a current flowing through it is converted *directly* into heat, Ohm's law holds in its simple form. Hence, if o be the resistance in ohms of the circuit, A the current flowing through it in amperes, and V be the P.D. between its terminals in volts,

$$V = Ao;$$
$$\text{so that } \; AVs = A^2os,$$

or the work in joules done by a current of A amperes in s seconds in heating a circuit of o ohms equals A^2os. But we know from the investigations carried out by Joule—which have been repeated subsequently, with even greater accuracy, by Prof. Rowland and others—that the heat required to raise the temperature of one pound of water by 1°C. when the water is at 15°C. is the equivalent of 1400·4 foot pounds of work. Therefore, if we take this as our unit of heat, it follows, since one joule equals 0·7372 foot pound very approximately, that H, the number of these heat units generated in s seconds in the circuit, is given by

$$H = 0{\cdot}000{,}526{,}4\ A^2 o s \text{ very approximately;}$$

or if m be the time in minutes,

$$H \doteq 0{\cdot}031{,}59\ A^2 o m \text{ very approximately.}$$

Lastly, if a "*calorie*" be defined as the heat required to raise the temperature of 1 gramme of water by 1°C. when the water is at 15°C., then C, the number of *calories* generated in s seconds by a current of A amperes in a resistance of o ohms, is given by

$$C = 0{\cdot}2388\ A^2 o s \text{ very approximately,}$$

or the number of calories generated in m minutes is given by

$$C = 14{\cdot}33\ A^2 o m \text{ very approximately.}$$

108. Measuring the Rate of Production of Heat by a Current.—The formulæ given in the last section may be verified by sending a known current for a certain time through a coil of wire of known resistance immersed in a measured weight of water, and by observing the rise of temperature with a delicate thermometer. As, however, a portion of the current passes through the water, the resistance in the circuit is a little smaller than that of the coil of wire; also the resistance may vary by warming during the course of the experiment. Hence greater accuracy will be obtained if, instead of attempting to measure the resistance of the circuit directly, we observe from time to time the current that flows, say A amperes, and the P.D. between the terminals of the coil, say V volts; then, if A′ and V′ be the mean value of the current and the pressure during a period of m minutes, the electric energy that has been given to the coil and water during that time is $A'V'm$ joules, which must therefore be proportional to the amount of heat produced in that time.

If the product AV be small, electric energy will be given to the circuit slowly; therefore the heat will be produced in it slowly, and it will not be possible to accurately ascertain the amount of heat generated in a given time without allowing for the heat that is lost by radiation, convection, and conduction during the experiment. If, however, the product AV be made fairly large, and the quantity of water employed in the experiment be not too great, the time taken for a rise of temperature to be produced that can be accurately read on a sensitive thermometer need not be long enough for any serious loss of heat to occur. Further, if the vessel containing the water be made of very thin glass, the heat absorbed in raising the temperature of the vessel may be neglected unless very great accuracy is desired; also, if the wire be composed of a substance of high specific resistance, not only will the change of resistance of the coil through warming become negligible, but its mass

may be small and still a considerable amount of power may be given to it. Hence the heat absorbed by the coil to raise its own temperature may be so small compared with the heat absorbed by the water that the former may be neglected unless very great accuracy be desired.

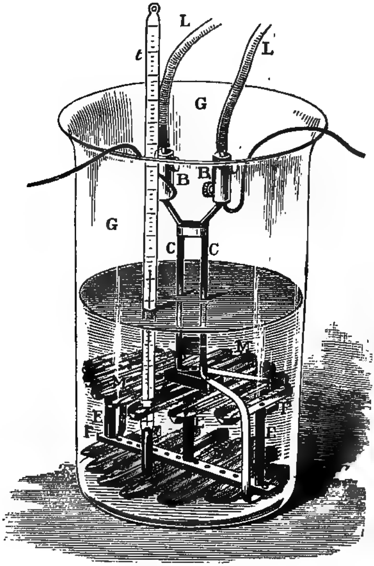


Fig. 152.—Calorimeter used in Measuring the Heat Equivalent of Electric Energy.

The problem of properly proportioning the parts, and of generally arranging the apparatus so that a beginner may obtain considerable accuracy by using it without it being necessary to make any corrections for the loss of heat by radiation, convection, and conduction, has been worked out by Mr. Haycraft, formerly one of the staff at the Central Technical College, and the author; and they find that with the apparatus illustrated in Figs. 152 and 153, which fulfils the conditions they have theoretically arrived at, students can easily obtain results not differing by as much as one per cent. from the truth.

A strip of manganin about $\frac{1}{4}$ inch wide, 0·03 inch thick, and about 10 feet long, is wound so as to form the top and bottom of a sort of cylindrical box, M M, about 5 inches across and 3 inches high (Figs. 152, 153), the

convolutions of the strip being kept from touching one another by being screwed to a light framework composed of two horizontal strips of vulcanised fibre, F, F, joined by three thin vertical rods of ebonite, E, E, E. The two ends of the strip are soldered to two stiff vertical copper wires, C, C, about 0·128 inch thick and 6 inches long, the soldered

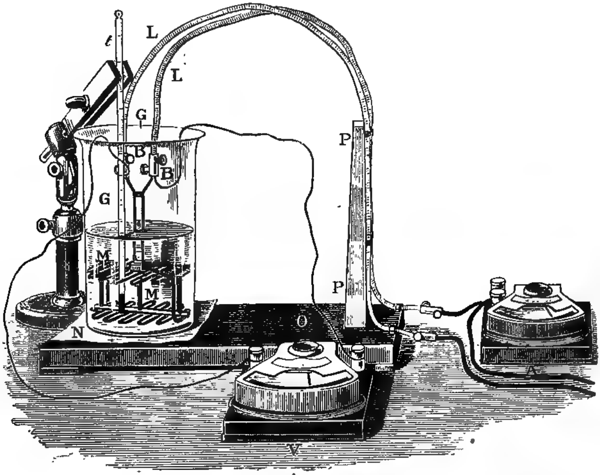


Fig. 153.—Apparatus for Measuring the Heat Equivalent of Electric Energy.

joints being covered over with varnish to prevent galvanic action taking place at the joint (*see* § 132, page 428), and the strip M M, and the upper wires C, C are also varnished to prevent electrolysis being produced by the current leaking through the water. The whole is immersed in about 122 cubic inches or 2 litres of water contained in a thin glass beaker, G G (Figs. 152, 153), which is just wide enough to take the framework of manganin strip, and, to diminish the risk of this beaker being broken, a piece of felt N is placed between it and the base board O O.

Electric connection is made with the stiff wires C, C by means of two insulated very flexible leads, L L, each composed of a strand of about 210 thin copper wires, the copper wires being each about 0·011 inch thick. The current is measured with an accurately-calibrated ammeter, A, and the P.D. set up between the upper ends of the stiff copper wires by means of an accurately-calibrated voltmeter, V (Fig. 153).

The object of using a flat conducting strip and forming it into the box shape seen in the figures is to enable the conductor itself to act as an efficient stirrer when it is moved up and down in the water, the flexible leads L, L, which are fastened to a wooden rod P P fixed to the base board O O, as shown in Fig. 153, serving as a handle to hold the box M M by. The heat generated in the strip is, therefore, given off fairly uniformly to the water, and the mean temperature can be read with considerable accuracy on a single stationary thermometer, *t*.

When the apparatus is constructed as described, a current of 30 amperes requires a P.D. to be maintained between the binding screws B, B of about 8·8 volts, corresponding with the energy given to the apparatus at the rate of about 264 joules per second. This is about the rate that theory shows leads to most accurate results when the amount of water used is 2,000 cubic centimetres, and this rate of expending electric energy causes a rise of the temperature of the water of about 7·5°C. in four minutes. The thickness of the copper wires C, C has been chosen so that a current of 30 amperes passing through them raises their temperature also by about 7·5°C., this rise of temperature being, however, a constant, and not increasing with time, except during the first few seconds after applying the current. The cross-section of the strand of copper wires in each of the covered leads L, L is chosen as being a little larger than that of each of the stiff copper wires C, C, the former being about 0·0192 square inch and the latter about 0·0141 square inch. Hence the rise of temperature in

the covered flexible leads is also about 7°C., and by this device any considerable transference of heat by conduction into, or out of, the manganin strip is automatically prevented when a current of about 30 amperes is passing for four minutes.

It is partly for the reason just given that the apparatus illustrated in Figs. 152 and 153, when constructed with the dimensions described, enables results to be obtained with currents of about 30 amperes which differ from the truth by less than 1 per cent.; a sample of a set of results actually obtained by students at the Central Technical College being given in the following table. But the error is not much greater when currents of 20 or 40 amperes are used corresponding with rates of production of heat respectively about half as great and twice as great as when a current of 30 amperes is employed.

Time in Seconds.	Temperature C°.			Current in Amperes.	Mean P.D. in Volts.	Calories per Joule.
	Initial.	Final.	Rise.			
120	18·40	22·02	3·62	30	8·634	0·2383
180	13·25	18·70	5·45	30	8·634	0·2390
180	13·60	19·00	5·40	30	8·648	0·2367
120	12·97	16·53	3·56	30	8·656	0·2375
120	12·64	16·26	3·62	30	8·698	0·2365
120	12·89	16·49	3·60	30	8·662	0·2364
120	12·11	15·72	3·61	30	8·666	0·2368
120	12·10	15·74	3·64	30	8·642	0·2395
120	13·13	16·75	3·62	30	8·692	0·2367

Mean 0·2375.

Average deviation from the mean = 0·001 = 0·42 per cent.

Now we saw in § 107 that the true number of calories per joule was about 0·2388, hence no one of the preceding results obtained by the students differs by more

than 1 per cent. from the truth, while the mean of the nine observations gives a result which has an error of only about one half per cent. Consequently the result aimed at in designing this apparatus has been achieved.

In carrying out the investigation we may vary either—

(1) The time during which the current is allowed to flow;
(2) The current made to flow through the strip;
(3) The resistance of the conductor by using similar stirrers made of somewhat thicker or thinner manganin strip;

and when a series of experiments is made varying each of these three conditions, one at a time, it is found that the rise of temperature of the water, and therefore the amount of heat produced, is proportional to the time, proportional to the square of the current, and proportional to the ratio of V to A—that is, to the resistance of the arrangement. Further, if we take as the calorie the heat required to raise the temperature of 1 gramme of water by 1°C. when the water is at a temperature of about 15°, we find that the relationship between the number of calories, the current in amperes, the resistance in ohms, and the time is practically that given in the last section.

Example 64.—A current of 30 amperes is passed through a coil of wire immersed in water for five minutes, a voltmeter reading 10·3 volts at its terminals. The volume of water is 2,000 cubic centimetres, and the temperature rises from 15·7° to 26·66°C. What result does the experiment give for the heat equivalent of one joule in calories?

Answer.—0·2364, a result about one per cent. too low, no corrections having been made for cooling during the experiment.

Example 65.—A temporary resistance is made by putting a coil of wire of 4 ohms' resistance into a wooden bucket containing 37 pounds of water. If a current of 40 amperes be sent through the coil, what about will be the rise of temperature of the water in the first three minutes? *Answer*.—16°C.

109. Power.—"*Power*" is the name given to the *rate of doing work*—that is, *the rate of transformation of one form of energy into another*—and it must be carefully distinguished from the amount of work done, there being the same sort of difference between *power* and *work* that there is between a *velocity* and a *distance.* The word *power* was, however, used in the older books on dynamics to stand for the applied force, and that is the meaning of the word *power* in such expressions as "the mechanical advantage of a machine is the ratio of the weight to the power." Again, the word *power* is sometimes wrongly used for energy, as in the expression the "storage of power." Beginners must, therefore, be on their guard against being misled by such loose expressions, and they should never employ the name *power*, or "activity," as suggested by Lord Kelvin, in any other meaning than the *rate of doing work.* In that sense, of course, power cannot be stored, for while a certain quantity of water in a reservoir at the top of a hill represents a certain store of energy, the power that this water can exert at any time when flowing out of the reservoir will depend on the rate at which it is allowed to flow.

When work is being done at a constant rate the power is constant, and it is measured by dividing the number which expresses the work done in any time by the number expressing the time. If, however, the rate of doing work at one moment is greater than at another—for example, when a person runs upstairs quickly at first and then more slowly—we do not mean by the power expended at any moment, the actual work done in a minute or even in a second, for the rate of doing work may be changing very rapidly. In such a case the power at any time is the limiting value of a ratio obtained thus:—Measure the work done in a *very short* time, a portion of which precedes, and the remainder of which follows, the instant at which we wish to measure the power; divide the work done in the *very short* time by that time, then this ratio more and more nearly represents

the power being expended at the moment in question, as we make the very short time shorter and shorter.

When, however, electric energy is being transformed into some other form of energy, the power may be very easily ascertained whether the rate of doing work is constant or not without it being necessary to measure a small time. For the work done in m minutes by a constant current of A amperes flowing through a circuit under a constant P.D. of V volts equals

$$44{\cdot}23\ \mathrm{AV}m \text{ foot pounds,}$$

provided that there is no apparatus in the circuit that gives energy to the current instead of receiving energy from it; therefore the rate of doing work in foot pounds per minute equals

$$44{\cdot}23\ \mathrm{AV},$$

or the rate of doing work in joules per second equals simply

$$\mathrm{AV}.$$

Hence, if at any moment we measure the current and the P.D. simultaneously, the product of the two measurements gives us the *instantaneous* value of the power being expended at that moment, and no measurement of time need be made. Hence the rate of transformation of electric into some other form of energy may be varying, but as long as it is not varying so rapidly as to prevent accurate readings of an ammeter and voltmeter being taken, the instantaneous value of the power can be ascertained at any moment.

110. Watt.—When work is being done at the rate of one joule per second the power exerted is called a "*watt*"; therefore the power of one *watt* is developed when work is done at the rate of

10^7 ergs per second,
or **1 joule per second,**
or **0·7372 foot pound per second very approximately,**
or **44·23 foot pounds per minute very approximately;**

and since *when work is being done at the rate of* 550 *foot*

pounds per second, or 33,000 *foot pounds per minute, one "horse-power" is said to be exerted,*

1 watt = 1/746th of a horse-power very approximately,
1 kilowatt = 1000/746th, or 1·340, of a horse-power very approximately,
∴ 1 kilowatt = $1\frac{1}{3}$ horse-power roughly.

Further, *if W be the power in watts expended in a circuit between the ends of which a P.D. of V volts is maintained and through which a steady current of A amperes is flowing,*

$$W = AV,$$

provided that the circuit contains no apparatus that gives energy to the current instead of receiving energy from it.

Also, *if W be the power in watts expended in heating a circuit of resistance o ohms through which a current of A amperes is flowing,*

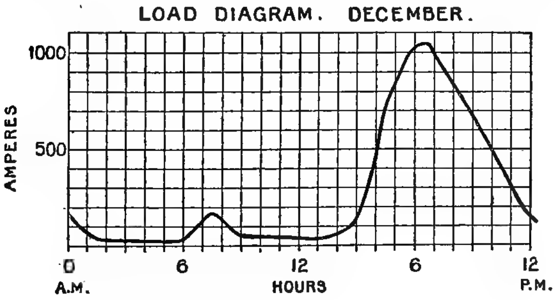
$$W = A^2 o.$$

Example 66.—What power in watts is expended in the arc lamp and in the resistance coil referred to in Examples 60 and 61?

Answer.—600 watts, and 0·096 watt respectively.

Example 67.—What power in kilowatts is expended in the coil in Example 65? *Answer.*—6·4 kilowatts.

Example 68.—The adjoining figure shows the "*load diagram*" of a central station for December—*i.e.*, the



curve giving the output of the station in amperes throughout the twenty-four hours. If the station pressure is 210 volts, what is the output in H.P. (horsepower) at 7 a.m., 12 noon, 5 p.m., and 10 p.m.?

Answer.—36 H.P., 11 H.P., 233 H.P., and 140 H.P. respectively.

Example 69.—Two glow lamps, each giving 16-candle power, take 2·75 and 3·5 watts per candle respectively. How many lamps can be supplied per horse-power expended in the two cases, and how many candles per horse-power will they give?

Answer.—Seventeen and thirteen lamps respectively; about 272 and 208 candles.

Example 70.—How many candles per horse-power are given by an arc lamp taking 11 amperes and 50 volts, and giving a mean candle-power of 570 in all directions?

Answer.—773 candles per horse-power.

111. Wattmeter: its Principle.—The power that is being given by a *steady* current to a circuit can be ascertained from simultaneous readings of an ammeter and of a voltmeter. Hence when, in addition to measuring the power, we desire to ascertain also the current flowing through the circuit or the P.D. between its terminals, the simplest method of measuring the power is to use an ammeter and a voltmeter. Consequently, when the "*wattmeter*"—an instrument which enables the power given to a circuit to be read off directly in watts from a single observation—was first described and constructed in England by Prof. Perry and the author in 1881, it did not appear that this instrument could have any very general application. Later on, however, it was realised that when a current is rapidly varying, as it is in the case of the distribution of electric energy by an "*alternating current*"—that is, a current whose direction varies many times per second—the mean value of the power given by the current to the circuit cannot generally be measured by taking simultaneous readings of an ammeter and voltmeter. For, although the power in

watts that is being given to the circuit at *any instant* is always equal to the product of the *instantaneous* value of the current in amperes into the *instantaneous* value of the P.D. in volts, the *mean* value of the power during any time is generally *not* equal to the product of the *mean* value of the current during that time multiplied by the *mean* value of the P.D.

In all cases, however, no matter in what way the current may periodically vary in strength, the *mean* value of the power is given accurately by the reading of a properly-constructed *wattmeter*, provided, of course, that, although the current may be rapidly varying, the mean value of the power remains constant for a sufficient length of time for a reading of the wattmeter to be accurately ascertained. The *wattmeter* has, therefore, acquired a new importance, since not only in the case of a steady current is it possible from a single reading of a wattmeter to measure what the simultaneous readings of two instruments together give us, but in the case of rapidly-varying currents the wattmeter measures directly what generally cannot be ascertained at all from simultaneous readings of an ammeter and voltmeter.

A *wattmeter* contains two coils, C C, *c c* (Fig. 154), one being fixed and the other movable. One of the coils, C C, which consists of a few turns of thick wire, is inserted in the main circuit; while the other coil, *c c*, consisting either of many turns of fine wire, or, better, of a few turns of fine wire in series with a stationary high resistance, W, is connected as a shunt to that portion of the circuit L M the power given to which we desire to measure. The current passing through *c c* is therefore proportional to the P.D. between the ends of L M, while the current passing through C C is the sum of the currents flowing through L M and through *c c*. If, however, the resistance of the fine-wire circuit of the wattmeter is very large, the current passing through it will be very small compared with the current flowing through L M, so that the current passing through C C will be practically

that flowing through L M. Hence the part of the wattmeter between the terminals T_1, T_2 acts as an ammeter, while that between the terminals t_1, t_2 serves as a voltmeter. Consequently the product of the currents in C C and *c c* is proportional to the power given to L M. But this product is directly proportional to the couple exerted between these two coils if the coils be always brought into the same position relatively to one another. Hence *the power to be measured is proportional to the torque*

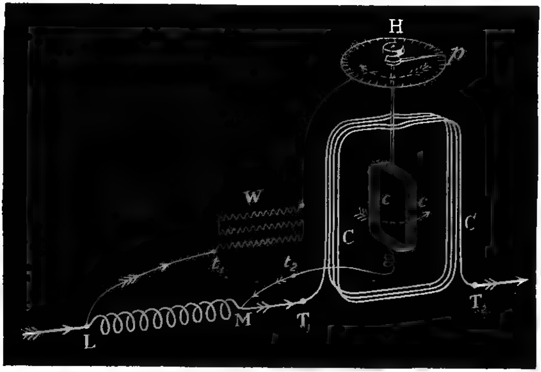


Fig. 154.—Diagram of Wattmeter.

that must be exerted on the movable coil of the wattmeter to keep it in a fixed position relatively to the stationary coil.

The torque required to be exerted on the suspended coil *c c* in order to maintain it in a fixed position relatively to the stationary coil C C, may be conveniently produced by turning the head H and the pointer *p* attached to it. This twists the vertical wire supporting the movable coil as the upper end of this wire is rigidly fastened to the head H. And, since the angle through which one end of a wire is twisted relatively to the other end is directly proportional to the torque exerted,

the power given electrically to the portion of the circuit L M will be directly proportional to the angle through which the pointer p has been turned to keep the coil $c\ c$ in the position it occupied when no current was passing through the coils.

Another way of joining up a wattmeter is to connect t_1, t_2, the terminals of the fine-wire circuit, to L and T_2 respectively, so that the fine-wire circuit is a shunt to both L M and the thick-wire coil C C of the wattmeter. In that case the current passing through C C will be accurately the current that flows through L M, but the current passing through $c\ c$ will now be proportional to the P.D. between the points L and T_2, and not between the points L and M. The difference between these two P.Ds. will, however, be very small if the power spent in sending the current through C C is very small compared with the power spent in sending it through L M, and this result can be practically attained by making the resistance of the coil C C as small as possible.

112. Commercial Forms of Wattmeters. — Commercial wattmeters based on the principle described in the last section have been constructed by several people. A compact form, designed by Mr. Swinburne, is seen in Fig. 155, which shows the instrument with the outer cylindrical cover removed so that the interior may be visible. The stationary coil C is made in two sections, the front one having been removed in the figure so that the suspended coil c can be better seen. The position of this suspended coil is sighted by means of a small pointer, which is rigidly attached to the bottom of the vertical rod hanging down from the small moving coil c, and when a measurement is made the milled head H is turned until the small pointer is exactly over a black line marked on a polished brass plate, which is fixed to the base of the instrument just under the little pointer. Parallax is avoided by the pointer and this wire being looked at through a small window $w\ w$ in the dial plate at the top of the wattmeter.

Instead of measuring the torque that has to be exerted to keep the suspended coil in its initial position as in using the wattmeters shown in Figs. 154 and 155, we may observe the angle through which the moving coil is turned against the action of a spring or gravity. A gravity deflectional wattmeter based on this principle, and constructed by Lord Kelvin, is seen in Fig. 156. The power to be measured will not be directly proportional to the angle through which the pointer attached to the movable coil turns, but the degree scale seen in Fig. 156 can be calibrated absolutely by sending various known currents through the two coils respectively, and a direct reading scale can then be constructed by a process similar to that described in § 15, page 58.

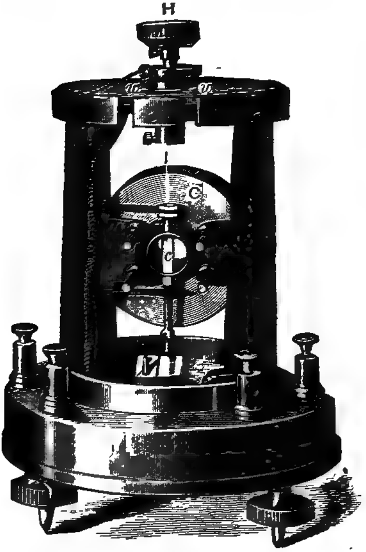


Fig. 155.—Swinburne Wattmeter, with cover removed.

Example 71.—In a certain wattmeter it is found that the head must be turned through 125° to bring the pointer to zero, when the current in the main coil is 20 amperes and the P.D. between the ends of the shunt coil is 120 volts. How much must the head be turned to bring the pointer to zero if the wattmeter is measuring the power taken by a resistance

of 7·3 ohms through which a current of 30 amperes is passing?

The wattmeter reading is proportional to the product of the currents in the two coils, and the current in the shunt coil is proportional to the P.D. between its terminals,

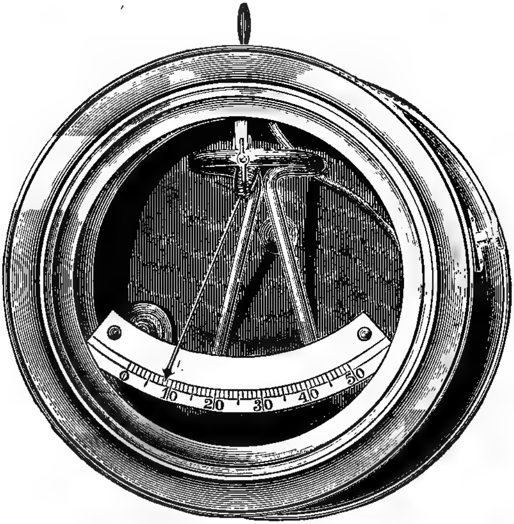


Fig. 156.—Kelvin Deflectional Wattmeter.

which is 7·3 × 30, or 219 volts in the second case. Hence, if θ is the angle through which the head must be turned,

$$\frac{\theta}{125} = \frac{30 \times 219}{20 \times 120}.$$

Answer.—The head must be turned through 342°.

Example 72.—If the resistance of the shunt coil in the above wattmeter is 6,542 ohms, what additional resistance in the shunt circuit will make the constant of the instrument 20 watts per degree? *Answer.*—272·6 ohms.

Example 73.—If the current passing through the circuit, the power given to which we desire to measure, is 20 amperes, while the P.D. maintained between its terminals is 30 volts, and if the resistances of the thick-wire coil and of the fine-wire circuit of a wattmeter are 0·01 and 1,000 ohms respectively, calculate the error that will be made by using the wattmeter when joined up in the two ways described in § 111.

Answer.—When the wattmeter is joined up as shown in Fig. 154 the current passing through the thick-wire coil C C will be $20 + \frac{30}{1000}$ amperes instead of 20 amperes —that is, will be 0·15 per cent. too large; therefore the power measured by the wattmeter will be 0·15 per cent. greater than the power given to the circuit L M. If, on the other hand, the wattmeter be joined up as described at the end of § 111, the current passing through the fine-wire circuit of the wattmeter will be produced by a P.D. of $30 + 20 \times 0{\cdot}01$, or 30·2 volts instead of 30 volts, the 0·2 volt being the P.D. expended in sending the current through C C. Hence the current through the fine-wire circuit, and therefore the power measured by the wattmeter, will be 0·67 per cent. too large. Consequently the former method of joining up the wattmeter would give the more accurate result in this particular case.

113. Joulemeter: Clock Form.—As shown by Professor Perry and the author in 1882, any pendulum clock can be easily converted into a "*joulemeter;*" that is, into an instrument which records the energy given to an electric circuit in any definite time, and Fig. 157 illustrates the first electric energy meter, called originally an "*ergmeter,*" that was constructed in this way.

The ordinary pendulum bob is replaced by a bobbin B, on which is wound a coil of fine wire, the coil being wound on in two parts, *c*, *c*, for convenience of attachment of the bobbin to the pendulum rod. These two halves of the fine-wire coil are joined in series with one another, and the terminals of this coil, t_1, t_2, are connected as a shunt

with that portion of the circuit L M, the energy given to which we desire to record. Fixed to the clock case in the position shown is a coil consisting of a few turns of thick wire; this coil being also constructed in two parts C, C, so that the pendulum coil may swing symmetrically between them. These two halves of the thick-wire coil

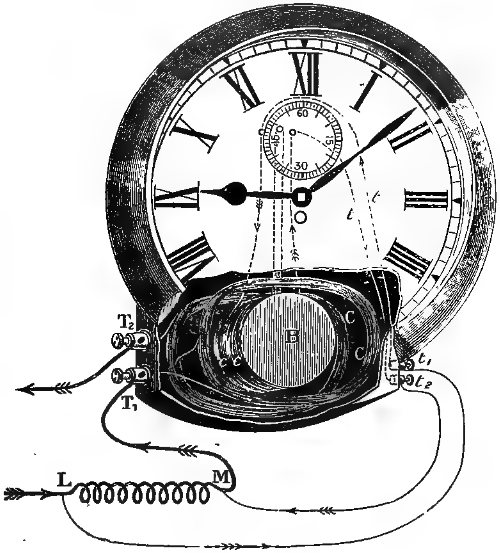


Fig. 157.—Ayrton and Perry's Original Gaining Clock Joulemeter.

are joined in series with one another, and the terminals of this coil, T_1, T_2, are connected up as shown, so that the main current through L M passes through the stationary coil C C. Currents, then, flow both round the moving coil c c and the stationary coil C C, and produce, therefore, an attraction or a repulsion between these coils, depending on whether the coils are joined up so that the currents circulate round them in the same direction or

in opposite directions. The force exerted between the coils will vary with their relative positions, but its mean value will be proportional to the product of the currents flowing in the two coils; that is, it will be proportional to the power given to the part of the circuit L M.

The action of this force on the swinging pendulum will be approximately the same as if the action of gravity had been increased or diminished; hence, if the coil be joined up so that there is an attraction, the clock will be caused to gain, whereas if the ends of the fine-wire coil be interchanged the clock will lose. And if this force is small compared with the weight of the pendulum bob it may be shown in the following way, that the total gain or loss of the clock in any period is directly proportional to the energy given electrically to the circuit in that period.

The number of semi-vibrations N that a pendulum makes in s seconds—

$$\text{equals } \frac{s}{\pi}\sqrt{\frac{M}{I}},$$

where I is the moment of inertia of the pendulum about its axis of rotation, and M has such a value that M sin. θ equals the torque tending to pull the pendulum into a vertical position when it has been deflected through any angle θ from that position. Now M consists of two parts M_1 and $\pm M_2$, the former being due to the action of gravity alone, while the latter is approximately proportional to W, the power in watts given to the circuit, and may, therefore, be written as k W.

Hence, if the coils be joined up so that the clock gains, and if W is constant during the s seconds—

$$N = \frac{s}{\pi}\sqrt{\frac{M_1 + k\,W}{I}};$$

or, expanding by the use of the binomial theorem,

$$N = \frac{s}{\pi\sqrt{I}}\left(M_1^{\frac{1}{2}} + \frac{k}{2}M_1^{-\frac{1}{2}}W + \frac{k^2}{8}M_1^{-\frac{3}{2}}W^2 + \ldots\right).$$

Now the number of convolutions of wire on the two coils, and their distance apart, are arranged so that even when the maximum power that the *joulemeter* is intended to be used with is given in the circuit, M_2 is small compared with M_1. Hence, for practical purposes the terms involving the square and higher powers of M_2 may be neglected, and approximately we may say—

$$N = \frac{s}{\pi \sqrt{I}} \left(M_1^{\frac{1}{2}} + \frac{k}{2} M_1^{-\frac{1}{2}} W \right).$$

If, therefore, N_1 be the number of semi-vibrations which the joulemeter pendulum would make in s seconds when no current is flowing round the coils, and N_2 be the number it makes in the same time while a power of W watts is being furnished to the circuit under consideration,—

$$N_2 - N_1 = \frac{s\,k}{2\,\pi \sqrt{I}} M_1^{-\frac{1}{2}} W.$$

But, if the clock be initially adjusted to keep true time when no current is flowing through the coil, $N_2 - N_1$ is proportional to the gain of the clock in s seconds, also s W is proportional to the energy in joules supplied to the circuit in s seconds; therefore it follows that, as long as the power which is supplied remains constant and is not too large, the gain in the clock is directly proportional to the energy supplied. Now during the next period of s' seconds when the power supplied is, say, W′ watts, the gain in the clock will bear the same proportion to s' W′ (the energy supplied in s' seconds) that the previous gain bore to the energy supplied in s seconds, and so on for any number of periods short or long during any one of which the power remains constant. Therefore, adding together all the gains and all the amounts of energy supplied in any interval—a day, a week, or a month—the total gain of the joulemeter will be directly proportional to the total amount of energy that has been given to the circuit in that interval, whether the energy

has been supplied at a constant or at a variable rate, provided that the rate of supply has at no time been so great as to make M_2 more than a small fraction of M_1. And experience shows that this *direct* proportionality between the gain of the clock and the amount of energy supplied remains sufficiently true for practical purposes as long as the gain of the clock produced by the electro-magnetic attraction on the pendulum coil does not exceed about 2·6 minutes per hour.

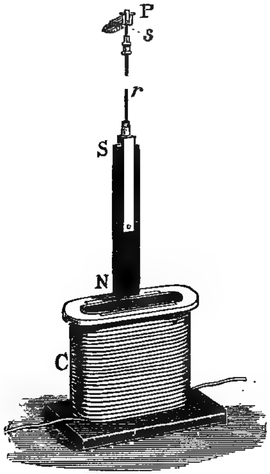


Fig. 158.— Swinging Permanent Magnet-Pendulum accelerated by a Current flowing around the Coil.

To measure, then, the energy given to the electric lamps, motors, &c., in a house during any interval, we have merely to insert the thick-wire coil C C of the joulemeter in one of the house mains and connect the fine-wire coil *c c* across the house mains, when the gain or loss of the joulemeter in the interval (after allowing, of course, for any gain or loss that arises from the two clocks not being exactly "*synchronised,*" or adjusted to keep exactly the same time when no currents are flowing in the coils) multiplied by the constant of the particular instrument will give the energy in joules that the house has received.

If the P.D. between the ends of the swinging coil be kept constant—that is to say, if the energy be supplied at practically constant pressure, which is the condition aimed at in the parallel method of supplying a district with electric energy—the pendulum coil acts as if it were a permanent magnet, and it may therefore be replaced

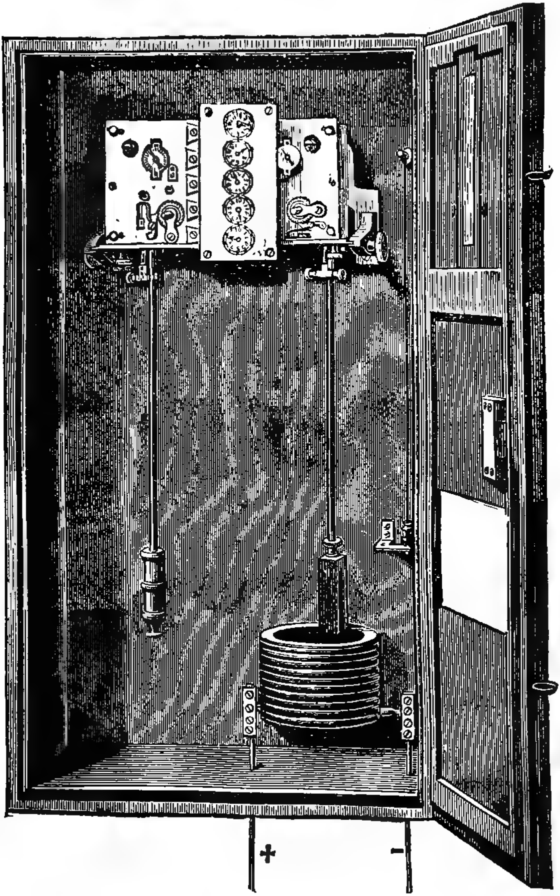


Fig. 159.—Aron type of Ayrton and Perry's Gaining Clock Joulemeter.

by a permanent magnet N S, as in Fig. 158. This magnet is supported by a stiff wire *r*, which is joined by a piece of flat spring *s* to an ordinary pendulum attachment P, and the magnet swings pendulum-wise over a stationary coil C round which the main current flows.

Further, if instead of observing the gain or the loss of the meter in any interval by comparing its indication at the beginning and end of the interval with a good clock or

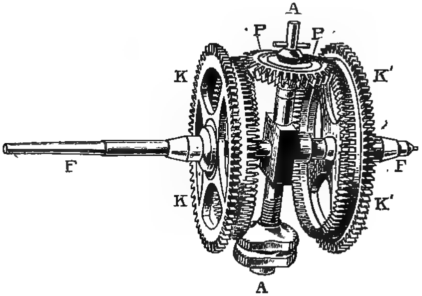


Fig. 160.—Differential Gearing of the Aron Supply Meter.

watch, we may place two clocks inside the "*supply meter*," as seen in Fig. 159, one clock being an ordinary one and the other a clock having for its pendulum a permanent magnet swinging near a stationary coil, through which passes the main current flowing through the house, the energy given to the house in any time will then be directly proportional to the difference between the number of vibrations that the two clocks have made in the interval. This difference can be read off on a counting mechanism like that used on a gas meter if the staff F F driving this counting mechanism be connected by means of "*differential gearing*" (Fig. 160) with the two clocks.

The staff F F (Fig. 160) is rigidly connected with the balanced arm, A A, which carries at one end a pinion P P.

This pinion gears into two crown wheels K K, K′K′, turning loosely on the staff F F. These crown wheels have also teeth cut on their circumferences like ordinary toothed-wheels, and these toothed-wheels are geared with the two clocks respectively, one wheel being rotated by one of the clocks righthandedly and the other left-handedly. When no electric energy is being supplied to the circuit L M (Fig. 157, page 335), the crown wheels are driven by the clocks at equal rates, the pinion P P therefore is simply turned round on the arm A A, but the arm itself is not moved. But when energy is supplied to the circuit the clock with the magnetic pendulum goes faster, the crown wheel driven by it, therefore, also rotates faster than the other crown wheel, and the pinion P P not only is rotated on the arm A A, but the arm itself, and the staff F F attached to it, are driven round, and move on the dial hands, at a rate depending on that at which electric energy is supplied to the circuit.

The arrangement just described has been employed by Dr. Aron in the construction of some hundreds of thousands of *supply meters* during the last few years.

114. Joulemeter: Motor Form.—In the last section was described the method of recording the sum of the products of the power into the time; that is, the total amount of electric energy given to a circuit by using the attraction between the current and pressure coils of a wattmeter to alter the rate of going of a clock. But, as pointed out by Prof. Perry and the author in 1882, in the same patent specification, this attraction may, instead, be employed to drive the counting mechanism, and give a direct record of the energy supplied to any circuit if the current and pressure coils be made to form the stationary and moving parts respectively of an electromotor *without iron*, and if the rotation of the motor be resisted by a torque proportional to the velocity of rotation. This principle has been used by Prof. Elihu Thomson in the construction of a very large number of joulemeters. For some reason this instrument as constructed by Prof.

Elihu Thomson has been called a "*recording wattmeter;*" this name is, however, a misnomer, since it is the total amount of energy in joules, and not the variations of the power in watts, which the instrument records.

It is impossible to obtain *continuous* motion by the mutual action of the currents in two coils unless the current in one of the coils, at any rate, be periodically reversed. For, suppose currents flow round two coils in such directions that the coils attract one another, the coils, if one or both of them be free to move, will approach one another, the force of attraction will rapidly increase, causing them to finally rush together, when they will press against one another, and any further motion will be clearly impossible. On the other hand, if the directions of the currents be such that the coils tend to repel one another, either it will happen that one of the coils will turn round, when they will approach as before, or if neither of the coils be free to turn they will recede from one another until the distance separating them becomes so great that the force of repulsion is too small to overcome any frictional resistance that may oppose the motion.

To keep up a continuous motion, then, of one coil relatively to another there must be employed some form of "*commutator*" or arrangement for reversing the current through one of the coils; further, if we wish that the force producing the motion shall remain fairly constant, either the moving or stationary part of the motor must consist of a number of coils so arranged that, as the rotation of the motor changes the position of each coil in the magnetic field, its place in the field is taken by the next coil. This part of the motor is called the "*armature*," while the other part is called the "*field*," and if the *armature* has a sufficient number of coils on it the torque exerted between the *field* and the *armature* remains practically constant, in spite of the motion of the one relatively to the other.

The armature of the Elihu Thomson joulemeter is

the rotating portion, and it consists of eight coils, $c_1, c_2 \ldots . c_8$, wound on a light framework, as seen in Fig. 161, which shows the armature detached from the complete meter (Fig. 162) in order that the construction of the former may be clearly seen. The end of each coil is electrically connected with the beginning of the next, and is also connected by means of one of the wires $w_1, w_2 \ldots . w_8$ (Fig. 161) with one piece of the *eight-part commutator* $k_1, k_2 \ldots . k_8$. The armature, which is in series with a stationary resistance, is joined as a shunt to the portion of the circuit the energy given to which it is desired to record, and this shunt current is led into and out of the commutator by two *stationary "brushes,"* B, B, the current dividing into two parts at each brush and following the paths shown by the arrows (Fig. 161*a*).

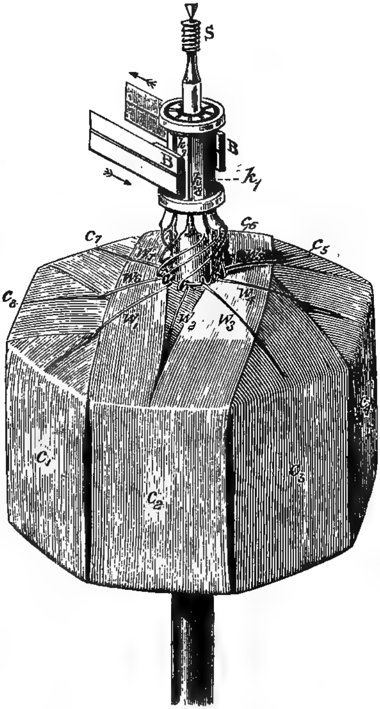


Fig. 161.—Rotating Armature of the Elihu Thomson type of Motor Joulemeter.

This figure, 161*a*, is a diagrammatic sketch of the

armature, commutator, and brushes at the moment when the two pieces k_7 and k_3 of the commutator are touching the brushes, and to avoid confusion in this sketch only these two pieces of the commutator are shown connected

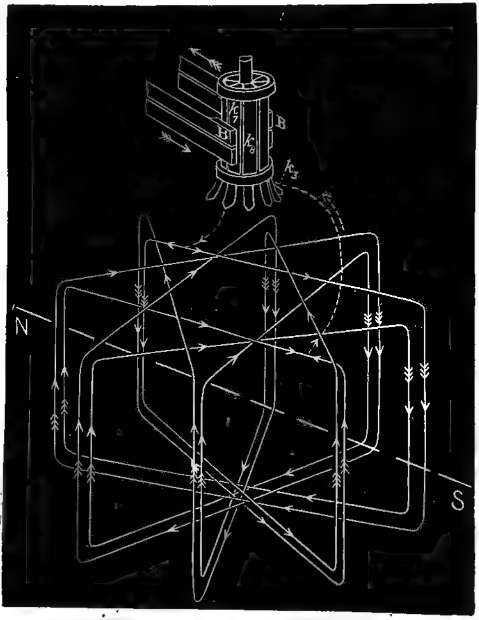


Fig. 161*a*.—Diagram showing the Directions of the Currents in the Armature of the Elihu Thomson type of Joulemeter.

with the coils. But in reality each of the eight commutator pieces $k_1 \ldots . k_8$ is joined respectively with the end of one coil and the beginning of the next, and, since the brushes are stationary while the armature and commutator revolve, the direction of the currents in the coils would appear exactly the same whether the pair of com-

mutator pieces touching the brushes were $k_1\,k_5$, $k_2\,k_6$, $k_3\,k_7$, or $k_4\,k_8$. The result is that, although the armature rotates, the current flowing round it produces a magnetic field, in a nearly fixed position, indicated by the dotted line N S.

The stationary *field* coils C C, seen in perspective in Fig. 162 and in sectional elevation in Fig. 162*a*, are

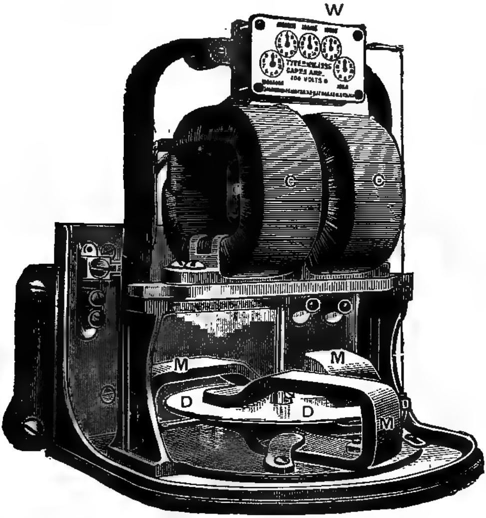


Fig. 162.—Interior of the Elihu Thomson Motor Joulemeter, with case removed.

placed in series with the portion of the circuit the power given to which we desire to measure, so that the main current passes through these field coils and produces another stationary magnetic field, which is almost at right angles to that produced by the armature, and the action of the one field on the other causes a continuous rotation of the armature.

As these two fields have always the same relative

position, the torque exerted will be directly proportional to the product of the strengths of the fields, and, as no iron is used in either the armature or the field coils, the magnetic fields will be directly proportional to the currents producing them; hence the torque producing the rotation will be directly proportional to AV, the

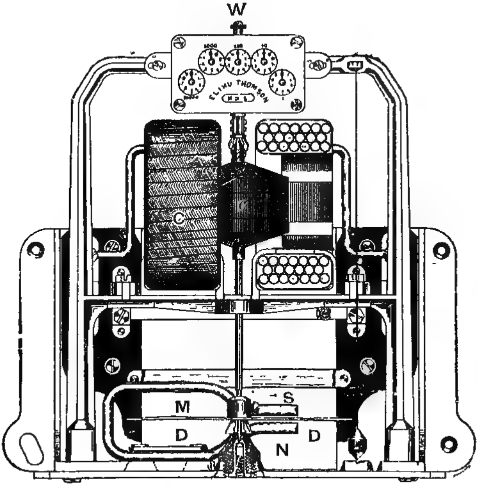


Fig. 162*a*.—Sectional Elevation of the Elihu Thomson type of Joulemeter.

power in watts given to the portion of the circuit under consideration.

The motion of the armature *c* (Figs. 162 and 162*a*) is resisted by the horizontal copper disc D D, which is rigidly attached to the armature spindle, being rotated in the magnetic field produced by three stationary horseshoe permanent magnets M, M, M. The south pole S of each of these magnets (Fig. 162*a*) is above, and the north pole N below, the copper disc, so that the lines of force produced by these permanent magnets are vertical

and at right angles to the plane of rotation of the copper disc. This causes Foucault currents to be induced in the rotating copper disc, and the attraction between these currents and the stationary magnets impedes the turning of the armature. Now, the strength of these induced currents is proportional to the angular velocity a, so that the torque which resists the motion is proportional to a.

We have, therefore, as shown by Prof. Perry and the author, a driving torque proportional to AV and a retarding torque proportional to a; hence, if the frictional resistance to motion introduced by the bearings of the armature, the rubbing of the commutator against the brushes B, B (Fig. 161), and the train of wheels in the counting mechanism W (Fig. 162), driven by the screw or worm S (Fig. 161), be very small, the armature must rotate at such a speed that the electromagnetic driving torque, which is proportional to AV, is exactly equal to the electromagnetic retarding torque, which is proportional to a, or

$$AV \propto a.$$

If, now, during any time s seconds, the power supplied to the circuit be constant, AV will be constant for that time, and so also will a; therefore

$$AVs \propto a\,s,$$

but AVs is the energy in joules and as is the angle turned through by the armature in that time. Consequently for each period of time during which the energy is supplied at a constant rate the angle turned through by the armature, and therefore the advance of the counting mechanism, is directly proportional to the energy supplied in that time. Therefore, adding together all the amounts of advance of the counting mechanism and all the amounts of energy supplied for each of the periods during which energy is supplied at various constant rates, we may conclude that the total advance of the counting mechanism in any interval will be directly proportional to the total amount of energy supplied in that interval,

whether the energy has been supplied at a uniform or at a variable rate.

The friction at the bearings of the armature may be rendered small by using a very light armature, and by forming the ends of the armature spindle of hard metal, carefully pointed, and by supporting them in jewels, as is done in a watch. The friction and inertia of the counting mechanism can be overcome by making the parts small and light; and the friction of the commutator k_1, k_8 against the brushes B, B Prof. Elihu Thomson finds can be reduced to a workable limit by constructing the commutator of silver, as well as the parts of the brushes that rub against it, and by making the diameter of the commutator very small.

In the case of the clock form of joulemeter, we saw that the pressure coil could be dispensed with and a simplification introduced when it was known that the electric energy would be supplied under *constant pressure;* so also in the case of the motor joulemeter, the armature which constitutes the pressure coil can be modified when the condition of supply is constant pressure. The instrument then becomes a "*coulombmeter*," and its description will be found in the chapter of Vol. II. which deals with "Quantity of Electricity."

The clock type of meter has the great advantage over the motor form that, no matter how small be the rate at which electric energy is supplied to a circuit, the clock meter actually records the total amount of energy supplied, whereas, in consequence of friction, a motor meter will not start until the currents passing round its coils reach a certain value. Hence, if the electric power that a circuit receives be always very small, the armature of a motor meter may never move, and so the meter will record no energy received, even though the period during which this very small amount of power has been supplied has been so long that the total amount of energy that ought to be recorded is considerable.

The clock meter, on the other hand, has the disadvantage that it requires to be periodically wound up. Hence, if such a meter becomes for a time inaccessible—for example, in a London flat which has been left locked up—one of the clocks may stop before the other, and the record be rendered quite erroneous. Indeed, if it be the clock that has the magnetic pendulum which stops first, the meter may record that electric energy has been supplied by the householder to the electric mains under the streets, and, therefore, that he ought to receive payment for electric energy that has apparently been delivered by him to the company.

115. Board of Trade Unit of Energy.—"*The Board of Trade unit,*" the words "*of energy*" being generally omitted, is the name given to the work done in a circuit when the power exerted in watts multiplied by the time during which it is exerted in hours equals 1,000, or

1 Board of Trade unit = 1,000 watt hours,
" " " = 3,600,000 joules,
" " " = 36×10^{12} ergs,
" " " = 2,653,800 foot pounds very approximately,
" " " = 1·340 horse-power hour very approximately,
" " " = $1\frac{1}{3}$ horse-power hour roughly.

A *Board of Trade unit* is, therefore, a thing that can be bought and sold at a specified price, like a ton of iron, and this price can be regulated by agreement or by law, as cab fares are so regulated. When an "*Electric Supply Company*" obtains an Act of Parliament to lay wires under the streets of London for the purpose of supplying electric energy to the houses, one of the regulations always imposed is that the price for a Board of Trade unit delivered to a consumer must not exceed eightpence. The company may, however, charge as much less as it likes, provided it gives equal advantages to all its customers,

and the average price per Board of Trade unit in London to large consumers is probably about sixpence. In Newcastle it is as low as fourpence, and in Eastbourne ninepence.

Example 74.—The magnet pendulum of a double clock joulemeter (Fig. 159) swings 4 per cent. faster when a current of 50 amperes is passing than it does when no current is passing. If the two clocks are not quite equally rated for zero current—the magnet pendulum gaining one minute in a day—what number of units will be registered if 10 amperes at 100 volts' pressure pass through the meter for seven days; also what number of units would the meter record if it had been accurately synchronised?

Answer.—Since the magnet pendulum swings 4 per cent. faster when 50 amperes is passing, it should gain 0·8 per cent. on the other pendulum when the current is 10 amperes. But in consequence of the imperfect synchronism it gains one minute per day, or 0·069 per cent., in addition. The meter record is proportional to the gain of the one pendulum relative to the other, and is therefore greater than it should be in the ratio $\frac{0{\cdot}869}{0{\cdot}800}$.

If the meter were accurate it would register $\frac{10 \times 100 \times 7 \times 24}{1000}$, or 168 Board of Trade units; hence the number of units recorded is $168 \times \frac{0{\cdot}869}{0{\cdot}800}$, or 182·5.

Example 75.—If the magnet pendulum of a double clock joulemeter is losing when no current passes, the number of units registered is too small. What loss per day will make the number of units registered 30 per cent. less than they should be if (*a*) the average current is 10 amperes; (*b*) if the average current is 50 amperes?

Answer.—(*a*) Loss of 3·46 minutes per day; (*b*) loss of 17·3 minutes per day.

Example 76.—If electrical energy is supplied at 6d. per Board of Trade unit, determine whether it is more economical to use 16-candle power lamps taking 2·5 watts per candle and lasting practically unimpaired for 500 hours, when the "*filament*" breaks, or 16-candle power lamps taking 3·5 watts per candle and lasting 900 hours, the cost of a new lamp being in each case 1s. 3d.

Using 2·5 watt lamps :—

Cost for *energy* per candle hour $= \frac{2 \cdot 5}{1000} \times 6$

$= 0 \cdot 015$ penny.

Cost for *lamp renewals* per candle hour $= \frac{15}{16 \times 500}$

$= 0 \cdot 0019$ penny.

Total cost per candle hour $= 0 \cdot 0169$ penny.

Using 3·5 watt lamps :—

Cost for *energy* per candle hour $= \frac{3 \cdot 5}{1000} \times 6$

$= 0 \cdot 021$ penny.

Cost for *lamp renewals* per candle hour $= \frac{15}{16 \times 900}$

$= 0 \cdot 001$ penny.

Total cost per candle hour $= 0 \cdot 022$ penny.

Therefore, in this particular case, it is more economical to use the lamp having a shorter life but taking less power.

Example 77.—Is the same conclusion true if the lamps are 8-candle power, all other things remaining the same?

Answer.—The total costs per candle hour become 0·0188 and 0·023 penny respectively, so that the shorter-life lamp is still the cheaper.

Example 78.—Compare the cost for equally lighting the same area with gas at 2s. 6d. per 1,000 cubic feet (the burners used giving 12 candles for 5 cubic feet per hour) with incandescent lamps using electric energy at

6d. per unit (the lamps taking 3·92 watts per candle), and with arc lamps supplied with electric energy also at 6d. per unit (the lamps taking 1 watt per candle).

The cost of renewal for broken glow lamps, and the carbons for the arc lamps, not to be included.

Answer.—Relative costs:—Gas, 2·08; incandescent lamps, 3·92; arc lamps, 1.

Example 79.—What is the reduction in a consumer's bill of £80 per annum for electric energy supplied (*a*) if the price of a unit is reduced from 7d. to 6d.; (*b*) if lamps taking 2·8 watts per candle are used instead of lamps taking 3·5?

Answer.—(*a*) A reduction of £11 9s.; (*b*) a reduction of £16.

Example 80.—How many Board of Trade units are consumed by a 100-volt 8-candle power lamp taking 28 watts burning continuously for one year? What is the cost at 6d. per unit?

Answer.—245 units; £6 2s. 6d.

116. Electric Transmission of Energy.—In order to maintain a steady electric current we must have a *closed* electric circuit such as K L M N (Fig. 163), and any complete circuit always consists of two essentially distinct parts. In the one part K L M N the current flows in the direction in which the potential diminishes—that is to say, the potential at K is greater than that at L, the potential at L greater than that at M, and so on—and at every point throughout this portion of the circuit electric energy is being turned into heat, or into heat and also into some other form of energy, such as chemical or mechanical energy. This part of the circuit corresponds with the overhead telegraph wires and the telegraph instruments which are placed at the ends of the wires used to receive the telegraph messages, or it corresponds with the electric-light insulated copper mains under the streets, the wires, the glow and arc lamps in the houses, and the electro-motors used to do work in houses and factories which are supplied with current from the street mains. And in all

the calculations which have been made in this book hitherto regarding current, P.D., energy, and power, it is this part of the circuit K L M N only that we have been dealing with. So in the same way we might have been studying the flow of water in the water mains under the streets or in the water pipes in our houses, or the flow of water along a river where the water moves under the action of gravity.

The water which produces the stream may be obtained from a reservoir or an elevated cistern, or from some pond at the top of a hill; but, unless there be some contrivance for keeping the reservoir filled by raising the water from a low level to a high level against the action of gravity, the reservoir will run dry and the water stream will cease.

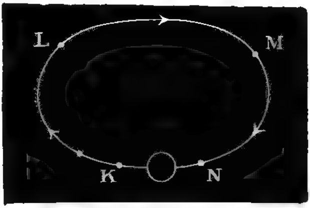


Fig. 163.

Hence, to maintain a *continuous* stream, the water must continuously, or at any rate from time to time, be carried up in buckets, or be raised by some form of pump, or by the evaporating power of some hot body like the sun; in fact, as much work must be done on the water in raising it as it does in its descent through the pipes or along the river bed. So in the same way in some part N K of any complete electric circuit there must be some apparatus for raising the electricity from a *low* to a *high* potential, and the energy which this apparatus thus puts into the electric circuit must be withdrawn by the apparatus from some outside store of energy. In the sending of currents to produce telegraphic signals, or to ring an electric bell, the battery forms this pump which raises the electricity from a low to a high potential as the current passes through it, and the chemicals placed in the battery constitute the store

of energy on which the battery draws; while in the sending of a current to produce the electric light or to work electromotors in a town the *dynamo* at the "*Electric Light Central Station*" is the pump, and the coal in the cellars at the "*Generating Station*" which is used to drive the steam engines is the store of energy on which the dynamo indirectly draws through the medium of the steam engine.

A complete electric circuit is, therefore, something like a bell rope with a man pulling the rope at one end and a bell ringing at the other. By pulling the rope energy is given to it, this energy travels to the other end of the rope and is there given out to the bell; or a complete electric circuit is something like one of the pipes of the London Hydraulic Company under the streets with a pump at one end and a water-motor at the other. The pump takes energy from some outside source and gives it to the water, this energy is partly wasted in heating the running water and the pipes in consequence of friction, but the greater part of the energy is given out by the water to the water-motor at the other end of the pipe. Here the pipe corresponds with the electrical conductor, the pump with the battery or the dynamo, and the water-motor with the electromotor. There is no doubt an important difference in the two cases, the water which flows out through the water-motor at one end of the pipe need not be *immediately* returned to the pump at the other end, indeed it may not be the same water at all which is pumped up again by the pump to maintain the water stream, whereas in the electric circuit the *same* electricity must be regarded as flowing round and round the circuit. But there is this important similarity, that, just as the water is not the energy, so electricity is not energy, in spite of erroneous statements that have been sometimes made to the contrary. Pressure is given to the water by the pump at one end of the pipe, and pressure is given out by the water to the motor at the other end, so potential is given to the electricity at the

battery at one end of the wire, and potential is lost by the electricity at the other end of the wire, where the electricity flows through the electromotor.

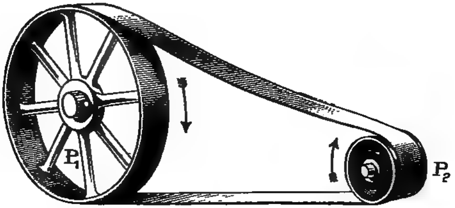


Fig. 164.—Transmission of Power with an endless belt.

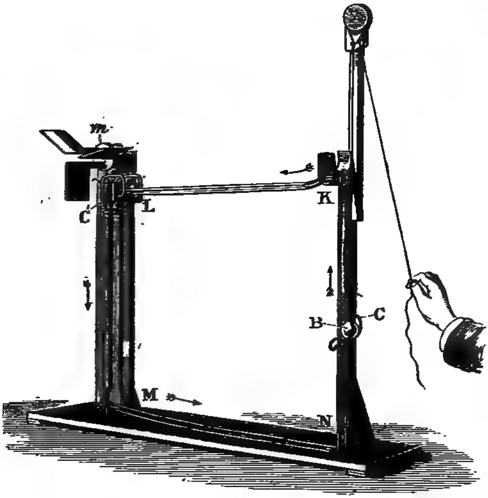


Fig. 165.—Mechanical Model illustrating the Transmission of Energy from a Generator, N K, to a Motor, L M.

Probably the closest analogy with the electric transmission of energy is the driving of one pulley by another

by means of an endless belt (Fig. 164). Energy is put into the belt as it passes round the driving pulley P_1, energy is given out by the belt as it passes round the driven pulley P_2. The running belt corresponds with the electric current, the driving pulley P_1 with the battery or dynamo, and the driven pulley P_2 with the electro-motor. The model (Fig. 165) shows in a rough symbolical way what takes place in the transmission of energy with pressure-water, compressed air, an endless belt or electricity. The working stuff, water, air, belt, or electricity, is first raised in pressure, and has energy given to it symbolised by the ball, B, being raised in the carrier C through the height N K *against* the action of gravity; the ball then gradually loses pressure as it proceeds along the tube or wire K L which conveys it to the other end of the system, shown by the ball falling as it proceeds from K to L, and the energy thus lost is spent in heating the tube or wire. At the other end there is a great drop of pressure as the ball falls, in the carrier C′, through L M, corresponding with a transference of energy to the motor *m* which drives a little air-fan, and finally the ball comes back along the return pipe or wire M N, losing, as it returns, all that remains of the energy given to it initially in the pump or elevator at N K. The ball has, in fact, come back to its original level.

If the circuit external to the battery is simply a resistance containing no electromotor nor electrolytic cell, then the circuit is analogous with the model seen in Fig. 166, the balls B, B falling by gravity along the rails K L M N, and being raised *against* the action of gravity through the height N K. The balls are lifted by their being picked up by the hooks attached to the endless belt *b b*, the right-hand side of which is made to continuously rise by the handle H being turned.

There is another way of transmitting energy through a pipe which is wholly different from the methods previously considered, and that is by means of coal gas, but in this case the *quality* of the material sent through the

pipe and not its pressure is the important consideration. The energy contained in coal gas is not pressure-energy, but chemical energy; therefore, as long as the pressure of the gas is sufficient to make it come out of the pipe at a suitable rate, it does not matter, as far as the amount of energy contained in a given weight of gas is concerned, whether the pressure be small or large. But the chemical constitution of the coal gas is of great importance. On the other hand, when energy is transmitted by water, or

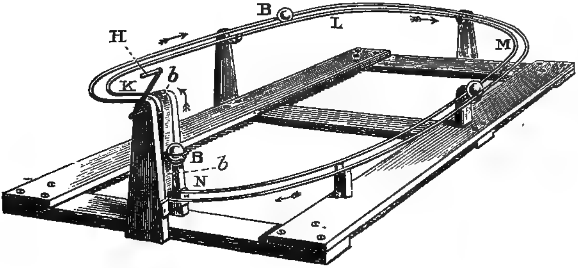


Fig. 166.—Mechanical Model illustrating an Electric Circuit composed of a Current Generator and an External Resistance.

by air or electricity, the pressure is as important a factor in estimating the amount of energy delivered as the quantity of the working substance. Apart from the action of the tide, water at sea-level is quite useless for working machinery, no matter how much water be available, so electricity at zero potential is useless for working an electromotor or producing an electric light. It is, therefore, all important to the user of the water supplied by the London Hydraulic Company whether its pressure is 750 pounds per square inch or 500 pounds per square inch, but it is of no importance to him whether the water be ordinary river water or be chemically pure.

Hence, while practically no restriction is imposed by law on the pressure that the Gas Companies must maintain in the gas as supplied to a house, the public Electric

Light Companies are prevented by law from allowing the P.D. between the electric light mains, where they join the house mains, from varying more than 4 per cent. from the standard pressure.

This fundamental difference between the transmission of energy by coal gas and by electricity must be fully grasped, for it is probably a want of appreciation of this fundament-difference that has led people to make such erroneous statements as that electricity is a form of energy.

117. Power Developed by a Current Generator.—If A be the current in amperes flowing round the circuit K L M N (Fig. 163), and if V be the P.D. in volts between the points K and N, the work done per second on the part of the circuit K L M N equals AV joules. In addition, if the resistance of the portion of the circuit between N and K be b ohms, the current will do work in heating this resistance at the rate of $A^2 b$ joules per second. Hence, the total power developed by the current equals

$$AV + A^2 b \text{ watts.}$$

Now, from the conservation of energy it follows that the work done per second by the current on the circuit must equal the work done per second on the current by the apparatus between N and K, which converts some form of energy into electric energy. Hence, whatever be the construction of this apparatus, the rate at which the transformation of energy takes place in it, the rate, in fact, at which it introduces electric energy into the circuit, must equal

$$AV + A^2 b \text{ watts.}$$

There are three distinct classes of apparatus that may be employed for introducing electric energy into a circuit, viz. :—

(1) A battery, which transforms chemical energy into electric energy ;

(2) A "*thermo-pile*," which transforms heat into electric energy (*see* the note on page 563) ;

(3) A dynamo, which transforms mechanical energy into electric energy;

and in all cases, whether the current generator be of the battery, thermo-pile, or dynamo type, the rate at which the current generator withdraws energy from some outside source and introduces it into the electric circuit equals

$$\mathrm{AV} + \mathrm{A}^2 b \text{ watts.}$$

118. Electromotive Force of a Battery.—A battery consists of a number of galvanic or electrolytic cells, each cell consisting of two plates of *different* metals immersed in one liquid, or of two plates of the same metal immersed in *different* liquids, or of two plates of *different* metals immersed in *different* liquids, the combination being such that when a current passes through the cell the work corresponding with the chemical action that takes place in one portion of the cell is *not balanced* by the work corresponding with the chemical action that takes place in the same time in the other part of a cell.

Fig. 167 shows a battery composed of five cells of the very simplest form, each cell consisting of a plate of zinc, z, and a plate of copper, c, dipping into dilute sulphuric acid. Such a cell is frequently called a "*simple Voltaic element.*" The copper plate of one cell is joined by means of a copper wire to the zinc plate of the next, so that the cells are in series; and on joining the two terminal copper wires marked + and − in the figure, directly together, or to the terminals of a galvanometer, voltameter, or other indicator of the direction of the current, the current is found to flow in the direction of the arrows. (*See* "Definition of the Direction of a Current," § 7, page 31.)

From the convention we decided to adopt in § 6, page 23, regarding the meaning of the strength of a current, it follows that the rate at which chemical action takes place in an electrolytic cell is a very approximate measure of the current flowing, and therefore the rate at which chemical action takes place in a battery of electro-

lytic or galvanic cells in series such as is indicated in Fig. 167 must be approximately proportional to the current flowing through them, and to the number of cells in series through which it flows, provided that the cells are so constructed that no appreciable chemical action goes on in them when they are at rest, and no current is flowing through them. Consequently, the expression given above for the power in watts developed by the current must be very approximately proportional to the product of the current into the number of cells in series, therefore

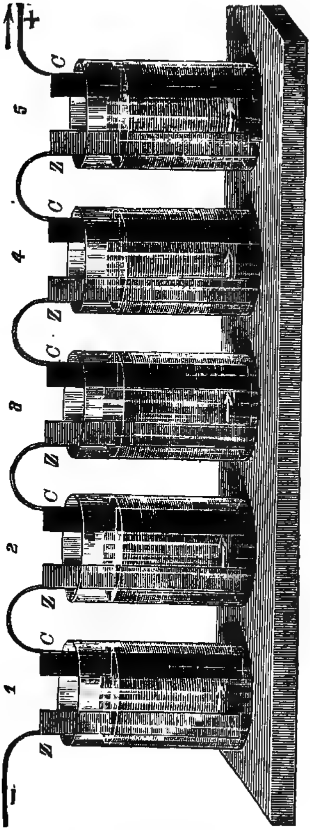


Fig. 167.—Simple Voltaic Elements joined in series.

$$AV + A^2 b = D A n$$

very approximately,

or dividing by A—

$$V + A b = D n$$

very approximately,

where n is the number of cells in series in the battery, and D is a constant depending on the nature of the chemical action taking place in a cell, that is, on the type of cell employed.

The words "very approximately" have been used in the preceding paragraph because, although Faraday's law, that the rate at which chemical action is produced in a given electrolytic cell is proportional to the current passing, is nearly true, the more accurate experiments that have been made recently show that this rate is influenced somewhat by the size of the plates and by the temperature of the cell; *see*, for example, the result of Prof. Gray's tests on the copper voltameter given in the note, § 6, p. 28.

The term Ab in the last equation equals the P.D. in volts required to send the current through b, the resistance of the part of the circuit N K (Fig. 163). Let this be called V′ volts, then,

$$V + V' = D\,n.$$

Hence the P.D. employed in sending the current through K L M N, plus the P.D. employed in sending it through N K, is equal to a constant $D\,n$, which depends only on the battery, and not on the resistance of the circuit or the current flowing through it. Hence, we may conclude generally that *when a current is produced by a battery, the P.D. employed in sending the current right round the circuit is practically a constant for the particular battery, whatever be the resistance of the circuit or of the current flowing.* This constant is called the "*electromotive force*" of the battery, and is frequently designated by the letters "*E.M.F.*"

The *electromotive force* of the battery then depends mainly on the *chemical constitution* of the cells, and on the *number* of cells *in series n* comprising the battery; the constant D in the last equation being, in fact, the *E.M.F* of one of the cells. We may, therefore, define *the electro motive force of a cell as the rate in watts at which chemical energy is converted into electrical energy in the cell per ampere of current flowing, or as the total P.D employed right round any complete circuit through which a current is sent by the cell.*

It follows, therefore, that the E.M.F. of a cell is independent of the size and shape of the cell, for, as shown in § 5, page 19, there is the same amount of chemical action produced per second per ampere in an electrolytic cell whatever be its size and shape; consequently, the rate in watts at which chemical energy is converted into electric energy per ampere flowing—that is, *the E.M.F. of the cell—must be independent of its size and shape.*

For experimentally testing the truth of this result, we may conveniently use the cell shown in Fig. 168. The liquid is contained in a long wooden trough lined with Griffith's anti-sulphuric enamel, and the copper and zinc plates C and Z are supported by stout wires, W, W, sliding in the screw clamps S, S. On loosening the nut P the clamp which supports the wire attached to the copper plate can be slid along the groove and the distance between the plates varied, while on loosening the screws S, S the depths to which the plates are immersed in the liquid can be altered. Now, it is found that, no matter how the distance between the plates be altered, or whether they be immersed more or less in the liquid, or whether they be raised up so much that only the little projections at the bottoms of these plates are in contact with the liquid, as seen in Fig. 168*a*, the E.M.F. of the cell remains exactly the same.

A very convenient method of measuring the E.M.F. of this cell is to employ a high-resistance voltmeter, as described in the next section; but it must be remembered that, as the resistance of the cell when the plates are raised up so that only a small bit of each plate just touches the liquid may be as high as 50 ohms, the resistance of the voltmeter used should not be less than about 500 ohms. (*See* § 119, page 365.)

Further details regarding the construction of the cell illustrated in Fig. 168 will be found under the description of the "Daniell's Cell," § 131, page 423.

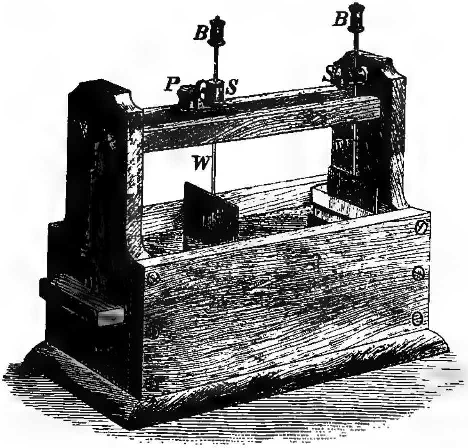


Fig. 168.

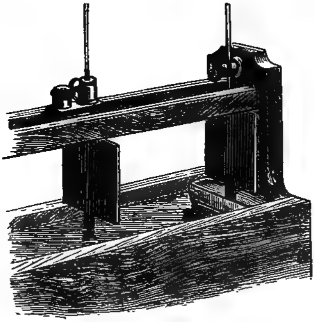


Fig. 168*a*.

Figs. 168, 168*a*.—Cell arranged for proving that the E.M.F. is Independent of the Distance Apart of the Plates and of the Areas Immersed in the Liquids.

The E.M.F. of a cell, then, depends only on the nature of the materials used in constructing it, and not on the amount of the materials employed. If, however, the current passing through a cell be increased so much that the nature of the chemical action taking place in it becomes changed, the E.M.F. no longer remains constant and the cell is said to be "*polarised.*" With certain types of cells this limiting strength of current above which "*polarisation*" takes place is very small, much smaller than the cell can produce if its terminals be joined by a conductor of low resistance.

119. Connection between the E.M.F. of a Battery, the P.D. between its Terminals, the Resistance, and the Current.—If E be the E.M.F. of a battery in volts, b its resistance in ohms, V the P.D. between its terminals in volts, x the external resistance in ohms, and A the current in amperes produced by the battery, we have from the last section

$$\mathrm{E} = \mathrm{V} + \mathrm{A}\,b;$$

therefore, since V equals A x, we have

$$\mathrm{E} = \mathrm{A}\,(x + b),$$

$$\text{or} \quad \mathrm{E} = \frac{\mathrm{V}}{x}\,(x + b).$$

These equations can be most conveniently written in the following form :—

$$\mathrm{V} = \mathrm{E} - \mathrm{A}\,b.$$

$$\mathrm{V} = \frac{x}{x + b}\,\mathrm{E}.$$

$$\mathrm{A} = \frac{\mathrm{V}}{x}.$$

$$\mathrm{A} = \frac{\mathrm{E}}{x + b}.$$

From the last equation it follows, since E is a constant for a given battery, that when x is very large A is

very small, and from the first equation we see that when A is very small V is equal to E. Hence *to find the E.M.F. of a battery we must measure the P.D. between the terminals when the battery is sending no current at all; or but an extremely small one.* A voltmeter whose resistance is very high compared with that of the battery must, therefore, be used in measuring E, and the only current that the battery is allowed to send must be that passing through the voltmeter.

Next, let x diminish, then from the last equation we see that A increases, and from the second equation of the four that V diminishes. Finally, let x become nought—that is, let the battery be short-circuited—then A becomes equal to $\frac{E}{b}$, which is the maximum value the current can attain for a particular battery, and V becomes nought.

The preceding is all given concisely in the following table:—

Values of		
x	V	A
Infinity.	E	Nought.
Great compared with b.	Very little less than E.	Small.
a, say.	$\frac{a}{a+b}$ E.	$\frac{E}{a+b}$.
Small compared with b.	Small.	Great.
Nought.	Nought.	Maximum, and equal to $\frac{E}{b}$.

The apparatus shown in Fig. 169, consisting of a battery, B, a delicate ammeter, A, a voltmeter, V, and a variable resistance, x, enables the preceding results to be tested experimentally.

First, make x equal to infinity, then the reading of the voltmeter gives E.

Secondly, make x have any suitable value, so that the current can be easily read accurately on the ammeter. Let it be A_1 amperes, and let the corresponding P.D. between the terminals of the battery be V_1 volts; next, give x some other value, and let the current and P.D. be now A_2 amperes and V_2 volts respectively; then, if

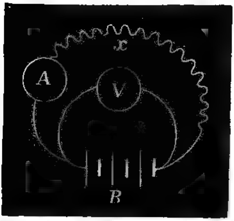


Fig. 169.

$$V_1 = E - A_1 b$$
$$\text{and} \quad V_2 = E - A_2 b,$$

as stated above,

$$\frac{E - V_1}{E - V_2} \text{ must equal } \frac{A_1}{A_2},$$

and the accuracy of this relationship can be tested experimentally without knowing the value of b, the resistance of the battery.

Or, having determined E by measuring V when x is infinite, we may find b from either

$$b = \frac{E - V_1}{A_1} \text{ ohms}$$

$$\text{or} \quad b = \frac{E - V_2}{A_2} \text{ ohms};$$

and the value of b, found from these two equations, being the same, constitutes a proof that the equation

$$V = E - A\,b$$

is correct.

Lastly, having found b, we may give x various values, and try experimentally whether the current always equals $\frac{E}{x + b}$ amperes.

As a rough analogy, the P.D. between the terminals of a battery may be likened to the force exerted by a locomotive engine in dragging the railway carriages, which is, of course, equal to the pull on the coupling connecting the engine with the first carriage, while the current strength may be likened to the speed of the train, and the external resistance to the mass of the carriages composing the train. If the train be long and heavy, corresponding with a great external resistance, the pull exerted by the engine is great, but the speed of the train is slow; whereas if there be only a few carriages the pull is less but the speed is greater, and in the extreme case when the engine is running alone the pull exerted on the coupling, which is now hanging loose, is nought, and the speed of the train is the greatest. Also the pull exerted by the engine on the first carriage is always less than the total force exerted by the engine (unless the engine is attempting to pull so heavy a train that it does not move, corresponding with infinite external resistance and current nought), because if the engine is moving at all some of its pulling power is employed in moving itself. And so with a battery, if any current at all is flowing, the terminal P.D. must always be slightly less than the E.M.F.

If a current A amperes be sent through a battery of resistance b ohms, in the direction *opposed* to that in which the battery would itself send a current, then the P.D. of V volts maintained between the battery terminals has to send this current against the battery resistance b, as well as to overcome the E.M.F. of the battery, say E volts. Hence in this case

$$V = E + A\,b.$$

Example 81.—A Daniell's cell has an E.M.F. of 1·07 volt, and an internal resistance of $2\frac{1}{2}$ ohms; what

current will it send through an external resistance of 32 ohms? *Answer.*—0·031 ampere nearly.

Example 82.—A battery having an E.M.F. of 15 volts, and an internal resistance of 25 ohms, is sending a current through an external resistance of 5 ohms; what is the P.D. between the battery terminals?

Answer.—$2\frac{1}{2}$ volts.

Example 83.—What current must the battery in the last question send so that its terminal P.D. may be 7·5 volts? *Answer.*—0·3 ampere.

Example 84.—If a battery having an E.M.F. of 8 volts have its terminal P.D. reduced to 2 volts on sending a current of 2 amperes, what is its internal resistance?

Answer.—3 ohms.

Example 85.—The P.D. between the terminals of a battery is 15 volts when the battery is sending a current of 2 amperes, and 12 volts when the current is 3 amperes; what is its internal resistance?

If E be the unknown E.M.F. of the battery, and b its resistance,

$$\begin{aligned} \text{we have } 15 &= E - 2\,b, \\ \text{also } 12 &= E - 3\,b, \\ \text{or } \quad b &= 3 \text{ ohms.} \end{aligned}$$

Answer.—3 ohms.

Example 86.—A battery having an E.M.F. of 55 volts, and an internal resistance of 0·25 ohm, is sending a current of 20 amperes through an external resistance. How many watts are spent in the external resistance, and in the battery itself?

Answer.—The total watts developed are 20×55, or 1,100. The watts taken by the battery itself, due to its resistance, are $20^2 \times 0{\cdot}25$, or 100.

Hence, the watts spent in the external circuit are 1,000.

Example 87.—A battery having an E.M.F. of 2·2 volts, and a resistance of 0·18 ohm, is opposing a current sent through it by a more powerful battery. If the

current passing through it is 15 amperes, what is the P.D. between its terminals?

$$\text{Since, generally, } V = E + A\,b,$$
$$\text{we have } V = 2{\cdot}2 + 15 \times 0{\cdot}18\,;$$
$$\therefore\; V = 4{\cdot}9.$$

Answer.—4·9 volts.

120. Electromotive Force of any Current Generator.—If b be the internal resistance of any current generator, and V be the P.D. in volts between its terminals, when the current that the generator is producing, or is helping to produce, is A amperes, it is customary to call the expression

$$V + A\,b \text{ volts,}$$

the E.M.F. of the generator, even when the expression is not independent of the value of A. In such a case the E.M.F. of the generator is not a constant, as it is very approximately in the case of a battery, but varies with the current passing, and it must then be regarded merely as a name for the value that $V + A\,b$ may happen to have. A dynamo is an example of a very important type of current generator, the E.M.F. of which often varies greatly with the current passing, and the name E.M.F., which, like the name resistance, originally came into existence to designate a constant property which was not altered by varying the current, is now used in an extended sense in connection with a dynamo, as is the name "resistance" when speaking of the apparent resistance of the electric arc (*see* § 71, page 233).

When the E.M.F. of a current generator varies with the current, we cannot find its value, as we did in the case of a battery, by stopping the current and measuring the P.D. between the terminals of the generator, since the stoppage of the current would alter the value of the thing to be measured. The values of V and A can, however, be measured at any moment by means of a voltmeter and an ammeter, and if the generator be, for example, a dynamo, whose resistance is practically inde-

pendent of its E.M.F. and of the current passing (except in so far as the current warms the coils of the machine), we can stop the rotation of the armature, which reduces the E.M.F. to nought, and then measure b the resistance of the dynamo by means of a Wheatstone's bridge, as we would measure the resistance of any other coil of wire.

In § 117, page 358, it was proved that the power developed by a current generator equalled

$$AV + A^2b \text{ watts,}$$

therefore, if we decide in all cases to call the expression $V + Ab$ the E.M.F. of the generator, whether it be constant and independent of the current or not, it follows that *the electric power developed by any current generator equals the product of the current, that is flowing, into the E.M.F. of the generator at the time.* Hence, we may *define the E.M.F. of any current generator in volts as the ratio which the electric power developed by the generator, in watts, bears to the current flowing through it, in amperes,* this ratio being a constant in the case of a good battery, but varying greatly with the current in the case of other types of current generators, such as dynamos.

121. Power Absorbed in the Circuit Exterior to the Generator. Back E.M.F.—When power is given by a current of A amperes to a circuit between the ends of which a P.D. of V volts is maintained, the power so given equals AV watts. Of this a portion, A^2a watts, will be spent in heating the circuit where a is its resistance in ohms, and if

$$A^2a = AV$$

the circuit acts as a simple resistance, the whole of the electric energy given to it being converted *directly* into heat.

If, however, no thermo-pile be in circuit, and if

$$A^2a < AV,$$

there must be some apparatus in the circuit which transforms electric energy into some form of energy other than heat, and the rate at which this transformation takes place equals

$$AV - A^2a.$$

Two classes of apparatus may be employed for removing electric energy from a circuit without directly converting it into heat, viz. :—

(1) An electromotor ; *

(2) A cell, or battery, placed in the circuit so that its E.M.F. opposes the current,

and we know from § 119, page 367, that, if the E.M.F. of an opposing cell has a constant value of E volts, then

$$E = V - A\,a;$$

so that $AV - A^2a$, which represents the rate at which electric energy is withdrawn from the circuit by the cell and not converted into heat, equals

$$A\,E$$

where E is the "*back E.M.F.*" in the circuit, or the E.M.F. of the cell opposing the current.

When there is an electromotor, or thermo-pile, in the circuit the expression $V - A\,a$ will not usually be a constant and independent of the current, as it is in the case of a good cell, but we are led by analogy to call the expression $V - A\,a$ in all cases the *back E.M.F.* in the circuit, whether it be constant and independent of the current or not. So that in all cases, apart from the heating due to resistance, *the rate of conversion of electric energy in a circuit into some other form of energy equals the product of the current into the back E.M.F. in the circuit at the time* ; or we may *define the back E.M.F. of any apparatus, in volts, as the ratio which the rate of conversion, in watts, of electric energy into some other form of energy bears to the current, in amperes, flowing through the apparatus.*

If the *back E.M.F.* is independent of the current, when, for example, it is produced by a battery which is inserted in the circuit so as to oppose the current, we can find its value by stopping the current and measuring

* The type of electromotor dealt with throughout in this chapter is the "*series*," or single circuit, electromotor having its armature and the field-magnet *in series* with the main circuit.

the P.D. between the ends of the circuit containing the back E.M.F. When, however, the back E.M.F. varies with the current while the resistance of the apparatus producing it does not, as, for example, in the case of a motor, the value of the back E.M.F. can be ascertained at any moment by taking simultaneous observations of a voltmeter and ammeter, to determine the values of V and A, then, having stopped the rotation of the armature of the motor to reduce the back E.M.F. to nought, the resistance of the motor, a, can be measured with a Wheatstone's bridge, or in any other convenient way.

If, however, both the back E.M.F. and the resistance of the apparatus producing the back E.M.F. vary with the current, it may be impossible by a direct measurement to determine the exact values of either, and it is for this reason that there has been so much doubt about the value of the back E.M.F. in an electric arc, and, indeed, doubt as to whether there is any back E.M.F. at all.

When there is a *back E.M.F.* of E volts in a circuit of resistance a ohms, and between the ends of which a P.D. of V volts is maintained, the current

$$A = \frac{V - E}{a} \text{ amperes.}$$

If, now, a and V be kept constant, and E be increased, A will diminish; when E becomes equal to V the current will be nought; when E is made larger than V the current becomes negative, the change of sign meaning that the current begins to flow in the opposite direction; and the apparatus that previously had a back E.M.F., and was withdrawing electric energy from the circuit and transforming it into some other form of energy, begins to act as a generator, exerting a forward E.M.F., and introduces electric energy into the circuit.

Example 88.—A current generator having a resistance of 0·3 ohm, maintains a P.D. of 100 volts between its terminals when producing a current of 45 amperes. What is its E.M.F.? *Answer.*—113½ volt.

Example 89.—A current generator has an E.M.F. of 67 volts, and maintains a P.D. of 63 volts between its terminals when it is producing a current of 12 amperes. What will be the current when the E.M.F. is 105 volts and the terminal P.D. 98 volts?

Answer.—If the resistance of the generator is constant the difference between the E.M.F. and the terminal P.D. is proportional to the current, therefore the required current is $\frac{7}{4} \times 12$ or 21 amperes.

Example 90.—A dynamo is sending current to a motor of 0·35 ohm resistance, connected with the dynamo with leads, having a resistance of 1 ohm. The P.D. between the terminals of the dynamo is 98 volts when the current is 5 amperes; what is the back E.M.F. of the motor and the P.D. maintained between its terminals?

Answer.—E.M.F. = $91\frac{1}{4}$ volts.
P.D. = 93 volts.

Example 91.—A battery of 3 cells in series, each having 1·08 volt E.M.F., is joined up in circuit with two lead plates immersed in dilute sulphuric acid. The resistance of the whole circuit, including the battery and the lead cell, is 2·7 ohms, and the current is found to be 0·385 ampere. What is the back E.M.F. of the lead cell?

Answer.—2·2 volts.

Example 92.—A battery sends current through a cell consisting of two lead plates in dilute sulphuric acid, the cell having a back E.M.F. of 2 volts. What is the resistance of the cell if the P.D. between the terminals is 5 volts and the current 1·5 ampere? *Answer.*—2 ohms.

Example 93.—The resistance of a motor is 0·64 ohm, and when a P.D. of 83 volts is maintained between its terminals a current of 25 amperes passes. What is the back E.M.F. of the motor? *Answer.*—67 volts.

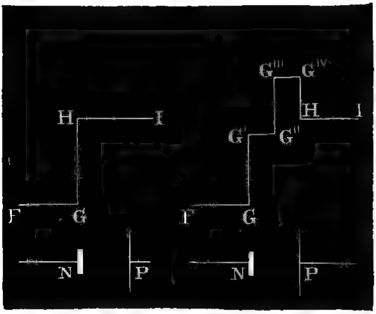
Example 94.—If the resistance of a motor is 2 ohms, and when a P.D. of 100 volts is maintained between its terminals it runs at such a speed that its back E.M.F. is 85 volts, what is the current flowing through the motor?

Answer.—$7\frac{1}{2}$ amperes.

Example 95.—A current of 30 amperes is flowing through a motor of $\frac{1}{2}$ ohm resistance, and it is running at such a speed that its back E.M.F. is 76 volts. What is the P.D. that is maintained between the motor terminals?

Answer.—91 volts.

122. Distribution of Potential in a Battery.—If the terminals of a cell be insulated from one another the P.D. between the terminals, as we have seen, is a constant for the particular type of cell. The way in which the potential varies inside the cell, the exact way in which it changes from the lower value at one terminal of the cell to the higher value of the other, is not certain. By some it is believed that the major part of the P.D. is produced at a junction of one metal with another in the cell, whereas it is thought by others that it is at a junction of an oxidisable metal with a liquid that the P.D. is set up, and that the P.D. between two metals in contact is very small. As regards the parts of the circuit external to the cell it is unimportant how the potential varies inside the cell, for the E.M.F. and resistance of the cell, together with the arrangement of the outside circuit, determine the variation of potential from point to point along this outside circuit. We may, therefore, represent the distribution of potential in a cell whose terminals are insulated from one another by the line F G H I (Fig. 170), the potential at the positive terminal, P, of the cell exceeding that at the negative



Figs. 170, 171.—Distribution of Potential in a Single Cell.

terminal, N, by an amount G H, which therefore represents the E.M.F. of the cell. The line F G H I (Fig. 170) is drawn as if the E.M.F. of the cell were wholly produced at the contact of the zinc plate with the copper wire attached to it, and, as far as points *outside* the cell are concerned, it is immaterial whether the potential line is F G H I (Fig. 170), or whether, as the experiments of Prof. Perry and the author appear to show, it is like F G G^{I} G^{II} G^{III} G^{IV} H I (Fig. 171) in the case of a cell consisting of a plate of zinc and of copper immersed in a solution of common salt, having a specific gravity of 1·18 at 20·5°C. For, as already stated, the distribution of the potential along the outside circuit depends simply on three things—one, the outside circuit; two, the resistance of the cell; and three, the height that the line H I is above the line F G when the terminals are insulated from one another. Therefore, as long as the height G H is constant we may, as far as points outside the cell are concerned, neglect the various steps in the potential curve inside the cell, and assume that the E.M.F. is wholly produced by one single step. But while the line G H represents a fixed E.M.F. in volts for an insulated cell of a given type, the actual value of the potential of either terminal of the cell relatively to the earth may be anything we like. If the terminal N be put to earth its potential will be zero, so that the line of zero potential will be F G, while on the contrary, if the terminal P be put to earth, the line of zero potential will be H I, and F G will then represent a potential below zero. Lastly, the whole cell may be insulated and charged until the potential of either terminal relatively to the earth has any value such as plus or minus 1,000 volts; the P.D. between the terminals P and N will remain as before as long as they are insulated from one another, and the chemical constitution of the cells remains unchanged.

If three similar cells, each having an E.M.F. of 0·93 volt, be joined up in series and the terminals of the battery be insulated from one another, then, dis-

regarding the distribution of potential from point to point inside each cell, the general distribution will be given by the line F G H I J K L M (Fig. 172), the lengths of the lines G H, I J, and K L, which respectively represent the E.M.Fs. of the cells, being equal to one another. If, on the other hand, the cells be of different types, then the lengths of the lines G H, I J, and K L will differ.

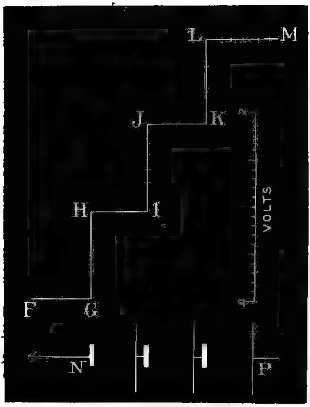


Fig. 172.—Distribution of Potential in Three Cells in Series.

From these figures it will be seen that, while the copper wire P attached to the copper plate of a cell is the "*positive terminal*" of the cell, the zinc plate is the "*positive plate,*" for the potential suddenly rises in passing from the copper wire N to the zinc plate, which is metallically fastened to this wire. This is indicated symbolically in the figures.

Next, let three cells of different E.M.Fs. and resistances be employed in series to send a current through an external resistance. Let the short thick and the long thin lines which stand for the zinc and the copper plates of the battery be drawn in Fig. 173 so that the horizontal distances between these lines are proportional to the resistances of the cells and of the wires connecting them together, some convenient length being taken to represent 1 ohm. On the same scale, let the line P Q represent the resistance of the circuit external to the cells, so that the end Q of this external circuit is in reality attached to N, the zinc plate of the first cell. Next, along a horizontal line whose level is taken to indicate

the potential of the point Q, mark off lengths G R, R S, S T, and T U to represent respectively the resistances of the three cells and of the conductor P Q, which is external to the battery. On a vertical line passing through G mark off lengths G H, H V, and V W to represent respectively the E.M.Fs. of the cells, some convenient length being taken to represent 1 volt; join W and U. Next, through the points R, S, and T draw vertical lines, and through the points H and V draw sloping lines parallel to W U; then the points of intersection I, J, K, L, and M give the potentials at the terminals of the cells relatively to the potential at Q, and the line M U gives the potential of any point in P Q relatively to that of Q. For example, the P.D. between X and Q is represented by the line Y Z, on the same scale that the lines G H, H V, and V W represent the E.M.Fs. of the cells.

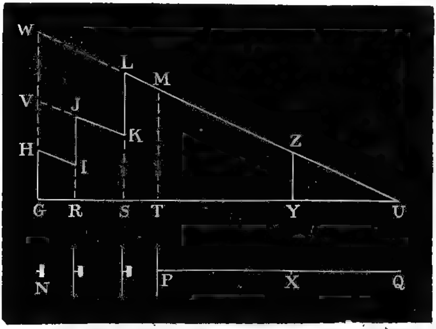


Fig. 173.—Distribution of Potential in a Battery and External Circuit.

The preceding follows from the fact, first, that when a steady current is flowing through any single circuit (that is, a circuit without branches) the current in all parts of the circuit is the same; secondly, that when a steady current is flowing through a conductor the ratio of the P.D. between any two points to the resistance of the conductor between those points is constant. Hence, the slopes of the lines H I, J K, and of L U to the horizontal line G U must be the same, and, if the same unit of length be used vertically for 1 volt as is used horizontally for

1 ohm, the current flowing in amperes is numerically equal to the tangent of the angle which any one of these sloping lines makes with G U. Or generally, whatever be the lengths employed to represent an ohm and a volt respectively, the current in amperes equals the tangent of the angle that W U makes with G U multiplied by the ratio of the lengths taken to represent an ohm and a volt respectively.

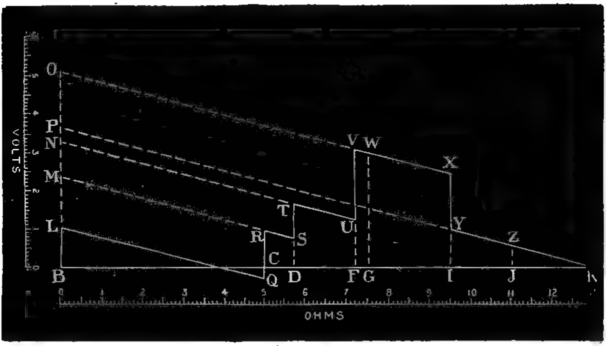


Fig. 174.—Distribution of Potential in a Battery and External Circuit containing an opposing E.M.F.

As an example of this general method of constructing the potential line, let us consider the case of using four cells in series having E.M.Fs. of 1, 1·3, 0·9, and 1·85 volt respectively, and resistances of 5, 0·7, 1·5, and 0·3 ohm respectively, to send a current through an external circuit consisting of a wire of 4 ohms' resistance, and having a cell of 1·5 volt E.M.F. and 1·5 ohm resistance inserted at its centre so as to *oppose* the current. Along a horizontal line (Fig. 174) mark off lengths B C, C D, D F, F G, G I, I J, and J K to represent to scale the resistances of the first, second, third, and fourth cells, the resistance of half the external wire, the resistance of the opposing cell, and the resistance of the other half of the

external wire. Along a vertical line passing through B mark off BL, LM, MN, NO, and OP to represent to scale the E.M.Fs. of the five cells—viz. 1, 1·3, 0·9, 1·85, and − 1·5. BP, therefore, represents the total E.M.F. in the circuit, while BK represents the total resistance. Hence the line joining P and K gives the slope of the potential, and the tangent of the angle PKB multiplied by the ratio of the lengths used to represent an ohm and a volt respectively equals the current in amperes that flows through the circuit, which in this case is 0·273 ampere.

Through the points on BK erect vertical lines, and through the points on BO draw sloping lines each parallel to PK; then the points of intersection Q, R, S, T, U, V, W, X, Y, Z give the potentials at the various junctions. For example, the potential at the copper plate of the third cell relatively to the potential at the end of the circuit K is represented by FU; the P.D. between the point I, which is at the end of the first half of the external wire, and the point K, at the end of the second half, is represented by IX.

If we consider any pair of points like T and U, or W and X, or Y and Z, &c., such that the potential line falls steadily between the points without discontinuity, the difference between the ordinates of the potential line at the two points equals the product of the current in amperes into the resistance between the points. For example,

$$\begin{aligned} \mathrm{DT} - \mathrm{FU} &= 0{\cdot}273 \times 1{\cdot}5 \text{ volt.} \\ \mathrm{GW} - \mathrm{IX} &= 0{\cdot}273 \times 2 \quad ,, \\ \mathrm{IY} - \mathrm{JZ} &= 0{\cdot}273 \times 1{\cdot}5 \quad ,, \end{aligned}$$

While DT−FU represents the potential lost in the second cell, it does not, however, represent the P.D. between the terminals of this cell, for the potential of the copper wire attached to the zinc plate of this cell is not DT, but DS; so that the P.D. between the terminals of this cell, measured in the direction in which the current flows, is DS−FU, a negative quantity. The current, in fact, is forced by the cell to flow from a point of low potential to a point of high.

It is to be remembered that the preceding figures, as already explained, are drawn as if the E.M.F. of each cell was generated at one contact—viz. between the zinc plate and the copper wire attached to it. The potential, therefore, at the points D, F, &c., has in each case two values, depending on whether we refer to the potential of the zinc plate or the potential of the copper wire attached to it. The P.D. available for sending the current through the third cell and overcoming its resistance is $DT - FU$, but the P.D. that would be measured on a voltmeter attached to the terminals of the third cell would be $FU - DS$. If, therefore, V be the P.D. measured by the voltmeter, E be the E.M.F. of the cell, c its resistance, and A be the current flowing, we have

$$\begin{aligned} Ac &= DT - FU, \\ V &= FU - DS, \\ E &= ST, \\ \text{or} \quad V &= FU - (DT - ST) \\ &= ST - (DT - FU), \\ \therefore \ V &= E - Ac, \end{aligned}$$

as stated in §119, page 364.

For this cell, E is by hypothesis 0·9 volt, and c 1·5 ohm, therefore

$$\begin{aligned} V &= 0{\cdot}9 - 0{\cdot}273 \times 1{\cdot}5 \\ &= 0{\cdot}4905 \text{ volt.} \end{aligned}$$

Hence, if W is the rate in watts at which electric energy is *given to* the outside circuit by this cell,

$$\begin{aligned} W &= 0{\cdot}273 \times 0{\cdot}4905 \\ &= 0{\cdot}1339 \text{ watt.} \end{aligned}$$

Also, if W_1 represents the rate in watts at which chemical energy is converted into electric energy in the cell,

$$\begin{aligned} W_1 &= 0{\cdot}273 \times 0{\cdot}9 \\ &= 0{\cdot}2457 \text{ watt;} \end{aligned}$$

and, if W_2 represents the rate in watts at which electric energy is converted into heat in the cell,

$$\begin{aligned} W_2 &= W_1 - W \\ &= 0{\cdot}1118 \text{ watt,} \end{aligned}$$

or W_2 may be found by multiplying the square of the current by the resistance of the cell, so that

$$\begin{aligned} W_2 &= \overline{0{\cdot}273}^2 \times 1{\cdot}5 \\ &= 0{\cdot}1118 \text{ watt as before.} \end{aligned}$$

123. A Current Generator may Abstract Energy from a Circuit even when its E.M.F. Helps the Current.—When a current is passing through a generator in the direction of its E.M.F. the *excess* of the potential at the terminal by which the current *leaves* the generator over the potential at the terminal by which it *enters* it represents that portion of the E.M.F. of the generator which, not having been used in sending the current through the generator itself, is available for sending it through the external circuit. If, however, the potential at the terminal of the generator by which the current *leaves* it is *lower* than the potential at the terminal by which the current *enters* it, electric energy is *absorbed by the generator*, whether the generator be joined up so as to help or to oppose the current.

For example, the first, second, third, and fourth cells in Fig. 174 (page 378) are all joined up the same way, and in each of them there is a conversion of chemical energy into electric energy; but in the case of the first cell the potential at the leaving terminal C is CQ, a negative quantity, and is, therefore, lower than the potential at the entering terminal K or B, which has been arbitrarily taken as our zero of potential. Hence the drop of potential in this first cell is actually greater than BL the E.M.F. of the cell, for it is equal to CQ + BL. So that, if V is the P.D. in volts measured by a voltmeter attached to the terminals of this first cell,

$$\begin{aligned} \text{Also} \qquad V &= \text{CQ}, \\ E &= \text{BL}, \\ \therefore \qquad Ac &= \text{CQ} + \text{BL}, \\ &= V + E. \\ \text{Hence} \qquad V &= Ac - E \end{aligned}$$

for this first cell, and *not* $V = E - Ac$, which is the

usual relationship when a current is passing through a cell in the direction of its E.M.F.

Let W be the rate in watts at which electric energy is *taken from* the outside circuit by this cell, then

$$W = AV.$$

Also, if W_1 is the rate in watts at which chemical energy is converted into electric energy in this cell,

$$W_1 = AE;$$

and if W_2 is the rate in watts at which electric energy is converted into heat in the cell,

$$W_2 = A^2 c.$$

Therefore, combining these last three equations with the equation given above for V, we have

$$W = W_2 - W_1,$$
$$\text{or} \quad W_2 = W + W_1.$$

Hence, the electric energy which is converted into heat in this cell is greater than that produced by the whole chemical action taking place in it, and, although the current is passing through this first cell in the direction of its E.M.F., the current is greater than would be produced if the cell were short-circuited; hence, the cell abstracts electric energy from the circuit instead of giving electric energy to it, which is what a cell usually does when the current is passing through it in the direction of its E.M.F.

If we now substitute in the preceding equations the values of c and E, which are 5 ohms and 1 volt for this first cell, and of A, which is 0·273 ampere for the whole circuit, we have

$$\begin{aligned} V &= 0{\cdot}273 \times 5 - 1 \\ &= 0{\cdot}365 \text{ volt}, \\ W &= A\,V \\ &= 0{\cdot}273 \times 0{\cdot}365 \\ &= 0{\cdot}0996 \text{ watt}, \\ W_1 &= 0{\cdot}273 \times 1 \\ &= 0{\cdot}273 \text{ watt}, \end{aligned}$$

$$\text{and} \quad W_2 = 0{\cdot}273^2 \times 5$$
$$= 0{\cdot}3726 \text{ watt};$$
$$\text{or} \quad W_2 = W + W_1$$
$$= 0{\cdot}0996 + 0{\cdot}273$$
$$= 0{\cdot}3726 \text{ watt},$$

as given above.

Next, let us consider the fifth cell, which has been inserted between the points I and J so as to oppose the current. In this case, not merely is the potential J Z at the terminal J by which the current *leaves* the cell lower than the potential I X at the terminal I by which the current *enters* the cell, but the E.M.F. of the cell *opposes* the current. Hence, in the cell electric energy will be withdrawn from the circuit and converted into heat, and electric energy will also be withdrawn from the circuit and converted into chemical energy.

If V is the P.D. measured by a voltmeter attached to the terminals of the cell,

$$V = \text{I X} - \text{J Z},$$
$$\text{also} \quad E = \text{Y X},$$
$$\text{and} \quad A\,c = \text{I Y} - \text{J Z};$$
$$\text{hence} \quad V = \text{I Y} + \text{Y X} - \text{J Z},$$
$$\therefore \quad V = E + A\,c,$$

which we have already seen (§ 119, page 367) is the formula for a cell of resistance c ohms when a current of A amperes passes it in opposition to its E.M.F. of E volts.

For this cell, E is by hypothesis 1·5 volt and c 1·5 ohm, therefore,

$$V = 1{\cdot}5 + 0{\cdot}273 \times 1{\cdot}5$$
$$= 1{\cdot}909 \text{ volt}.$$

Hence, if W is the rate in watts at which electric energy is *given to* this cell,

$$W = 0{\cdot}273 \times 1{\cdot}909$$
$$= 0{\cdot}5211 \text{ watt};$$

also if W_1 represents the rate in watts at which electric energy is converted into chemical energy in the cell,

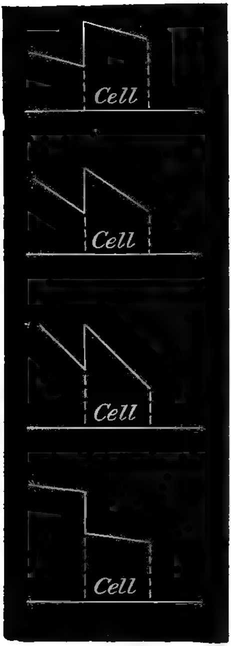
$$W_1 = 0{\cdot}273 \times 1{\cdot}5$$
$$= 0{\cdot}4095 \text{ watt},$$

and if W_2 represents the rate in watts at which electric energy is converted into heat in the cell,

$$\begin{aligned} W_2 &= W - W_1 \\ &= 0{\cdot}1116 \text{ watt as before,} \end{aligned}$$

or W_2 may be found by multiplying the square of the current by the resistance of the cell, so that

$$\begin{aligned} W_2 &= \overline{0{\cdot}273}^2 \times 1{\cdot}5 \\ &= 0{\cdot}1116 \text{ watt, as before.} \end{aligned}$$



Figs. 175, 176, 177, 178.

When a current is passing through a cell there are four possible distributions of potentials represented by Figs. 175, 176, 177, and 178, the current in each case flowing from left to right. In Fig. 175 the rate of transformation of chemical energy into electric energy exceeds the rate of conversion of electric energy into heat in the cell, and the cell gives electric energy to the external circuit.

In Fig. 176 the rate of transformation of chemical energy into electric energy is exactly equal to the rate of conversion of electric energy into heat in the cell, and the cell neither gives energy to the outside circuit nor receives energy from it. This is what happens when a cell is short-circuited with an extremely short thick copper wire, or when the current passing through a cell in a circuit containing other current generators is such that the potential at the terminal of

the cell by which the current leaves the cell is exactly equal to the potential at the terminal by which it enters.

In Fig. 177 the rate of transformation of chemical energy into electric energy is less than the rate of conversion of electric energy into heat in the cell, and the cell abstracts energy from the outside circuit.

In Fig. 178 the rate at which the cell receives energy from the outside circuit equals the sum of the rates at which electric energy is converted into chemical energy in the cell, and at which electric energy is converted into heat in the cell.

Fig. 175 may be likened to a pump which raises water, and also wastes some energy in friction; Fig. 176 to a pump which raises no water, but wastes all the energy it receives in friction in its own mechanism; Fig. 177 to a pump which raises water, wasting some energy in friction, and which is partly driven by allowing the water to subsequently fall to a *greater* distance than that to which the pump has raised it; Fig. 178 corresponds with a turbine which is driven by falling water, some of the energy of the falling water being wasted in friction of the mechanism.

Example 96.—What are the maximum currents that can be passed through the following cells, if they are not to abstract energy from the circuit when they are joined up so that their E.M.F. tends to assist the current?

Cell (*a*) has E.M.F. 1·1 volt and resistance ·75 ohm.
Cell (*b*) has E.M.F. 1·5 volt and resistance ·3 ohm.
Cell (*c*) has E.M.F. 2·1 volt and resistance ·1 ohm.

Answer.—(*a*) 1·47 ampere.
(*b*) 5·00 amperes.
(*c*) 21·0 amperes.

124. External Circuit that Receives Maximum Power from a Given Current Generator.—Let E be the E.M.F. of the current generator in volts, and *b* its resistance in ohms, then, if A is the current in amperes produced when the terminals of the generator are connected

to the ends of some external circuit, and if W is the power in watts given to this external circuit, W_1 the electric power in watts produced in the generator, and W_2 the power in watts wasted in heating the generator,

$$W_1 = AE$$
$$W_2 = A^2 b,$$
$$\therefore \quad W = A(E - Ab)$$

in all cases.

The change, however, produced in the value of W by varying the external circuit so as to alter the value of A will depend on whether the values of E and b are constant and independent of the value of A, or whether one or both vary with A. If the generator be one having a fixed E.M.F. and resistance an examination of the change of the value of W with a variation in the current, A, is quite simple. For when the external circuit is so selected that A is very small, *then* W is obviously very small; if, now, the circuit be gradually altered so as to make A increase then W increases; on the other hand, when A has nearly its maximum value, viz. $\frac{E}{b}$, which is, of course, attained on the generator being short-circuited, W is very small again. As A is continuously increased there must, therefore, be some value of A at which W ceases to increase and begins to diminish, or, in other words, there must be some value of A which makes the expression just given for W a maximum.

To ascertain this value of A we may employ various methods; for example, we may give arbitrary values to E and b, plot a curve connecting W and A, and find out by inspection the approximate value of A for which W is a maximum. Such a curve is seen in Fig. 179, the values of 2 volts and 3 ohms having been arbitrarily given to E and b respectively in calculating the values of the expression for W. From this curve we see that W is a maximum for some value of A between 0·32 ampere and 0·36 ampere, and that this value of A

is somewhat nearer 0·32 than 0·36 ampere. If the curve be drawn on a much larger scale, so that the value of A that makes W a maximum can be read off with still greater accuracy, it is found that this value of A is $0{\cdot}3\dot{3}$ or $\frac{1}{3}$ ampere. Now $\frac{1}{3}$ ampere is half $\frac{2}{3}$ ampere, which is

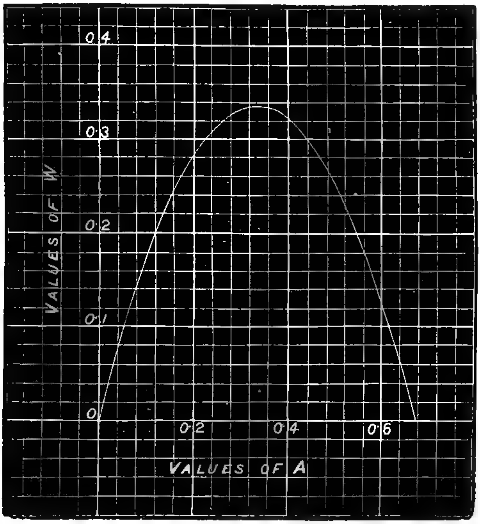


Fig. 179.—Curve showing the Value of the Current that gives the Maximum Power to an External Circuit.

the current that the generator would produce if short-circuited, and the same result would be arrived at whatever values were given to E and c; therefore, generally, we may conclude that *the external circuit which receives maximum power from a current generator, of fixed E.M.F. and resistance, is the circuit which makes the current half as great as it would be if the generator were short-circuited.*

The following is another way of obtaining the same result: —

$$\text{Since} \qquad W = A\,(E - A\,b),$$
$$W = -\,(A^2 b - A\,E),$$
$$\text{or} \qquad \frac{W}{b} = -\left(A^2 - \frac{A\,E}{b}\right);$$

therefore, subtracting and adding $\frac{E^2}{4b^2}$ we have

$$\frac{W}{b} = -\left(A^2 - A\,\frac{E}{b} + \frac{E^2}{4\,b^2}\right) + \frac{E^2}{4\,b^2},$$
$$= -\left(A - \frac{E}{2\,b}\right)^2 + \frac{E^2}{4\,b^2}.$$

Now, since $\left(A - \frac{E}{2\,b}\right)^2$ is a square it can never be negative, therefore $\frac{W}{b}$ will be a maximum when

$$A = \frac{E}{2\,b}.$$

This method of ascertaining the value of A which makes W a maximum is simpler than the preceding graphical method, but, unlike the former, it gives no indication of the way W diminishes from its maximum value as A is made greater or less than $\frac{E}{2\,b}$. A more detailed consideration of this point will be found in § 126, page 397.

$$\text{When} \qquad A = \frac{E}{2\,b},$$
$$\text{we have} \quad W_1 = \frac{E^2}{2\,b},$$
$$W_2 = \frac{E^2}{4\,b},$$
$$\text{and} \qquad W = \frac{E^2}{4\,b},$$

therefore *a current generator, of fixed E.M.F. and resistance, will give maximum power to an external circuit when half the electric power produced in the generator is wasted in heating itself and half is given to the external circuit;* or we may conclude that *the greatest power a generator of fixed E.M.F. and resistance can give to any external circuit is one quarter of the power which the generator would develop if short-circuited.*

A third method for ascertaining the conditions of the external circuit so that it may receive the maximum power is the following:—Along two axes O Y and O X, at right angles to one another (Fig. 180), set off lengths O P and O Q respectively, to represent to scale E the constant E.M.F. of the generator in volts and A the current in amperes produced with some particular arrangement of the outside circuit. Through the point O draw a line O R, making with O X an angle ROX, such that

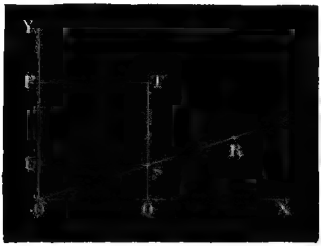


Fig. 180.

tan. ROX = b, the constant resistance of the generator.

Through the point Q draw a line parallel to O P, cutting O R in S, and through the point P a line parallel to O Q, meeting Q S in T.

Then since

$$\text{QS} = \text{A} \times \text{tan. ROX}$$
$$\text{QS} = \text{A}b,$$

therefore Q S represents to scale the P.D. employed in sending the current through the generator, hence S T represents V the P.D. in volts between the terminals of the generator. Through the point S draw a line parallel to O Q, and meeting O P in U.

Then the area of the rectangle

$$\text{O Q T P} = AE$$
$$= W_1,$$

the area of the rectangle

$$\text{O Q S U} = A \times \text{P.D. used in generator}$$
$$= W_2,$$

and the area of the rectangle

$$\text{U S T P} = AV$$
$$= W.$$

Next let the external circuit be altered so that the current is slightly increased; and let the current be now represented by O Q′ (Fig. 181). Complete a figure similar to the preceding, obtaining points S′, T′ and U′; then the power now wasted in the generator W_2' is represented by the area of the rectangle O Q′ S′ U′, and the power W' given to the external circuit is represented by the area of the rectangle U′ S′ T′ P.

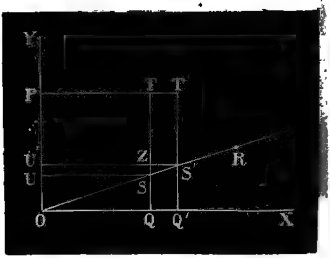


Fig. 181.

Now we wish to consider whether the power given to the external circuit has been increased by the increase that has been made in the current, that is, we desire to know whether the area U′ S′ T′ P exceeds the area U S T P. From Fig. 181 we see that

$$\text{area U}' \text{S}' \text{T}' \text{P} -$$
$$\text{area U S T P} = \text{area Z S}' \text{T}' \text{T} - \text{area U S Z U}'$$
$$= \text{Z S}' \times \text{Z T} - \text{S Z} \times \text{U S}.$$

But $\quad \text{Z S}' = \text{Q Q}';$

and $\quad \text{Z T} = \text{S T} - \text{S Z},$
$$= V - \text{S Z};$$

$$\text{also} \quad \frac{\text{SZ}}{\text{ZS}'} = \frac{\text{OU}}{\text{US}'},$$
$$= \tan.\ \text{ROX},$$
$$= b\,;$$
$$\therefore \quad \text{SZ} = b \times \text{QQ}'\,;$$
$$\text{and} \quad \text{US} = \text{OQ},$$

therefore, substituting these values for ZS′, ZT, SZ and US in the expression given above, we see that the increase in the power given to the external circuit by an increase in the current of QQ′ equals

$$\text{QQ}'(\text{V} - b \times \text{QQ}' - b \times \text{OQ}),$$

but QQ′ is small compared with OQ, since by hypothesis only a small change was made in the current, therefore the increase in the power given to the external circuit, produced by making a small increase in the current, equals approximately

$$\text{QQ}'(\text{V} - b \times \text{OQ}).$$

This expression will be positive as long as V exceeds $b \times$ OQ, and will become negative when V becomes less than $b \times$ OQ. But $b \times$ OQ represents the P.D. used in sending the current through the cell; hence making a number of changes in the circuit so as to produce small increments in the current will increase the power given to the external circuit as long as V, the P.D. between the terminals of the generator, exceeds the P.D. used in the generator itself; but when these two become equal any further increase in the current will make the increase in the power delivered negative—that is, will begin to diminish the power given to the external circuit. Consequently, *the external circuit that receives maximum power from a current generator, of fixed E.M.F. and resistance, is the circuit that makes the P.D. between the terminals of the generator equal to half its E.M.F.*

This is the form of the law that it is most easy to test experimentally, and the apparatus shown in Fig. 169, page 366, may conveniently be used for this purpose.

The product of the readings of the ammeter A and the voltmeter V gives the power furnished to the external circuit, and if x, the resistance of this circuit, be altered and a set of simultaneous readings of the ammeter and voltmeter be taken, it will be found that the product of these readings will be a maximum when the deflection of the voltmeter is half the deflection that is obtained on breaking the external circuit—that is, when the P.D. between the battery terminals is half its E.M.F.

To test the law in its two previous forms would require the battery to be short-circuited, but the E.M.F. of even a so-called "constant battery" would change somewhat if the battery were short-circuited; while in the case of many useful batteries, although the E.M.F. is fairly constant when the resistance of the external circuit is two, three, or more times as great as that of the battery itself, it would fall considerably on the battery being short-circuited, and therefore would render an experimental test of the law in its first two forms very difficult to carry out.

It is to be observed that the preceding results are all generally true, whatever be the nature of that portion of the external circuit which we desire shall receive maximum power. For example, the reasoning would be exactly the same whether the portion of the external circuit under consideration were composed of a variable resistance or whether it contained in addition a forward E.M.F. produced by some current generator that could be altered, or a back E.M.F. produced by some electrolytic cell, or by a running electromotor, the E.M.F. of which could be adjusted to bring the current to the required value.

From what precedes, then, we may conclude:—

(1) If an external circuit be a simple resistance of x ohms, then in order that it may receive maximum power from a generator having a fixed E.M.F. of E volts and a fixed resistance of b ohms

$$x \text{ must equal } b.$$

(2) If the external circuit contain in addition a forward E.M.F. of E′ volts,

$$\frac{E + E}{b + x} \text{ must equal } \frac{E}{2\,b},$$

$$\text{or} \quad E' \text{ must equal } \frac{x - b}{2\,b} \times E.$$

(3) If it contain instead a back E.M.F. of E′ volts,

$$E' \text{ must equal } \frac{b - x}{2\,b} \times E.$$

125. Arrangement of Part of the External Circuit to Receive Maximum Power.—If we desire that a current generator of fixed E.M.F. and resistance shall give maximum power to a *portion* of an external circuit—for example, if the generator be connected by long leads of fixed resistance l ohms to a motor or to lamps at a distance, and we desire to arrange the motor or the lamps so that they shall receive the maximum power—then the fixed resistance of the leads must be added to the fixed resistance of the generator; hence for b in what precedes we must substitute $b + l$.

The three equations given at the end of §124 state the conditions under which an external circuit of resistance x ohms shall receive maximum power, this power being wholly employed in heating the circuit if it be a simple resistance, and partly so employed if the external circuit has any resistance at all. If, however, there be an apparatus in the external circuit which has a back E.M.F. of E′ volts, and which, therefore, for example, produces a transformation of electric energy into mechanical or chemical energy, and if we desire to arrange this back E.M.F. so that this transformation of energy may be effected as rapidly as possible, and not merely that the apparatus shall receive the maximum power, then the solution is not the one previously given.

For if m be the resistance of this apparatus in ohms, b and l being the resistances of the generator and the

leads in ohms, then what we now desire is not that $A \left\{ E - A(b + l) \right\}$ shall be a maximum, but that $A E'$ shall be a maximum.

We know that

$$A = \frac{E - E'}{b + l + m},$$

$$\therefore \quad A E' = \frac{E - E'}{b + l + m} \times E';$$

and, by comparing this with the expression $A(E = Ab)$, which was shown in § 124 page 388, to be a maximum when A equalled $\frac{E}{2b}$, we may conclude that the expression just found for AE' will be a maximum when

$$E' = \frac{E}{2}.$$

Under those circumstances the expression for the rate of transformation of electric energy into non-heat energy in the apparatus becomes

$$\frac{E^2}{4(b + l + m)},$$

and this equals

$$\left(\frac{E - \frac{E}{2}}{b + l + m}\right)^2 \times (b + l + m),$$

which is the power expended by the generator in heating the circuit; so that here again the power utilised is half the total electric power which the generator develops.

Consequently, while it follows from the equation given at the end of § 124, page 239, that the apparatus in question will receive maximum power when

$$E' = \frac{b + l - m}{2(b + l)} \times E,$$

the rate of transformation of electric energy into

mechanical or chemical energy, on the contrary, will be effected most rapidly in the apparatus when

$$E' = \frac{E}{2}.$$

All the preceding conclusions are based on the assumption that the E.M.F. and resistance of the primary current generator are *fixed* and independent of the current flowing, and that the arrangement of the external circuit is the *only variable* under consideration. If, however, either the E.M.F. or the resistance of the primary generator change with alterations in the current, then the preceding conclusions will not be generally true, and the particular arrangement of the external circuit that will receive maximum power from the generator will depend on the exact way in which the E.M.F. and resistance of the generator varies with the current in each particular case.

Another class of problem also sometimes arises in practice—viz. one in which the conditions of the external circuit are fixed and cannot be altered, and it is the arrangement of the generator which gives maximum power to this external circuit that we desire to find out. In such a case it is clear that, since the conditions of the external circuit are fixed by hypothesis, the power given to it will depend on the current passing through it; hence the problem reduces itself to finding the arrangement of the generator that will send the greatest current through a given external circuit. To solve this problem some condition must necessarily be given limiting the power of the generator, since without such a condition all we can say is that the larger the E.M.F. and the smaller the resistance of the generator the greater will be the current it will send through the external circuit. When this condition is that the generator consists of a *fixed number* of galvanic cells of a *given type* the problem of finding the arrangement of these cells to produce the maximum current through the fixed external circuit will be found solved in § 163, page 548. Other

problems differing somewhat in character, but having a certain resemblance to the problem discussed in this and the preceding section, will be found solved in § 164, page 549, in § 165, page 553, and in § 166, page 555.

Example 97.—What is the maximum horse-power that can be given to any external circuit by a battery of 50 cells in series, each having a resistance of 0·05 ohm and an E.M.F. of 2·2 volts?

Answer.—The maximum power will be one-quarter of the power which the battery would develop if short-circuited, on the assumption that short-circuiting the battery did not affect its E.M.F. or resistance. Therefore the maximum power that can be given to any external circuit equals

$$\frac{1}{4} \times \frac{50 \times 2{\cdot}2}{50 \times 0{\cdot}05} \times 50 \times 2{\cdot}2, \text{ or } 1{,}210, \text{ watts,}$$

which equals 1·622 horse-power.

Example 98.—How many glow lamps, each requiring a current of $\frac{1}{3}$ ampere and a P.D. of 100 volts between its terminals to make it glow properly, can be used with the above battery of cells, and how should the lamps be arranged?

Answer.—In order that the battery may give maximum power to the lamps, the lamps, as they contain no forward or back E.M.F., must be grouped so that the resistance of the group equals the resistance of the battery. The latter is 2·5 ohms, while that of one lamp is $\frac{100}{\frac{1}{3}}$, or 300 ohms; hence the lamps must be placed in parallel, and, if p be the number of lamps arranged in parallel, the lamps will receive maximum power when

$$\frac{300}{p} = 2{\cdot}5,$$

that is, when $p = 120$.

It does not follow, however, that 120 lamps can be used in parallel and each receive $\frac{1}{3}$ ampere; indeed, all the preceding shows us is that arranging lamps in

parallel up to the number of 120 is the method for causing the group of lamps to receive the maximum power from the battery of 50 cells. To find the actual number of lamps, p, that can be employed in parallel, each lamp receiving a current of $\frac{1}{3}$ ampere, we have

$$\frac{p}{3} = \frac{50 \times 2{\cdot}2}{50 \times 0{\cdot}05 + \frac{300}{p}};$$

$$\therefore \quad p = 12,$$

or twelve is the greatest number of lamps that can be used, and they must be arranged in parallel.

Example 99.—A current generator having a fixed E.M.F. of 80 volts and a resistance of 0·7 ohm is used to drive an electromotor having a resistance of $\frac{1}{2}$ ohm, the electromotor being connected with the generator by mains having a resistance of 2 ohms. What should be the back E.M.F. of the motor so that it will receive the maximum power?

Answer. $\dfrac{0{\cdot}7 + 2 - 0{\cdot}5}{2\,(0{\cdot}7 + 2)} \times 80$, or 32·6 volts.

Example 100.—What should be the back E.M.F. of the motor in the last question so that it may develop the greatest mechanical power? *Answer.*—40 volts.

126. Way in which Power Received by External Circuit Varies from Maximum.—In all problems of maxima and minima it is important not merely to ascertain what is the value of the variable that makes the given expression a maximum or minimum, but to consider also how much change is produced in the value of the expression in question when the value of the variable is altered from that required to make it a maximum or minimum.

For example, we have seen that a current generator of constant E.M.F. of E volts and resistance of b ohms will give maximum power to an external circuit when the current flowing equals $\frac{E}{2\,b}$. Suppose, now, that the

external circuit is arranged so that the current is 10 or 20 per cent. larger or smaller than this value, then how much smaller will be the power given to this external circuit than the maximum power which the generator could give to it?

The power given to the external circuit

$$W = A\,(E - A\,b) \text{ watts.}$$

$$\text{If} \quad A = \frac{E}{2\,b},$$

$$\text{then } W = \frac{E^2}{4\,b}.$$

$$\text{If} \quad A = \frac{E}{2\,b} + C,$$

$$\text{then } W = \left(\frac{E}{2b} + C\right)\left(\frac{E}{2} - C\,b\right),$$

$$\therefore \frac{\text{diminution of power from maximum}}{\text{maximum power}} = \frac{\dfrac{E^2}{4\,b} - \left(\dfrac{E}{2\,b} + C\right)\left(\dfrac{E}{2} - C\,b\right)}{\dfrac{E^2}{4\,b}}$$

$$= \frac{C^2\,b}{\dfrac{E^2}{4\,b}}$$

$$= \left(\frac{C}{\frac{E}{2b}}\right)^2,$$

and exactly the same result would be obtained if A were diminished from $\frac{E}{2\,b}$ to $\frac{E}{2\,b} - C$. Hence *the proportional diminution in the power given to the external circuit from its maximum value equals the square of the proportional change in the current from the value that makes the power a maximum.*

For example, if $\quad E = 10$ volts,
and $\quad b = 1$ ohm,
the external circuit will receive the maximum power, viz. 25 watts, when the current is 5 amperes. If, now, the external circuit be such that the current is 7 or 3 amperes, that is, 40 per cent. greater, or 40 per cent. less, than 5 amperes, the power given the external circuit will be 21 watts, that is only $\left(\frac{40}{100}\right)^2$, or 16 per cent., less than 25 watts, the maximum.

Further, to produce a given percentage change in the current we must make a still greater percentage change in the external resistance, hence the power given to the external circuit will not differ much from the maximum even when the external resistance differs considerably from that which causes the external circuit to receive the maximum power. To find the exact change produced in the power let the external circuit consist of a simple resistance equal to $b - r$ ohms, then we have

$$\frac{\text{diminution of power from maximum}}{\text{maximum power}} = \left(\frac{C}{\frac{E}{2b}}\right)^2.$$

$$\therefore \frac{\text{diminution of power from maximum}}{\text{maximum power}} = \frac{\left(\frac{E}{2b - r} - \frac{E}{2b}\right)^2}{\left(\frac{E}{2b}\right)^2}$$

$$= \frac{r^2}{(2b - r)^2}$$

$$= \left(\frac{r}{b}\right)^2 \times \frac{1}{\left(2 - \frac{r}{b}\right)^2},$$

where $\frac{r}{b}$ is the proportional diminution in the resistance

of the external circuit from the value that causes the external circuit to receive maximum power.

If, instead of the external circuit having a resistance of r ohms, less than that of the battery, it had had a resistance of r ohms greater than b, then we should have found

$$\frac{\text{diminution of power from maximum}}{\text{maximum power}} = \left(\frac{r}{b}\right)^2 \times \frac{1}{\left(2 + \frac{r}{b}\right)^2}.$$

As an example let the resistance of the external circuit be only half that of the battery, that is, let

$$\frac{r_1}{b_1} = \frac{1}{2}$$

then
$$\frac{\text{diminution of power from maximum}}{\text{maximum power}} = \left(\frac{1}{2}\right)^2 \times \frac{1}{(1{\cdot}5)^2}$$
$$= \frac{1}{9},$$

so that a diminution of the resistance of the external circuit from b to $\frac{b}{2}$ causes the power received by the external circuit to be diminished by only $\frac{1}{9}$th.

Again, let the resistance of the external circuit be 50 per cent. greater than that of the battery, that is, let $\frac{r}{b}$ be $\frac{1}{2}$ as before, but now using the second form of the formula we have

$$\frac{\text{diminution of power from maximum}}{\text{maximum}} = \left(\frac{1}{2}\right)^2 \times \frac{1}{(2{\cdot}5)^2}$$
$$= \frac{1}{25},$$

so that an increase of the external resistance from b to $\frac{3}{2}b$ causes the power received by the external circuit to be diminished by only $\frac{1}{25}$th. And an examination of the second form of the formula shows that we may go on increasing r until r equals b, that is, we may go on

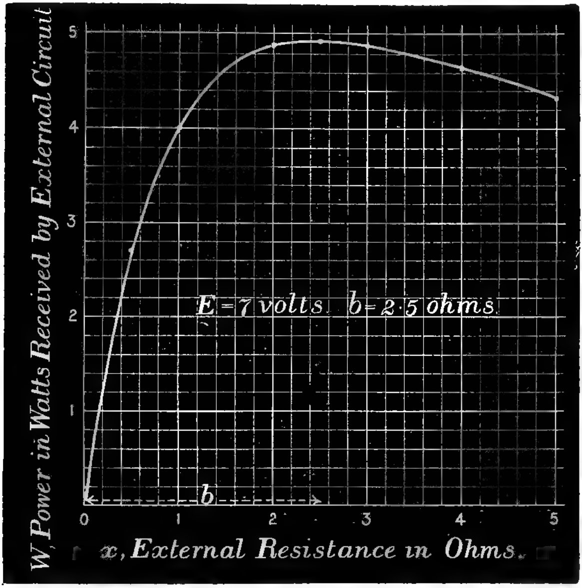


Fig. 182.—Curve connecting the Power received by an External Circuit and the Resistance of that Circuit.

increasing the external resistance from b to $2b$ without the power given to the external circuit being less than the maximum by more than $\frac{1}{9}$th.

Finally, then, we may conclude that, if the external circuit be a simple resistance, having *any* value between $\frac{b}{2}$ and $2b$ ohms, the power that it will receive will not

be less than 89 per cent. of the maximum power, viz. $\frac{E^2}{4b}$ watts.

The curve in Fig. 182 shows the connection between x, the external resistance, and W, the power received by the external circuit, equal lengths being taken to represent an ohm and a watt, and values of 7 volts and $2\frac{1}{2}$ ohms being arbitrarily taken for the fixed values of E and b. The curve is very flat in the neighbourhood of x equal to b, which illustrates what has been proved above, viz. that while a generator of fixed E.M.F. and resistance b ohms gives the maximum power to an external circuit, composed of a simple resistance x ohms, when x equals b, the power received by the external circuit will not be materially diminished if x has any value from about $\frac{3}{4}b$ to $1\frac{1}{2}b$.

If we were dealing with the power given by the generator to a portion only of the circuit, and if this portion consisted of a simple resistance of x ohms, then in all the preceding equations we should have to substitute $b + l$ for b where l ohms was the fixed resistance of the leads connecting the generator with the portion of the circuit under consideration. And the curve in Fig. 182 would then connect the power given to this portion of the external circuit when x was the resistance of this portion alone, and when the resistance of the battery and leads together was $2\frac{1}{2}$ ohms.

If the external circuit, instead of consisting of a simple resistance, contains an apparatus possessing a back E.M.F. of E′ volts, then we saw in § 125, page 394, that, apart from heating due to resistance, a generator having a fixed E.M.F. of E volts will cause the rate of transformation of electric energy in this apparatus to be a maximum when this back E.M.F. is adjusted so that it equals $\frac{E}{2}$. Now this rate of transformation of energy we have seen equals

$$\frac{E - E'}{b + l + m} \times E' \text{ watts,}$$

where b, l and m are the resistances in ohms of the generator, the leads and the apparatus in question, and by applying to this expression exactly the reasoning that was used in connection with the expression A (E — A b) at the beginning of the section, it can be shown that when

$$E' = \frac{E}{2} \pm F$$

$$\frac{\text{diminution of rate of transformation of energy from maximum rate}}{\text{maximum rate of transformation of energy}} = \left(\frac{F}{\frac{E}{2}}\right)^2.$$

Hence, *the proportional diminution of the rate of transformation of electric energy due to a back E.M.F. in an apparatus equals the square of the proportional change in the back E.M.F. of the apparatus from the value that makes the rate of transformation a maximum.*

For example, if $E = 20$ volts

then, apart from the production of heat due to resistance, the rate of transformation of electric energy will be a maximum when the back E.M.F. is 10 volts. If, however, the back E.M.F. is 15 or 5 volts, that is, 50 per cent. greater, or 50 per cent. less, than 10 volts, the rate of transformation of electric energy

will be $\frac{5}{b+l+m} \times 5$ watts and $\frac{15}{b+l+m} \times 5$ watts

respectively, which are each only $\left(\frac{50}{100}\right)^2$, or 25 per cent., less than

$$\frac{10}{b+l+m} \times 10 \text{ watts},$$

the maximum rate of transformation.

Example 101.—If a battery of 50 cells in series, each having an E.M.F. of 2·2 volts and a resistance of 0·05 ohm, be giving the maximum power to an external circuit, what is the current that flows, and by how much per cent. will the power given to the outside circuit be

reduced if the circuit be altered so that the current flowing is diminished by 20 per cent.?

Answer.—22 amperes;
By 4 per cent.

Example 102.—If the external circuit in the last question consist of a simple resistance, what is the value of this resistance when it receives maximum power, and by how much per cent. will the power given to the external circuit be reduced if its resistance is (*a*) 50 per cent. smaller, (*b*) 40 per cent. larger, than that which corresponds with maximum power?

Answer.—2·5 ohms;
By 6·25 per cent.,
By 2·778 per cent.

Example 103.—A generator having a fixed E.M.F. of 220 volts drives a motor. What should be the back E.M.F. of the motor so that it may develop the greatest mechanical power, and by how much will the power it develops be reduced if the back E.M.F. be increased by $\frac{2}{3}$rds above this value? *Answer.*—110 volts;
By $\frac{4}{9}$ths.

127. Efficiency.—When, by means of any machine, or contrivance, one form of energy is converted into another form, some heat is produced; hence, if heat energy is not the form in which the energy is required after the conversion, some portion of the energy which has been used up has been converted into a useless form as far as the object in question is concerned, and may, therefore, be regarded as wasted. Consequently, in all cases the amount of *useful* energy produced is less than the amount of energy used up. For example, when a machine is used to do work there is a waste of energy in the heating of the bearings of the machine; if falling water is employed to turn a water wheel there is in addition waste of energy in the eddies set up in the water, in the splash of the water against the blades of the wheel as well as in the friction of the water stream against the sides of the channel which guides it to the wheel. When oil, wax,

gas, &c., are consumed as illuminants only a very small fraction of the available energy is converted into the special form of energy, called light, which affects the retina of the eye, and the greater part is wasted in heat, whose action on the eye does not differ from its action on other parts of the body. Again, in a battery a certain amount of chemical energy is wasted in the heat produced by "*local action*" (*see* § 132, page 428), which goes on even when the battery is producing no useful current; further, on the battery being used to send a current through some external circuit a portion of the chemical energy that is converted into the electric energy is always wasted in heating the battery in consequence of its resistance.

The value of any machine or contrivance for effecting a conversion of one form of energy into another depends first on the rate at which energy in a useful form is evolved by it—that is, the *useful* power the machine develops, and which is sometimes called its "*useful activity*"—secondly, the value of the contrivance depends on the ratio of the amount of *useful* energy produced to the amount used up in the process, and this ratio is called the "*efficiency.*"

Efficiency, then, is expressed by a simple number, less than unity, such as $\frac{1}{2}$, 0·63, 75 per cent., 84 per cent. Sometimes, however, it is found convenient to employ different units of energy, or of power, in the numerator and denominator of the fraction which represents the efficiency. For example, while the true efficiency of a glow lamp does not generally exceed 0·01—that is, not more than one-hundredth of the electric energy supplied to it is converted into light—the efficiency of a glow lamp is sometimes spoken of as $\frac{1}{4}$ candle per watt, meaning that an electric power of 4 watts must be supplied to the lamp to produce an illumination of 1 candle.

When any current generator developing an E.M.F. of E volts and having a resistance of b ohms is sending a current of A amperes round any circuit, the ratio

which the power in watts given to the external circuit bears to the electric power in watts developed in the generator is

$$\frac{A\,(E - A\,b)}{A\,E};$$

therefore, this fraction is the efficiency of the generator.

The name "*electrical efficiency*" is sometimes given to the preceding expression to distinguish it from the "*commercial efficiency*" of the generator, which means the ratio of the useful electric power it produces to the *total* power it consumes. Now, the total power consumed is always greater than the total electric power the generator produces. For example, if the generator be a battery some of the chemicals will often be wasted in local action, or if it be a dynamo there will be power wasted in friction at the bearings of the machine, &c. Hence the *commercial efficiency* of a generator is always less than

$$\frac{A\,(E - A\,b)}{A\,E},$$

Similarly, if E′ be the back E.M.F. of a motor in volts, *m* its resistance in ohms, and A be the current in amperes flowing, the *electrical efficiency* of the motor is

$$\frac{A\,E'}{A\,(E' + A\,m)};$$

while its *commercial efficiency*, or the ratio of the *useful* mechanical power it produces to the electric power it receives from the mains, will be less than this, since some of the mechanical power which the motor produces will be wasted in the friction at its bearings, as well as in the friction between the rotating commutator and the brushes, &c. The *commercial efficiency*, however, of very large well-made dynamos and motors is as high as 93 per cent.

Example 104.—What must be the resistance of a current generator so that 95 per cent. of the power produced

by it shall be given to the outside circuit, consisting of a simple conductor having a resistance of 35 ohms?

$$\text{We have } \frac{35}{35 + b} = \frac{95}{100},$$

if b be the resistance of the generator;

$$\therefore \quad b = 1{\cdot}842$$

Answer.—1·842 ohm.

Example 105.—$10\frac{1}{4}$ horse-power is spent in driving a dynamo which maintains a P.D. of 100 volts between its terminals when it is generating a current of 60 amperes. What is the commercial efficiency of the machine?

Answer.—$10\frac{1}{4}$ horse-power equals $10\frac{1}{4} \times 746$, or 7646·5 watts, while a current of 60 amperes at a P.D. of 100 volts equals 6,000 watts; therefore, the commercial efficiency is 78·5 per cent.

Example 106.—A motor having $1\frac{1}{2}$ ohm resistance develops a mechanical power of $\frac{1}{2}$ a horse when a P.D. of 60 volts is maintained between its terminals and a current of 9 amperes is sent through it. What are the electrical and the commercial efficiencies of the motor?

Answer.—The power wasted on account of the resistance of the motor is 121·5 watts, while the power received is 540 watts; therefore, the electrical efficiency is $\frac{540 - 121{\cdot}5}{540}$, or 77·5 per cent. The mechanical power developed is 373 watts; therefore, the commercial efficiency is $\frac{373}{540}$, or 69·1 per cent.

Example 107.—An electromotor is required to work a pump raising water through a height of 120 feet. If 15,000 gallons are to be raised per day of ten hours, what current will the motor take at 200 volts' pressure, supposing the "*combined efficiency*" of motor and pump to be 60 per cent.?

A gallon of water weighs 10 lbs.; hence the work to

be done in ten hours equals 15,000 × 10 × 120 ft. lbs., so that the power exerted in foot pounds per minute equals 30,000. But as 40 per cent. of the power given to the motor is wasted in the machinery, the motor must receive $\frac{10}{6} \times 30{,}000$, or 50,000 ft. lbs. per minute.

$$\text{Hence} \quad 50{,}000 = 44{\cdot}23 \times A \times 200,$$
$$\therefore \quad A = 5{\cdot}65.$$

Answer.—5·65 amperes.

Example 108.—If electric energy is supplied by public mains to a factory at 4½d. per Board of Trade unit, and an electromotor works with an efficiency of 80 per cent., how much does the energy used to drive the machinery in the factory cost per horse-power hour?

Answer.—One Board of Trade unit equals 1·340 horse-power hour, and of this 80 per cent. is delivered by the motor to the machinery; therefore, 1·072 horse-power hour costs 4½d., or one horse-power hour costs 4·198d.

Example 109.—If a glow lamp gives an illumination of 16 candles when a current of 0·63 ampere passes and a P.D. of 100 volts is maintained between its terminals, how many watts are required per candle?

Answer.—3·938.

128. Efficiency of Electric Transmission of Energy. —If a stream of water be used to work a turbine that drives a dynamo producing a current which flows through long leads and works an electromotor at the other end of the leads, the commercial efficiency of the whole arrangement is the ratio of the *useful* mechanical power developed by the motor at the one end of the system to the power given by the falling water to the turbine at the other. The whole power given by the falling water to the turbine would not, however, be available for driving the machinery in a factory even if the factory were built close to the falling water, for some of the power will be wasted in the turbine itself; hence the

"*commercial efficiency of transmission*" from one end of the system to the other may be taken as the ratio that the *useful* mechanical power given out by the distant electromotor bears to the mechanical power given by the turbine to the dynamo at the near end. The "*electrical efficiency of transmission*" in such a case is the ratio that the electric power which is converted into mechanical power in the motor bears to the electric power which is produced in the dynamo, or the *electrical efficiency of transmission* equals

$$\frac{\mathrm{A\,E'}}{\mathrm{A\,E}} \text{ or } \frac{\mathrm{E'}}{\mathrm{E}}.$$

If b, l, and m be the resistances in ohms of the dynamo, the leads, and the motor respectively,

$$\mathrm{A} = \frac{\mathrm{E} - \mathrm{E'}}{b + l + m} \text{ amperes;}$$

therefore, eliminating E′, the electrical efficiency of transmission equals

$$\frac{\mathrm{E} - \mathrm{A}\,(b + l + m)}{\mathrm{E}}.$$

Now, whether E, b, and m are constant and independent of the current, or whether they change their values with the current, the preceding expression varies from zero when the external circuit is such that A equals $\frac{\mathrm{E}}{b + l + m}$ (which will happen when the motor is held at rest so that it has no back E.M.F. and acts simply like a resistance) to unity when the external circuit is such that A is zero, provided, of course, that neither b nor m become extremely large when A becomes very small.

The electrical efficiency of transmission is, therefore, the greater the smaller is the current. Diminishing the current, however, diminishes the power developed by the generator unless its E.M.F. be increased. Similarly,

diminishing the current diminishes the power that can be received by the distant motor unless its back E.M.F. is increased. Hence, *to electrically transmit a large amount of mechanical power over a long distance with high efficiency we must employ a dynamo producing a large E.M.F. at the one end and a motor producing a large back E.M.F. at the other, and the current that flows must be kept very small.*

For precisely similar reasons, if we desire to employ water to transmit a large amount of power through a long pipe with high efficiency, the water must be at a high pressure and the stream must be very small. Hence the London Hydraulic Company uses water at 750 pounds per square inch pressure in their pipes, and Mr. Tweddle employs a pressure of as much as 1,400 pounds' pressure per square inch with his portable tools for riveting, &c., by hydraulic pressure.

It is interesting to consider how the E.M.F. of the generator must increase with the amount of power to be transmitted and with the resistance of the circuit, in order that the loss of power due to the resistance of the circuit may not exceed a certain percentage of the power to be transmitted.

The electric power W_1 developed in the generator $= \dfrac{E - E'}{b + l + m} \times E$ watts,

the electric power W'_1 converted into mechanical power by the distant motor $= \dfrac{E - E'}{b + l + m} \times E'$ watts,

therefore the power L lost on account of the resistance of the circuit $= \dfrac{(E - E')^2}{b + l + m}$ watts;

hence $$L = \left(\frac{W_1}{E}\right)^2 \times (b + l + m),$$

so that $\dfrac{100\,L}{W_1}$, the percentage of the power developed in

the generator which is *lost* on account of the resistance of the circuit, equals

$$100 \frac{W_1}{E^2} (b + l + m).$$

Consequently, if this percentage loss is to be a constant, E^2 must increase proportionately to the product of W_1 into $(b + l + m)$.

For example, if we desire to transmit 10,000 watts along a circuit having a resistance of 20 ohms, and to keep the loss of power down to 40 per cent.,

$$E = \sqrt{\frac{100 \times 10{,}000 \times 20}{40}}$$
$$= 709 \text{ volts},$$

or the generator must have an E.M.F. of 709 volts.

If in addition to, or instead of, the motor at the other end of the leads there be some apparatus of resistance r ohms in which we wish to develop heat or light, then this resistance r must not be included in the preceding expressions, for the heat developed in this resistance is what we desire shall be produced, and therefore must not be regarded as energy wasted in heat. For example, if the arrangement receiving energy at the other end of the leads be simply a group of glow lamps, having any resistance of r ohms, it follows, from what precedes, that the percentage of the power developed by the generator, which is *lost* on account of the resistance of the circuit, equals

$$100 \frac{W_1}{E^2} (b + l).$$

Although the transmission of signals by electricity through wires many hundreds of miles in length has been successfully carried on for nearly half a century, the history of the electric transmission of considerable amounts of power is all comprised within the past twelve years. In the following table are given the results of the most successful of the earlier attempts as well as the

result of the most striking of the later ones to accomplish this object, and it is seen how the employment of higher and higher P.Ds. has enabled larger and larger amounts of power to be transmitted over longer and longer distances with higher and higher efficiencies.

	1882. Hirschau to Munich.	1883. Vizille to Grenoble.	1886. Creil to Paris.	1891. Lauffen to Frankfort.
Distance in miles . .	3½	8¾	35	108·7
Diameter of line wire in inches . . .	0·18	0·079	0·2	Three wires of 0·158
Material of line wire .		Silicium bronze.	Copper.	Copper.
Horse-power delivered by electromotor .	5·8	7	52	Horse-power delivered to lamps 114·2
Commercial efficiency of transmission .	36 %	62 %	45 %	75·3 %
P.D. at transmitting end in volts . .	700	3,000	6,000	25,000

129. Connection between Electrical Efficiency of Transmission and Ratio of the Power Received to the Maximum Power Receivable.—When the current generator has a fixed E.M.F. and resistance like a battery, we have seen, in §§ 124 and 125, pages 385, 393, that whether we desire the whole of the external circuit, or a portion of the external circuit, to receive maximum power, or whether we desire that the transformation of electric energy into non-heat energy shall proceed most rapidly in an electromotor or electrolytic cell possessing a back E.M.F., the electric power usefully employed must be equal to the electric power wasted in heating the circuit so that the electrical efficiency of transmission must be one-half. If, however, the part of the circuit under considera-

tion be arranged so that it receives less than the maximum power receivable from the given current generator, the electrical efficiency of transmission may be greater than one-half, and the following calculation gives the connection between the electrical efficiency of transmission and the ratio of the power received to the maximum power that could be received.

If E is the fixed E.M.F. of the current generator in volts and b its resistance in ohms, x the resistance in ohms of the part of the circuit under consideration, and l the resistance in ohms of the leads connecting it with the generator, we have

$$\frac{\text{power received}}{\text{maximum power receivable}} = \frac{\left(\dfrac{E}{b+l+x}\right)^2 x}{\dfrac{E^2}{4\,(b+l)}}$$

$$= \frac{4\,(b+l)\,x}{(b+l+x)^2},$$

$$\text{also electrical efficiency of transmission} = \frac{\left(\dfrac{E}{b+l+x}\right)^2 x}{\dfrac{E}{b+l+x} \times E}$$

$$= \frac{x}{b+l+x};$$

$$\therefore \frac{\text{power received}}{\text{maximum power receivable}} = 4\,\frac{b+l}{x} \times (\text{efficiency})^2,$$

but

$$\frac{b+l}{x} \times \left(\frac{x}{b+l+x}\right)^2 = \frac{x}{b+l+x} - \left(\frac{x}{b+l+x}\right)^2,$$

$$\therefore \frac{\text{power received}}{\text{maximum power receivable}} = 4\,\{\text{efficiency} - (\text{efficiency})^2\}$$

$$= 4 \times \text{efficiency}\,(1 - \text{efficiency}).$$

Now this is a quadratic equation connecting the efficiency with the ratio of the power received to the maximum power receivable, which ratio we will call R for brevity, therefore each value of R will be given by two different values of the electrical efficiency of transmission, the sum of the two values being equal to unity; for example, whether the efficiency is $\frac{1}{5}$, or $\frac{4}{5}$, the preceding equation gives R equal to $\frac{16}{25}$. In the following table, however, which gives corresponding values of $\frac{E'}{E}$, the electrical efficiency, and of R, only the larger value of the efficiency is given corresponding with any particular value of R.

From this table we see that when x, the resistance of the part of the circuit under consideration, is increased until the electrical efficiency of transmission is 75 per cent., the power this part of the circuit receives is $\frac{3}{4}$ of the possible maximum, also that x may be increased until the electrical efficiency of transmission is over 85 per cent. without the power received being less than $\frac{1}{2}$ of the possible maximum.

$\frac{x}{b+l}$	1	2	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	5·828	6	9	50	∞
$\frac{E'}{E}$ or Electrical efficiency of transmission	$\frac{1}{2}$	0·67	$\frac{3}{4}$	0·78	$\frac{4}{5}$	0·82	$\frac{5}{6}$	0·85	0·8535	0·86	$\frac{9}{10}$	0·98	1
R or Ratio of power received to maximum power receivable.	1	0·9	$\frac{3}{4}$	0·70	0·64	0·60	$\frac{5}{9}$	0·51	$\frac{1}{2}$	0·48	0·36	0·08	0

The figures given in the first two rows of the preceding table are equally true whether the E.M.F. and resistance of the generator are constant or whether they vary with the current, but the figures in the third row for the ratio of the power received to the maximum power receivable are only true when both the E.M.F. and the resistance of the generator are constant; indeed, it may be shown that an external circuit receives maximum power from a dynamo when the external resistance is smaller than that of the dynamo, and when the electrical efficiency is therefore less than $\frac{1}{2}$.

If the external circuit, instead of being a simple resistance, contains an apparatus of resistance m ohms and back E.M.F. of E′ volts, and if it receives power through leads of fixed resistance l ohms from a generator having a fixed E.M.F. of E volts and resistance of b ohms, we know that the ratio which the rate of transformation of electric energy due to the back E.M.F. in this apparatus bears to the maximum rate of such transformation equals

$$\frac{\dfrac{E - E'}{b + l + m} \times E'}{\dfrac{E^2}{4\,(b + l + m)}} \quad \text{or} \quad 4\, \frac{E - E'}{E^2} \times E',$$

$$\text{also the electrical efficiency of transmission} = \frac{E'}{E};$$

therefore the ratio which the rate of transformation of electrical energy due to the back E.M.F. bears to the maximum rate of such transformation equals

$$4 \times \text{efficiency}\ (1 - \text{efficiency}).$$

This is exactly the same equation as was obtained in the previous case, and therefore must lead to the same numerical connection between the values of the electrical efficiency of transmission and the ratio which the rate of transformation of electrical energy due to back E.M.F.

bears to the maximum rate of such transformation, which ratio we will call R for brevity. It is not necessary, therefore, to give a second table; however, as the previous one was calculated by given regular increments to the value of $\frac{x}{b + l}$, it may be convenient to append a second table in which regular increments are given to the value of $\frac{E'}{E}$.

$\frac{E'}{E}$ or Electrial efficiency of transmission	$\frac{1}{2}$	0·6	$\frac{2}{3}$	0·7	$\frac{3}{4}$	$\frac{4}{5}$	0·8535	$\frac{9}{10}$	0·95	1
R	1	0·96	$\frac{8}{9}$	0·84	$\frac{3}{4}$	0·64	$\frac{1}{2}$	0·36	$0\cdot1\dot{9}$	0

Example 110.—A battery having an E.M.F. of 30 volts and a resistance of 4 ohms is sending a current through an outside circuit consisting of leading wires having a resistance of 1 ohm and 4 glow lamps arranged in parallel. If when a P.D. of 12 volts is maintained between the lamp terminals each lamp produces $3\frac{1}{2}$ candles, calculate the number of candles that is produced per watt, and the percentages of the power generated in the battery that are given to the lamps and wasted in the battery and leading wires.

Answer.—The current $= \frac{30 - 1}{4 + 12}$, or 3·6, amperes.

The power given to the 4 lamps

$$= 3\cdot6 \times 12, \text{ or } 43\cdot2, \text{ watts.}$$

Therefore, as the total illumination is $4 \times 3\frac{1}{2}$, or 14, candles, 0·324 candle is produced per watt.

Also the power generated by the battery

$$= 3{\cdot}6 \times 30, \text{ or } 108, \text{ watts.}$$

The power wasted in the battery

$$= (3{\cdot}6)^2 \times 4, \text{ or } 51{\cdot}84, \text{ watts.}$$

The power wasted in the leading wires

$$= (3{\cdot}6)^2 \times 1, \text{ or } 12{\cdot}96, \text{ watts.}$$

Therefore, of the 108 watts produced by the battery 43·2 watts, or 40 per cent., is given to the lamps, and 64·8 watts, or 60 per cent., is wasted in heating the battery and the leading wires.

Example 111.—A dynamo of 0·2 ohm resistance is supplying current to a group of glow lamps in parallel placed at the ends of leads having 1·8 ohm resistance. The lamps take 75 amperes, and a P.D. of 100 volts has to be maintained between their terminals. If 32 horse-power is spent in driving the dynamo, what are the electrical and commercial efficiencies of the transmission, and what are the electrical and commercial efficiencies of the dynamo alone?

Answer.—The power given to the lamps equals 7,500 watts, the power wasted in heating the dynamo and leads is $75^2 \times 2$, or 11,250, watts, therefore the dynamo must develop a power of 18,750 watts, hence the electric efficiency of transmission is 40 per cent. and the commercial efficiency of transmission 31·4 per cent. The electrical efficiency of the dynamo is 94 per cent., and its commercial efficiency 73·8 per cent.

Example 112.—A dynamo having a resistance of $2\frac{1}{2}$ ohms, and an E.M.F. of 1,000 volts, develops 40 horse-power. What may be the resistance of the leads so that 60 per cent. of the power developed by the dynamo is delivered to some apparatus at the other end of the leads? *Answer.*—10·9 ohms.

CHAPTER VI.

GALVANIC CELLS.

TYPES, CONSTRUCTION, CHEMICAL ACTION, RELATIVE ADVANTAGES, COST OF WORKING, TESTING, GROUPING.

130. Chemical Action in a Simple Voltaic Element.—When the terminals of a simple *voltaic element* are joined by a conductor a current flows, hydrogen is given

off at the copper plate, and the zinc plate is gradually turned into *zinc sulphate*. But, just as there is a doubt whether the oxygen in the sulphuric acid voltameter is formed by a primary or by a secondary action (*see* § 7, page 32), so it is not certain whether the *zinc sulphate* is formed directly by the passage of the current, or whether it is *zinc oxide* that is formed on the passage of the current and the *zinc sulphate* by a subsequent chemical action of the sulphuric acid on this zinc oxide. In the former case the sulphuric acid, H_2SO_4, would be directly split up into H_2 and SO_4, and, the zinc combining with the SO_4, would form zinc sulphate, $ZnSO_4$; while in the latter the water, H_2O, would be split up into hydrogen, H_2, and oxygen, O, by the passage of the current, the oxygen combining with the zinc to form zinc oxide, ZnO, while the zinc sulphate would be subsequently produced by the chemical action of this zinc oxide, ZnO, on the sulphuric acid, H_2SO_4, zinc sulphate, $ZnSO_4$, and water, H_2O, being produced.

The two processes may be represented respectively thus :—

First Case : Before sending a current—

$$k\,(Cu) + l\,(H_2SO_4) + m\,(H_2O) + n\,(Zn).$$

After sending a current—

$$\longleftarrow$$

$$k\,(Cu) + H_2 + (l - 1)\,(H_2SO_4) + m\,(H_2O) + (ZnSO_4) + (n - 1)\,(Zn),$$

k, l, m, and n being arbitrary quantities of copper, sulphuric acid, water, and zinc, and the arrow showing the direction of the current in the cell itself.

Second Case : Before sending a current—

$$k\,(Cu) + m\,(H_2O) + l\,(H_2SO_4) + n\,(Zn).$$

Effect produced on passage of a current—

$$\longleftarrow$$

$$k\,(Cu) + H_2 + (m - 1)\,(H_2O) + l\,(H_2SO_4) + (ZnO) + (n - 1)\,(Zn).$$

Effect produced subsequently—

$$k\,(\mathrm{Cu}) + \mathrm{H_2} + (m - 1)\,\mathrm{H_2O} + \mathrm{H_2O} + (l - 1)\,(\mathrm{H_2SO_4}) + (\mathrm{ZnSO_4}) + (n - 1)\,(\mathrm{Zn}).$$

Although the final result in the two cases is exactly the same, it might be imagined that we could ascertain which is the true action in the following way. In the first case the water apparently takes no part in the action, therefore it might seem that the action ought to proceed equally well whether or not water be present, and hence it might be thought that its being necessary, or its not being necessary, to dilute the sulphuric acid with water, in order that the cell might work satisfactorily, would constitute a test as to whether water did, or did not, enter into the reaction. Such a method of discriminating between the two actions is, however, not possible, for since zinc sulphate is but slightly soluble in strong sulphuric acid, and as a layer of zinc sulphate crystallising on the zinc plate would prevent the liquid coming into contact with the metal and would stop the action of the cell, water must be added on either hypothesis, for the purpose, at any rate, of dissolving the zinc sulphate that it formed.

The gradual replacing of the sulphuric acid in the liquid by zinc sulphate lowers the E.M.F. of the cell; but a more serious falling-off of the E.M.F. of a *simple voltaic element*, when sending a current, arises from the polarisation which is caused by some of the hydrogen gas which is liberated at the copper plate sticking to it and setting up an opposing or back E.M.F., in consequence of the tendency of the hydrogen to recombine with the SO_4, or with the oxygen from which it has been separated.

That the E.M.F. of a copper-zinc-dilute-sulphuric-acid cell falls when the cell is allowed to send a current can be tested by using a high-resistance voltmeter, V, and a suitable ammeter, A, placed in an external circuit whose resistance, x, can be varied (Fig. 183). We commence by making x infinite, so that the reading of the voltmeter

gives the E.M.F. of the cell. Next, x is made to have some convenient constant value, and the reading of the ammeter watched; gradually this will be found to fall, the fall being fairly rapid if the value of x be not large. The reading of the voltmeter also falls, and, since the value of x is constant, the ammeter and voltmeter readings fall at the same rate, each instrument telling us the same thing by the falling-off of its deflection—viz. that either the resistance of the cell has increased or its E.M.F. has diminished.

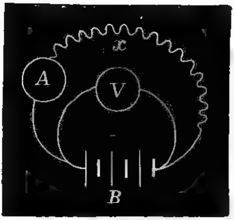


Fig. 183

If, however, we now again make x infinite, we can ascertain which of these two causes it was that made the current fall off; for, if the diminution of the current was due to an increase in the resistance of the cell *only*, then on making x infinite the reading of the high-resistance voltmeter V will be the same as it was originally; whereas, if the diminution observed in the current was wholly, or partly, caused by a falling-off in the E.M.F. of the cell and not entirely by an increase in its internal resistance, the voltmeter will read lower when x has been made infinite at the end of the experiment than it did when x was made infinite at the beginning. And this result is found to occur.

To ascertain at which of the plates of the cell the opposing E.M.F. is set up, we may use the cell seen in Fig. 184, consisting of two copper plates, C_1 and C_2, and two zinc plates, Z_1 and Z_2, dipping into dilute sulphuric acid. If the plates are all quite clean and no current has passed between any pair of them, the two copper plates will be practically the same, so that if they be joined together metallically through even a delicate galvanometer, G, no current will be observed, or if there be any current, arising from some minute difference in the two copper plates, it will be but a very slight one. And so with the two zinc

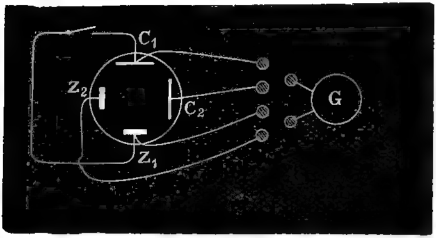


Fig. 184.—Cell arranged for Experiments on Polarisation.

plates, on joining them together through even a delicate galvanometer, no current, or at any rate but a very small current, will be observed.

If now, however, one of the copper plates, c_1, and one of the zinc plates, z_1, be used to send a current for a short time through some conductor, and then, after breaking the circuit, the two copper plates c_1 and c_2 be joined through the galvanometer, G, it will be found that a polarisation current flows for a short time from c_2 to c_1 through the external circuit, as if c_1, the copper plate

Fig. 185. Fig. 186.

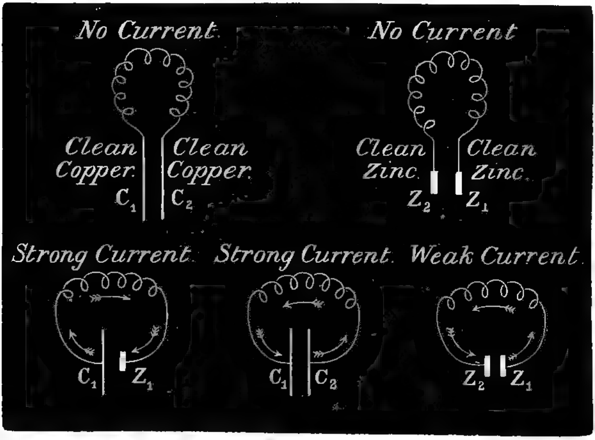


Fig. 187. Fig. 188. Fig. 189.

that has been used, were positive,* or like a zinc plate, relatively to C_2, the unused copper plate. Similarly, if the two zinc plates, instead of the two copper plates, be joined together through the galvanometer, a current will flow through the external circuit from Z_1, the zinc plate that has been used, to Z_2, the clean zinc plate ; but this polarisation current will be very small compared with the one obtained on joining the two copper plates. Indeed, it is so small that we may say without appreciable error that the diminution of the current in a single voltaic element is due entirely to polarisation at the copper plate. These tests, and the results obtained, are shown symbolically in order in Figs. 185, 186, 187, 188, and 189.

It is to be noticed that while the "*primary current*" flows from Z_1 to C_1 through the liquid, and the "*secondary current*" flows from C_1 to C_2 or from Z_2 to Z_1 through the liquid, the hydrogen gas in all three cases moves in the direction of the current, the result obtained with the sulphuric acid voltameter (*see* § 7, page 33).

131. Daniell's Use of a Depolariser: Two-Fluid Cell.—A number of devices was tried to prevent the hydrogen gas sticking to the negative plate: Smee, for example, used a roughened platinum plate instead of copper, the roughening being for the purpose of enabling the hydrogen bubbles to become detached. But no great improvement was introduced until Prof. Daniell, in 1836, hit on the idea of surrounding the negative plate† with a "*depolariser*" to prevent the hydrogen gas liberated from the dilute sulphuric acid reaching this plate. Instead of putting both the copper and the zinc plates in the dilute sulphuric acid, he surrounded the copper plate with a solution of *copper sulphate*, the two liquids being prevented from mixing together by a porous diaphragm

* For the definition of the positive plate in a cell, *see* § 122, page 376.

† For the definition of the negative plate of a cell, *see* § 122, page 376.

placed between them. As before, the dilute sulphuric acid, acting on the zinc plate, forms zinc sulphate and liberates hydrogen gas, but this hydrogen gas arriving at the *copper sulphate* solution forms sulphuric acid and deposits metallic copper on the copper plate. Omitting, for simplicity, the water used to form the solutions as well as the *water of crystallisation* of the copper and zinc sulphate crystals, this chemical action may be represented as follows :—

Before sending the current $k(\mathrm{Cu})+l(\mathrm{CuSO_4})$	Porous Partition.	$m(\mathrm{H_2SO_4})+n(\mathrm{Zn})$,
After sending the current	$\longleftarrow$	
$(k+1)(\mathrm{Cu})+(l-1)(\mathrm{CuSO_4})$		$(\mathrm{SO_4H_2})+(m-1)(\mathrm{H_2SO_4})+(\mathrm{ZnSO_4})+(n-1)\mathrm{Zn}$;

k and *n* being any arbitrary quantities of copper and zinc used to form the copper and zinc plates, *l* and *m* any arbitrary quantities of the copper sulphate and the sulphuric acid employed in the two portions of the cell, and the arrow showing the direction of the current in the cell itself. Substituting the *atomic weights* for the various substances employed, and remembering that the complete formulæ for crystals of copper and zinc sulphate are respectively $\mathrm{CuSO_4+5H_2O}$ and $\mathrm{ZnSO_4+7H_2O}$, we find that for every 26 ounces of zinc that are dissolved off the zinc plate, about 115 ounces of zinc sulphate crystals are formed, about 100 ounces of copper sulphate crystals are decomposed, and about 25 ounces of copper added to the copper plate of a Daniell's cell.

Hence, since we know that about 0·0003286 gramme of copper is deposited per second per ampere in a copper voltameter (§ 6, page 27), it follows that in each *hour* for each ampere flowing through a Daniell's cell about 0·042 ounce of copper is deposited, about 0·043 ounce of zinc

is used up, about 0·164 ounce of copper sulphate is consumed, and about 0·106 ounce of zinc sulphate is formed, which latter will become 0·189 ounce when crystallised out, since the complete formula for zinc sulphate is $ZnSO_4 + 7H_2O$.

Therefore, in twenty-four hours, for each ampere flowing through a Daniell's cell about 1 ounce of copper is deposited, about 1·03 ounce of zinc is used up, about 3·94 ounces of copper sulphate are consumed, about 2·55 ounces of zinc sulphate are formed, which become 4·54 ounces when crystallised out.

In the preceding no allowance is made for materials wasted on account of local action.

When a current is produced by a *Daniell's cell, copper is deposited on the copper plate, copper sulphaet is used up, the sulphuric acid remains unchanged in quantity, zinc sulphate is formed, and zinc is used up.* If, however, the copper sulphate solution is too weak, the water is decomposed instead of the copper sulphate, and hydrogen is deposited on the copper plate. This deposition of hydrogen lowers the E.M.F., and care should, therefore, be taken to keep up a sufficient supply of crystals of copper sulphate.

Daniell originally used a membranous tube made of ox gullet as his porous separator, but this was shortly replaced by a "*porous pot*" made of unglazed earthenware indicated by P in Fig. 190, which illustrates a common form of *Daniell's cell.* The zinc may be in the form of a rod, Z, placed in the dilute sulphuric acid which is put inside the porous pot, or in the form of a hollow cylinder surrounding the porous pot, in which case the dilute sulphuric acid is, of course, placed outside the porous pot and the solution of copper sulphate inside. The former arrangement is the more usual. Electric connection is made with the zinc by means of a copper wire, W, cast into it. The copper plate C, which is usually cut out of sheet copper, is placed in the solution of copper sulphate, and the whole is contained in a glass,

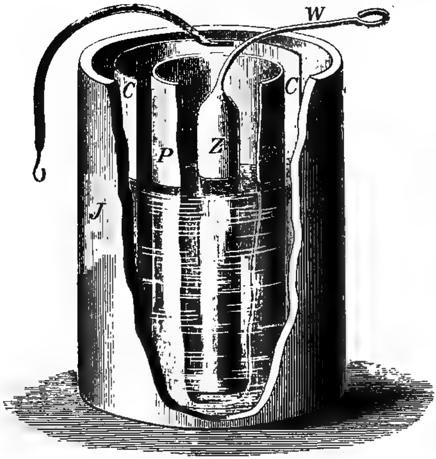


Fig. 190.—Porous Pot Daniell's Cell.

or glazed and highly vitrified stoneware jar, J. Electric connection is made with the copper plate by means of a copper wire insulated along its length with guttapercha or indiarubber, and having its end riveted, or soldered, to the top of the copper plate. If solder be used the joint should be covered over with wax, pitch, or with some adhesive matter to prevent the copper sulphate coming into contact with the joint. For if this were to happen the copper and solder being in metallic contact with one another, and also both coming into contact with the solution of copper sulphate, they would together form a little short-circuited cell, galvanic action would take place and the solder would be rapidly eaten away.

The E.M.F. of a Daniell's cell varies but little with changes of temperature, for the various contact P.Ds. which build up the total E.M.F. of the cell are oppositely affected by variations of temperature in such a way that the total change in the E.M.F. is practically zero.

The E.M.F. varies, however, from about 1·07 volt to 1·14 volt, depending on the density of the copper sulphate solution and on the amount of zinc sulphate present in the dilute sulphuric acid. As the copper sulphate is used up, and as the density of the copper sulphate solu-

tion is thereby diminished, when no steps are taken to maintain it constant, the E.M.F. of the cell falls. It also falls because the sulphate of zinc, which is formed by the eating away of the zinc rod, or plate, dissolves in the dilute sulphuric acid. The cell has, therefore, its highest E.M.F., 1·14 volt, when we start with the sulphate of copper solution saturated and no sulphate of zinc yet formed and dissolved in the dilute sulphuric acid. The falling off of the E.M.F. due to the weakening of the copper sulphate solution can be prevented by having crystals of the sulphate placed in the liquid to maintain the saturation, but we cannot so readily withdraw the sulphate of zinc from the dilute sulphuric acid. Hence, if we desire that the E.M.F. shall remain constant while the Daniell's cell is sending a current, it is better to start with *both solutions saturated.* The resistance of the cell will be higher and its E.M.F. lower than when dilute sulphuric acid is used, but this lower value of about 1·10 volt will be maintained constant while the cell is sending a current.

When a solution of zinc sulphate is used the chemical action is as follows, if, for simplicity, we omit, from the expression, the water used to form the solutions as well as the *water of crystallisation* of the two sulphates :—

Before sending a current

$k(Cu) + l(CuSO_4)$ Partition. $m(ZnSO_4) + n(Zn)$,

After sending a current

$$\longleftarrow$$

$(k+1)(Cu) + (l-1)(CuSO_4)$ Porous $(m+1)(ZnSO_4) + (n-1)(Zn)$.

When a Daniell's cell is used only to produce a P.D. and not to send a current which it is desired shall remain constant, solutions of copper sulphate and of zinc sulphate of any densities may be employed, and, since experience shows that the rise of E.M.F. produced by increasing the

density of the copper sulphate is almost exactly balanced by the fall of E.M.F. caused by an equal increase of density of the zinc sulphate solution, the E.M.F. of the cell when the solutions are *equi-dense* is practically constant, and has a value of about 1·10 volt whatever may be the actual densities.

As evaporation takes place from the zinc sulphate solution it increases in strength, and further, crystals of the sulphate are left on the sides of the jar previously wetted by the solution, the action being, of course, very marked when the solution is a nearly saturated one. The space between these crystals and the side of the jar acts as a number of capillary tubes, and draws up more liquid, which itself evaporates and deposits crystals above the former ones. So that finally the film of crystals passes over the edge of the jar and forms on the outside, thus making a kind of syphon, which draws off the liquid. This action may, to a great extent, be prevented by warming the edges of the glass, or stoneware, jars, and of the porous pots, before the cells are made up, and dipping them while warm into some paraffin wax melted in warm oil, a precaution that should always be carried out when a somewhat dense solution of zinc sulphate is employed in the cell.

132. Local or Prejudicial Action.—If a piece of *chemically pure* zinc be placed in strong, or in dilute, sulphuric acid no chemical action takes place, and no chemical action occurs if a piece of copper or carbon also be introduced into the liquid, provided that the zinc be not touched inside or outside the liquid by the other solid. If, however, the conducting solid be now touched against the zinc, either inside or outside the liquid, there is a rapid evolution of hydrogen bubbles from the solid, and the zinc is turned into zinc sulphate. We have, in fact, a short-circuited cell consisting of an oxidisable metal—zinc—in contact with a less oxidisable substance—copper or carbon—and both the oxidisable and the non-oxidisable substances in contact with the liquid.

Now, ordinary *commercial* zinc has impurities in it, such as lead, iron, and graphitic matter, so that when commercial zinc is placed in dilute acid a number of short-circuited galvanic cells is formed by the zinc, impurity, and liquid in contact, hydrogen gas is rapidly evolved, the zinc is speedily converted into zinc sulphate, and the energy that would be otherwise available for generating a useful electric current is frittered away in the heat produced by all these "*local currents.*" It is, in fact, this "*local action*" which enables the chemist to make hydrogen gas by placing scraps of commercial zinc in dilute sulphuric acid.

With a cell, on the contrary, we desire that the zinc shall only be used up when a *useful* electric current is produced—that is, a current that passes through the wire joining the zinc and copper plates *outside* the liquid. Or, in other words, we desire that no chemical action shall take place when the terminals of the cell are insulated from one another. We must either, therefore, employ chemically pure zinc, or in some way prevent *local action* taking place with commercial zinc. The price of commercial zinc is about 2d. a pound, while that of *redistilled chemically pure zinc* is from 3s. 6d. to 10s. a pound, the labour of effectively removing all the impurities from the zinc costing many times as much as the zinc itself. To employ such zinc for ordinary cells is, therefore, out of the question, and is indeed unnecessary since Sturgeon showed in 1830 that local action can be nearly as well prevented by coating the surface of the zinc with an "*amalgam*" of zinc and mercury, or "*amalgamating*" the zinc, as it is shortly called, as by employing the purest redistilled zinc.

To "*amalgamate*" a piece of zinc dip it into dilute sulphuric acid to clean its surface, then rub a little mercury over it by means of a piece of rag tied on to the end of a stick, and lastly, leave the zinc standing for a short time in a dish to catch the surplus mercury as it drains off.

The action of the amalgamated zinc is not well understood; by some it is considered that amalgamating the zinc prevents local currents by the amalgam *mechanically* covering up the impurities on the surface of the zinc and preventing their coming into contact with the liquid. By others it is thought that amalgamating the zinc protects it from local action by causing a film of hydrogen gas to adhere to it. This theory is based on the fact that while no action takes place when amalgamated zinc is placed in dilute sulphuric acid at ordinary atmospheric pressure, the creation of a vacuum above the liquid causes a rapid evolution of hydrogen, which, however, stops on the re-admission of the air.

Amalgamating zinc causes it to act as a somewhat more positive substance than zinc, therefore the E.M.F. of a cell containing amalgamated zinc is slightly higher than that of a cell constructed with unamalgamated zinc. The addition of a very small amount of zinc to mercury causes the mercury to act as if it were zinc alone, arising perhaps from the amalgam having the effect of bringing the zinc to the surface.

A second *prejudicial effect* is produced by the copper sulphate diffusing through the porous partition, coming into contact with the zinc, and being changed into zinc sulphate, the copper which is thus displaced from the sulphate being deposited on the zinc in a metallic form, or as black cupric oxide, CuO, with the evolution of hydrogen. This impairs the action of the cell, as the zinc partially coated with cupric oxide acts more like copper, and less like zinc, than if it were not so coated; the E.M.F. of the cell is, therefore, lowered. This diffusion can be retarded by constructing the porous partition so that it is only slightly porous, but this has the disadvantage of causing the cell to have a high resistance.

A formation of metallic copper is also produced in the pores of the porous partition at any spot where the zinc rod comes into contact with it, and, the copper so deposited being in metallic contact with the zinc rod,

while both are in contact with the liquid, the arrangement forms a short-circuited cell, leading to rapid waste of the battery material, growth of the metallic copper in the pores of the partition, and probable disintegration of the wall of the partition itself. To avoid this the partition must be rendered not porous, by being dipped into paraffin wax melted in warm oil, at any point where it is likely to be touched by the zinc. For example, the bottom of the porous pot P (Fig. 190, page 426), on which the zinc rod rests, should be so treated before the cell is put together.

The tendency of the copper sulphate solution to diffuse to the zinc plate, and the possibility of retarding this by diminishing the porosity of the partition at the expense of increasing the resistance of the cell, necessitates our considering, when we make a Daniell's cell, whether low resistance or constancy and portability are the conditions aimed at. And as examples of the two opposite types of Daniell's cells we may instance the "*gravity Daniell's cell*" and the "*Minotto's cell.*"

133. Gravity Daniell's Cells.—Figs. 191, 192, and 193 show three forms of Daniell's cells in which no porous partition is employed, the copper sulphate and the zinc sulphate solutions being kept separated solely by the action of gravity; and as the zinc sulphate solution is the lighter of the two, it is therefore put at the top. Figs. 191 and 192 are types of the "*Meidinger*" cell, in each of which the copper plate, C (Fig. 191), is put inside a small inner glass tumbler, *dd*, so that the particles of zinc which may become detached from the zinc plate, Z Z, shall fall clear of the copper plate and be prevented from coming into contact with it. In the type of *Meidinger* cell shown in Fig. 191, the crystals of copper sulphate are in a glass tube, *h*, with only a small hole at the bottom; while in the type illustrated in Fig. 192, the crystals are contained in an inverted flask open at the neck. In both contact is made with the copper plate by an insulated copper wire, *fg* (Fig. 191), and the zinc

plate, z z, which is in the form of a cylinder, is supported on a shoulder, *b b*, formed by a contraction at *b b* of the lower part of the outer glass vessel, A A. The "*Callaud*" cell (Fig. 193) is a simplification of the *Meidinger*, being without the reservoir for the copper sulphate crystals and the small glass tumbler to hold the copper plate.

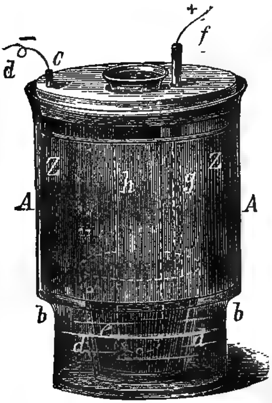


Fig. 191.—Meidinger Cell.

Fig. 192.—Meidinger Cell.

In some respects the *Callaud*, or some simple modification of it, is the most satisfactory form of *gravity Daniell's* cell, since the copper sulphate is supplied by a small crystal being dropped into the cell *daily*, and, the supply being adjusted to suit the demand, the plane of separation of the transparent zinc sulphate solution from the blue sulphate of copper solution may be kept sharp and well defined. To assist in maintaining this and so preventing the copper sulphate wandering to the zinc plate, it is well to allow the cell to send a weak current through an external circuit of considerable resistance

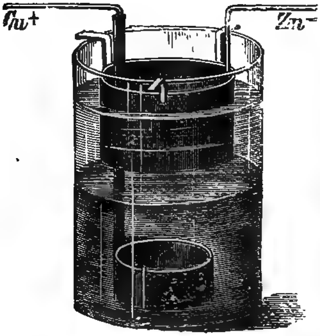


Fig. 193.—Callaud Cell.

even when the cell is not in ordinary use. A *gravity Daniell's* cell must, of course, not be moved about, or if moved great care must be taken to avoid the two liquids being mixed together.

134. Minotto's Cell.—In the "*Minotto's*" * cell the porous pot is replaced by a layer of sand or sawdust of comparatively high resistance, and it is constructed as shown in Fig. 194. At the bottom of a glass, or glazed and highly vitrified stoneware, jar, J, there is placed a disc of sheet copper, C, to which is riveted one end of an insulated copper wire, which passes up through the cell. Above this plate are placed some crystals of copper sulphate, C S, and on the top a piece of thin canvas, *c*, separating the copper sulphate from the layer of sand or sawdust *s*, and on the top of the sawdust rests the zinc plate Z, separated from the sand or sawdust by a piece of thin canvas, *c*. The cell is completed by pouring in some solution of zinc sulphate, so as to cover the zinc disc, but not so much as to reach up to the brass binding-screw B, cast into the top of a

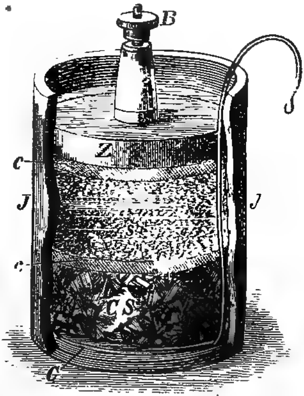


Fig. 194.—Minotto's Cell.

* Often wrongly spelt "*Menotti's.*"

little column of zinc, forming part of the zinc disc. Before putting in the sand or sawdust it should be soaked in a solution of zinc sulphate and squeezed partially dry, because if put into the cell quite dry a long time must elapse before the liquid will soak through the sand or sawdust, and until this happens the cell cannot come into action.

It is better to employ sand in stationary *Minotto's* cells, as it sinks down as the copper sulphate is consumed, but if the cells have to be moved about then it is better to use sawdust.

135. Resistance of Daniell's Cells.—The resistance of a cell varies with—

(1) The area of the plates immersed in the liquids;

(2) The distance apart of the plates;

(3) The composition of the liquids;

(4) The thickness and constitution of the walls of the porous partition.

With a porous pot Daniell's cell about 7 inches high, of the relative dimensions shown in Fig. 190, page 426, the resistance may be as low as $\frac{1}{3}$ of an ohm when the solution in which the zinc plate is immersed is dilute sulphuric acid of a specific gravity of about 1·15* at 15°C. and the porous pot has a very open grain. Such a cell must, however, be taken to pieces when not in use. If it has to be put on one side for only an hour or two, it will be sufficient to lift the porous pot with the contained zinc rod bodily out of the cell, and to place it in another empty jar, or stand it in a dish while out of use.

The porous pot Daniell's cells in the Muirhead type of battery seen in Fig. 195 may have a resistance of as much as 10 ohms apiece. Such cells are, however, frequently used in telegraph offices on account of the ease with which they can be coupled in series by means of the composite copper and zinc plates, and of the facility with which such a battery can be carried about. For, in

* For the percentage of sulphuric acid in solution corresponding with various specific gravities, *see* note, page 39.

addition to the cells being kept in place by the wooden box, the composite copper and zinc plates serve as clips to keep the porous pots in position, and so prevent them shaking about in transport.

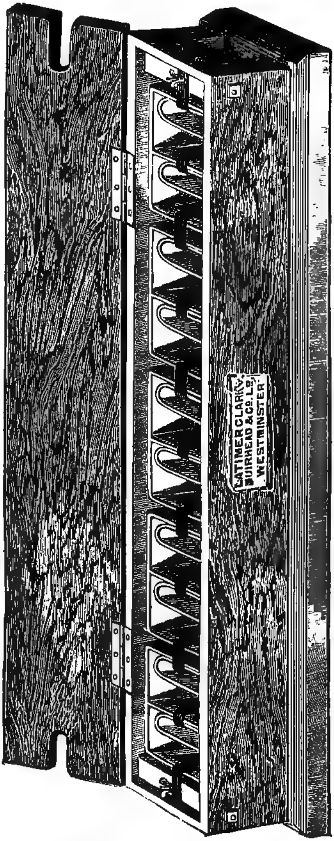


Fig. 195.—Muirhead's Telegraph Battery.

One of the plates is shown, in Fig. 196, flat as received from the manufacturer, z being the zinc plate, c the copper plate, and *c* a copper strip, one end of which is cast in the zinc plate and the other riveted to the copper plate. The dotted lines in Fig. 196 show the plates, with the strip bent, ready for insertion into the cells.

Cells of this type can be left joined up for several weeks, crystals of copper sulphate and water being added from time to time as required.

Gravity Daniell's cells have been constructed by Lord Kelvin so as to have a resistance of less than 0·1 ohm apiece. This result is attained by making the zinc and copper each in the form of a large plate, the plates being

placed horizontally one above the other at a short distance apart. On the other hand, *Minotto's* cells have frequently a resistance of 20 or 30 ohms apiece, this high resistance being of little importance, when the cells are employed to send a current through a large external resistance, compared with the constancy that is attained by employing a partition of sawdust some inches thick. Indeed, it is only necessary to pour a little water into such cells every few

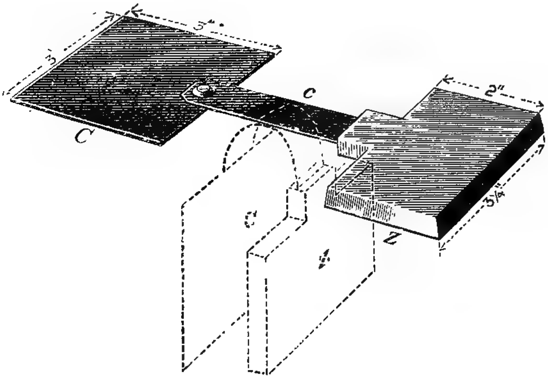


Fig. 196.—Composite Copper and Zinc Plates for Muirhead's Telegraph Battery. (Flat as received from manufacturer's, in dotted lines with connecting strip bent, for insertion in battery.)

days, to make up for that lost by evaporation, in order that they may be used for many months without any other attention being given to them.

The resistance of a Daniell's cell, like that of liquids generally, *diminishes* with *increase* of temperature; hence, as the E.M.F. is practically independent of changes of temperature, the current sent by a Daniell's cell through a constant resistance increases as the temperature rises.

Example 113.—Calculate the weight of zinc sulphate formed during $2\frac{1}{2}$ hours in a Daniell cell when a steady

current of 0·5 ampere passes through it, assuming that no zinc is consumed by local action.

Answer.—3·76 grammes.

Example 114.—In the last question it is found that 6·47 grammes of copper sulphate have been used up. Calculate how much per cent. of the copper sulphate has been wasted through local action.

Answer.—11·6 per cent.

Example 115.—How many pounds of zinc would be consumed in a battery of 26 Daniell's cells in series, sending a mean current of 0·15 ampere for 136 hours, if 10 per cent. zinc were wasted by local action?

Answer.—1·58 lb.

Example 116.—If 4 lbs. of zinc have been consumed in a week in 200 Daniell's cells in series, and the cells have been continually sending current, what has been the mean current, if no zinc has been wasted by local action?

Answer.—0·0450 ampere.

Example 117.—If copper sulphate costs 16s. 6d. per cwt. and zinc, cast in the form of battery plates and amalgamated, 3d. per lb., find the least cost of generating a Board of Trade unit in a wire of 40 ohms' resistance by means of a battery of 40 Daniell's cells in series, each cell having an E.M.F. of 1·1 volt and a resistance of 0·25 ohm.

Answer.—The current will equal $\frac{40 \times 1 \cdot 1}{40 \times 0 \cdot 25 + 40}$, or 0·88, ampere; therefore the P.D. between the battery terminals will be 44 − 0·88 × 10, or 35·2, volts, so that the power given by the battery to the wire will be 0·88 × 35·2, or 30·98, watts. Hence, to produce a Board of Trade unit in the wire the current must flow for $\frac{1000}{30 \cdot 98}$, or 32·28, hours. Consequently, we know from page 425 that the amount of copper sulphate used up is about $\frac{40 \times 0 \cdot 164 \times 0 \cdot 88 \times 32 \cdot 28}{16}$, or 11·6, lbs., costing

1s. $8\frac{1}{2}$d., and the amount of zinc changed into zinc sulphate is about $\frac{40 \times 0{\cdot}043 \times 0{\cdot}88 \times 32{\cdot}28}{16}$, or 3·06, lbs., costing about 9d. Hence, the cost of generating a Board of Trade unit in the wire is about 2s. 5d.

From this we must deduct the value of the copper deposited on the copper plates, allowing, however, for the labour of removing this copper, or of providing new copper plates for the battery. About 2·97 lbs. of copper will be deposited, and the value of this, allowing for the labour, may be 1s. So that the minimum cost of the Board of Trade unit given to the wire comes to about 1s. 5d., when no allowance is made for waste of material through local action.

136. Grove's Cell.—In the "*Grove's*" cell a zinc plate is placed in dilute sulphuric acid, as in the Daniell's, but the copper plate is replaced by one of *platinum* and the copper sulphate solution by *strong nitric acid*, HNO_3, which is generally said to act as the depolariser. The chemical action may be as follows, omitting the water used to dilute the sulphuric acid :—

	Porous Partition.	
Before sending a current $k(Pt) + l(HNO_3)$		$m(H_2SO_4) + n(Zn)$,
After sending a current $k(Pt) + (l-2)(HNO_3) + 2(NO_2) + 2(H_2O)$	$\longleftarrow$	$(m-1)(H_2SO_4) + (ZnSO_4) + (n-1)(Zn)$.

Zinc, sulphuric acid, and nitric acid are, therefore, used up; zinc sulphate, water, and *nitric peroxide*, NO_2, are formed, the latter dissolving in the liquid and colouring it red. After the cell, however, has been producing a current for some time more nitric peroxide is produced than the liquid can dissolve, and the gas comes off as dark brown fumes, which are extremely unpleasant and unhealthy if breathed for any time. A *Grove's* battery

should, therefore, be placed either in the open air, or under a chimney, when in use.

In the chemical action just described *pure* nitric acid is regarded as the depolariser, but since nitric acid, when freed from nitrous compounds, does not dissolve silver, although it will do so when nitric oxide, NO, is present, Dr. Armstrong * is of opinion that it is not the nitric acid which is the depolariser in a Grove's cell, but the nitric peroxide, a trace of which is always present in commercial nitric acid.

Under these circumstances some of the nitric peroxide will be reduced to nitric oxide on the passage of the current, and the oxygen thus liberated will combine with the hydrogen set free from the sulphuric acid and form water, in accordance with the following formula :—

	Porous Partition.	
Before sending a current $k(\mathrm{Pt})+l(\mathrm{HNO_3})+x(\mathrm{NO_2})$		$m(\mathrm{H_2SO_4})+n(\mathrm{Zn})$,
After sending a current	$\longleftarrow$	
$k(\mathrm{Pt})+l(\mathrm{HNO_3})+(x-1)\mathrm{NO_2}$ $+\mathrm{NO}+\mathrm{H_2O}$		$(m-1)(\mathrm{H_2SO_4})+(\mathrm{ZnSO_4})$ $+(n-1)(\mathrm{Zn})$.

Next, if the nitric acid be strong, the nitric oxide so produced electrolytically will be converted back into nitric peroxide, thus :—

$$2\mathrm{HNO_3} + \mathrm{NO} = 3\mathrm{NO_2} + \mathrm{H_2O},$$

so that with this way of looking at the subject the nitric acid is not used up to produce nitric peroxide electrolytically, but in converting the nitric oxide into nitric peroxide as a secondary chemical action.

The final result, however, is the same, viz. two equivalents of nitric acid are used up, and two each of nitric peroxide and of water are formed for each equivalent of sulphuric acid that is used up in forming one equivalent of zinc sulphate.

* "The Nature of Depolarisers," Proc. Chem. Soc., No. 125, 1893, p. 148, and No. 127, p. 188.

Consequently, in each hour for each ampere flowing through a Grove's cell there will be used up about 0·043 oz. of zinc, about 0·065 oz. of sulphuric acid, and about 0·083 oz. of nitric acid if of specific gravity 1·51, or 0·128. oz. if the nitric acid has a specific gravity of 1·4, *i.e.* contains about 65 per cent. of HNO_3; while about 0·106 oz. of zinc sulphate, about 0·061 oz. of nitric peroxide, and about 0·024 oz. of water are formed, no allowance being made for waste of material through local action. As with the Daniell's cell, this 0·106 oz. of zinc sulphate in solution will become 0·189 oz. when in the form of crystals.

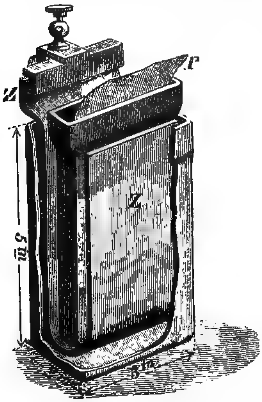


Fig. 197.—Grove's Cell.

The E.M.F. of the cell is high, being about 1·9 volt, when the nitric acid has a specific gravity of 1·41, and the dilute sulphuric acid a specific gravity of 1·14, whereas if the nitric acid be fuming, of a specific gravity 1·51, the E.M.F. rises to about 1·95 volt. The resistance also is low, that of a *Grove's* cell of the shape and dimensions indicated in Fig. 197 being about 0·15 ohm when the zinc plate z z is cast in the shape shown, so as to embrace both sides of the porous pot, when the nitric acid is strong and the dilute sulphuric acid has a density of about 1·15 without containing any zinc sulphate dissolved in it, and when, in addition, the sides of the porous pot are thin and of open grain.

The large E.M.F., combined with the small resistance, makes Grove's cells very valuable when a very strong current has to be produced for a few hours in a circuit of small resistance; hence, before the perfection of the dynamo and of secondary batteries, they were generally used when the electric light had to be produced. But the

production of the zinc sulphate, and of the water, on the passage of the current, both lowers the E.M.F. and increases the resistance of the cell, so that the current which a Grove's cell can send through a small fixed resistance falls off considerably, even after an hour or two.

When the specific gravity of the nitric acid is reduced by the production of the water to 1·33, and the dilute sulphuric acid has a specific gravity of 1·14, the E.M.F. of the cell falls to about 1·8 volt. Further, as the nitric acid becomes more and more dilute the nitric oxide is no longer converted into nitric peroxide in accordance with the last equation given above, and the reddish-brown colour of the liquid turns to a greenish-blue.

A Grove's battery must be taken to pieces at the end of each day's use, since the mixing of the liquids through the walls of the very porous pots, used to separate them, would render the battery practically useless the next day. The porous pots should be placed in water and left to soak all night, so that all the zinc sulphate solution may be dissolved out of the pores of the earthenware, for, otherwise, when the pots are dried the zinc sulphate solution will crystallise in the pores and cause the pots to tumble to pieces.

If we take the E.M.F. of a Grove's cell as 1·9 volt, and its resistance as 0·15 ohm, then a battery of Grove's cells in series will produce a current of a little over 6 amperes when the external resistance is equal to that of the battery—the condition (as was explained in § 124, page 392) that leads to the external circuit receiving the maximum power from a given battery. And in each hour in each cell with this current of 6 amperes

about 0·26 oz. of zinc,
,, 0·39 oz. of sulphuric acid,
and ,, 0·77 oz. of nitric acid (sp. gr. 1·4) will be used up;
while ,, 0·64 oz. of zinc sulphate in solution,
,, 0·37 oz. of nitric peroxide,
and ,, 0·14 oz. of water will be formed,

without any allowance being made for waste of material through local action.

Example 118.—If 4 lbs. of zinc have been consumed in a Grove's battery, how much sulphuric acid has been used up, assuming that no local action has taken place?

Answer.—6 lbs.

Example 119.—25 Grove's cells in series are sending a current of $8\frac{1}{2}$ amperes. In what time will 2 lbs. of nitric acid be consumed?

Answer.—Since about 0·128 oz. of nitric acid (sp. gr. 1·51) is consumed in a Grove's cell per hour per ampere, the time required will be about $\frac{2 \times 16}{0{\cdot}128 \times 25 \times 8{\cdot}5}$, or about 1 hour 11 minutes.

Example 120.—A battery of 12 Grove's cells in series, when sending an average current of 4 amperes for 6 hours 40 minutes, consumes 1lb. of zinc. What percentage of the zinc is wasted in local action?

Answer.—16·2 per cent.

Example 121.—A Grove's cell, having an E.M.F. of 1·9 volt and an internal resistance of 0·1 ohm, is used in series with a Daniell's cell, having an E.M.F. of 1·1 volt and an internal resistance of 3 ohms, to send a current through an external resistance of 2 ohms. Calculate the power in watts developed by each cell and the power wasted in heating each cell.

Answer.—

Power developed by the	Grove's	cell equals	1·117 watt.
,, ,, ,,	Daniell's	,, ,,	0·647 ,,
Power wasted in the	Grove's	,, ,,	0·036 ,,
,, ,, ,,	Daniell's	,, ,,	1·038 ,,

Example 122.—If zinc costs £18 a ton, sulphuric acid $1\frac{1}{2}$d. a lb., and nitric acid $3\frac{1}{2}$d. per lb., what is the least cost of supplying a Board of Trade unit of energy to a wire of 3 ohms' resistance by means of 20 Grove's cells in series, each having an E.M.F. of 1·9 volt and a resistance of 0·15 ohm?

Answer.—The current equals $\frac{20 \times 1\cdot9}{20 \times 0\cdot15 + 3}$ or 6·333, amperes, therefore the power given to the wire equals $(6\cdot333)^2 \times 3$, or 120·3, watts. Consequently, the current must flow for $\frac{1000}{120\cdot3}$, or 8·313, hours in order that the wire may receive a Board of Trade unit of energy. Hence, the zinc consumed will cost about 5½d., the sulphuric acid about 6d., and the nitric acid about 2s. 5d., or about 3s. 5d. altogether, when no allowance is made for waste of material through chemical action.

Example 123.—In the preceding question, how many Board of Trade units of energy were actually developed by the battery, and what was the cost of developing the Board of Trade unit?

Answer.—2; for, since the resistance of the battery was equal to that of the external circuit, 1 Board of Trade unit of energy was wasted in heating the battery, when 1 Board of Trade unit was given to the external circuit. Hence, the minimum price of the chemicals consumed in a Grove's battery when 1 Board of Trade unit is developed altogether is about 1s. 9d., when zinc, sulphuric acid, and nitric acid can be bought at the rates mentioned in example 122.

Example 124.—If the resistance external to a Grove's battery be very large compared with that of the battery itself, what will be the least cost of supplying a Board of Trade unit of energy to the outside circuit, with the prices of materials mentioned in example 122?

Answer.—In this case practically the whole of the energy equivalent to that of the chemicals consumed in the battery will be given to the outside circuit; hence the cost of 1 Board of Trade unit of energy given to the outside circuit will be about 1s. 9d.

137. Bunsen's Cell.—The "*Bunsen's*" cell differs from the Grove's only in the platinum plate being replaced by a cylinder, or by a block of *carbon* shown by *c* in Fig. 198, which represents a common form of *Bunsen's* cell much

used on the Continent. Its E.M.F. is practically the same as that of the Grove's cell. Carbon is a much cheaper material than platinum, but, as the nitric acid soaks into the carbon, more of it must be used to fill a cell if the negative plate be carbon than if it be composed of platinum. Hence, the first cost of a *Bunsen's* cell is less than that of a Grove's, but the cost of working it is

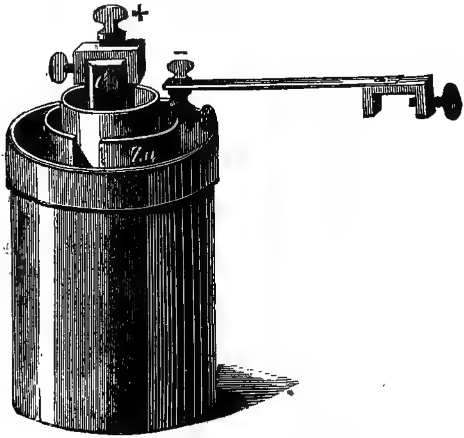


Fig. 198.—Bunsen's Cell.

greater. If, then, the cells are to be frequently employed to send a current it is more economical to purchase Grove's cells, whereas if they are to be only used occasionally it may be cheaper, on the whole, to obtain those of the *Bunsen* type.

The carbons for the Bunsen's cells are either cut out of *retort carbon*, or are made by baking in a furnace fine coke-dust and caking coal in an iron mould; then, in accordance with a process invented by Bunsen, the baked mass is soaked repeatedly in thick syrup or gas-tar, and re-baked to impart solidity and conducting power to it.

138. Potassium Bichromate Cell.—This is a form of cell devised by Prof. Poggendorff, in which the depolariser is *chromium trioxide* (CrO_3), popularly called *chromic acid*, since chromium trioxide dissolved in water has a strong acid reaction. But, as the chromium trioxide used formerly to be prepared, by the user of the cell, by acting on *potassium bichromate*, $K_2Cr_2O_7$, with sulphuric acid the cell is frequently called the "*potassium bichromate*" cell. Now, however, crystals of chromium trioxide containing 5 per cent. of water of crystallisation can be purchased ready prepared, and when these are used the cell may be shortly called a "*chromic acid*" cell.

Fig. 199.—Potassium Bichromate Cell without Porous Pot.

The cell is constructed in two forms, one without and one with a porous pot, seen in Figs. 199 and 200 respectively. The plates employed are of carbon, K, and amalgamated zinc, Z (Fig. 199), two carbon plates being generally used with the former type of cell to diminish its resistance. The zinc plate Z is supported by the rod *a*, and should be pushed into the liquid only when the cell is required to give a current, and withdrawn directly the current is interrupted, otherwise an insoluble chromium salt forms on the surface of the zinc and interferes with the action of the cell.

The chemical change which takes place when a current passes through a single fluid chromic acid cell, containing chromium trioxide dissolved in dilute sulphuric acid, is as follows :—

Before sending a current $\quad k(C)+l(CrO_3)+m(H_2SO_4)+n(Zn)$,

After sending a current

$$\longleftarrow$$

$$k(C)+(l-2)(CrO_3)+(Cr_2 3SO_4)+(m-6)(H_2SO_4)+6(H_2O)$$
$$+3(ZnSO_4)+(n-3)(Zn),$$

chromium trioxide, sulphuric acid, and zinc being used up, while *chromium sulphate*, Cr_23SO_4, water, and zinc sulphate are formed.

With the type of *potassium bichromate* cell, having a porous pot, the zinc, z (Fig. 200), is frequently cast, in the form of a block, on to a stout copper wire, carrying the binding screw, and both the block and the wire are well amalgamated, or the rod is coated with guttapercha to insulate it. In the porous pot containing the zinc, there is put a small quantity of mercury to maintain the amalgamation, and either dilute sulphuric acid, in which case the chemical action is the same as in the cell without the porous pot, or, instead, a solution of *common salt*, NaCl, when *zinc chloride*, $ZnCl_2$, is formed instead of zinc sulphate, and *sodium sulphate*, Na_2SO_4, in addition to the *chromium sulphate*. The complete chemical action is in this latter case:—

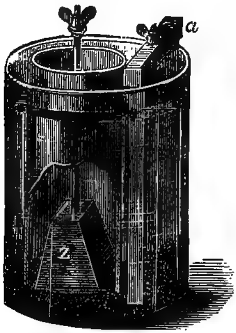


Fig. 200.—Potassium Bichromate Cell with Porous Pot.

Before sending a current

$k(C)+l(CrO_3)+m(H_2SO_4)$ Pot. $n(NaCl)+p(Zn)$,

After sending a current

$k(C)+(l-2)(CrO_3)+(Cr_23SO_4)+(m-6)(H_2SO_4)+6(H_2O)$ Porous $3(Na_2SO_4)+(n-6)(NaCl)+3ZnCl_2+(p-3)(Zn)$.

The *potassium bichromate* cell gives rise to no disagreeable fumes, has a high E.M.F. of over 2 volts when the cell is new, and a low internal resistance. The E.M.F., however, rapidly falls when the cell is employed to send a strong current continuously, but nearly recovers its original value when the cell has remained out

of action for some time. *This type of cell is, therefore, very suitable for producing a strong current for a short time.*

When the supply of chromium trioxide becomes exhausted, the orange colour of the solution turns blue, and when this change of colour is observed, more chromium trioxide, or potash bichromate, should be added. If, however, the cell begins to fail when the orange colour still remains, then more sulphuric acid is needed.

When chromium trioxide crystals are not available, so that a suitable mixture of chromic and sulphuric acids cannot be directly made by dissolving the crystals in water and adding about three and a half times their weight of sulphuric acid, it may be prepared in the following way:—Add with constant stirring 1 lb. of pulverised commercial potassium bichromate to $2\frac{1}{3}$ lbs., or about 1 pint, of sulphuric acid, having a specific gravity of about 1·84, and, when the formation of the chromium trioxide and the potassium sulphate, K_2SO_4, produced by the mixture, is completed, pour in slowly 12 lbs., or about $9\frac{1}{2}$ pints, of cold water. The liquid will become gradually warm, and the crystalline precipitate be entirely dissolved.

The chemical action produced by this mixing may be represented by—

$$K_2Cr_2O_7 + 7H_2SO_4 = 2CrO_3 + 6H_2SO_4 + K_2SO_4 + H_2O.$$

One-seventh of the oxygen contained in the potassium bichromate is, therefore, employed in forming water, and is useless to serve as a depolariser in the proper action of the cell. Further, one part of the sulphuric acid supplied is used in forming the potassium sulphate, while the equations previously given show us that three more parts of the sulphuric acid are used to produce the chromium sulphate, so that only three-sevenths of the total amount of sulphuric acid consumed are usefully employed in forming the zinc sulphate.

As the cell becomes saturated with the potassium and chromium sulphates a double salt, chrome alum, $K_2Cr_24SO_4$, crystallises out and sticks so firmly to the bottom of the cell that it is somewhat difficult to remove.

If chromic acid be not available it can be formed more cheaply and easily from *sodium bichromate* than from potassium bichromate. For, as Professor Carhart * points out, there is 11 per cent. more oxygen available as a depolariser in *sodium bichromate* than in the same weight of potassium bichromate; *sodium bichromate* is also much more soluble in water, so that a stronger solution, and containing, therefore, more oxygen, and consequently less quickly exhausted, can be used if *sodium bichromate* be employed; lastly, a double sulphate of sodium and chromium does not crystallise out as does chrome alum.

Example 125.—A single fluid potassium bichromate cell is used to produce a current of 1 ampere for 10 hours. How much sulphuric acid is consumed in the preparation of the necessary amount of chromium trioxide and in the working of the cell, and how much zinc sulphate and water are formed? Allow 33 per cent. additional for waste.

Answer.— Sulphuric acid, about 2 oz.
Zinc sulphate „ 1·4 oz.
Water „ 0·27 oz.

Example 126.—What is the mean value of the current that a chromic acid cell has been producing for 4 hours if the zinc, which originally weighed 8 oz., has been reduced to $7\frac{1}{2}$ oz.? Also, how much sulphuric acid and chromium trioxide have been used up?

Answer.—Current, about 2·9 amperes.
Sulphuric acid, about 1·27 oz.
Chromium trioxide crystals, about 0·53 oz.

Example 127.—How much zinc, sulphuric acid, and chromium trioxide would be consumed in a chromic acid cell having an E.M.F. of 1·8 volt and an internal resist-

* "Primary Batteries," Professor H. S. Carhart.

ance of 0·75 ohm, if used for 3 hours to send a current through an external resistance of $1\frac{1}{2}$ ohm?

Answer.—Zinc, about 0·103 oz.
Sulphuric acid, about 0·308 oz.
Chromium trioxide crystals, about 0·11 oz.

Example 128.—What would be the cost of the materials consumed in a two-fluid potassium bichromate battery, when supplying a Board of Trade unit to an external circuit having a resistance equal to that of the battery, if the mean E.M.F. of each cell were 1·8 volt, and waste due to local action were neglected? Take the price of zinc as 2d. per lb., of sulphuric acid 1d. per lb., of potassium bichromate 7d. per lb., and of salt $\frac{1}{2}$d. per lb.

Answer.—
Cost of zinc, about 6d.
„ sulphuric acid, about $10\frac{1}{2}$d.
„ potassium bichromate, about 2s. $7\frac{1}{2}$d.
„ salt, about $2\frac{3}{4}$d.

Hence, the total cost, if there be no waste due to local action, is about 4s. 3d.

Example 129.—What will be the reduction in cost in the last question if chromium trioxide at $7\frac{1}{2}$d. per lb. be used instead of potassium bichromate at 7d. per lb.?

Answer.—Cost of zinc, about 6d.
„ sulphuric acid, about 9d.
„ chromium trioxide, about 2s.
„ salt, about $2\frac{3}{4}$d.

Hence, the total cost, if there be no waste due to local action, will now be about 3s. 6d.

Example 130.—With the preceding prices of materials, what would be the cost of supplying a Board of Trade unit to an external circuit of very large resistance by means of a two-fluid chromic acid battery, neglecting waste due to local action.

Answer.—About 1s. 9d.

139. Leclanché Cell.—Hitherto we have been dealing with cells in which the liquid acting on the positive plate is an acid, and the depolariser is a fluid, but an

important type of cell was devised by Leclanché in 1866, in which the liquid acting on the zinc, or positive, plate was a neutral liquid, viz. a solution of *ammonium chloride*, popularly called *sal ammoniac*, NH_4Cl, and the depolariser was a solid *manganese peroxide*, MnO_2, packed with bits of *gas carbon* round the carbon or negative plate.

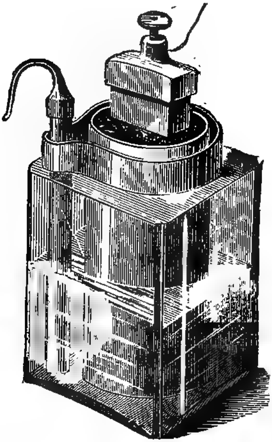


Fig. 201.—Leclanché Cell with Porous Pot.

The "*Leclanché*" cell is, therefore, a single-fluid cell, the porous pot seen in Fig. 201, which illustrates one of the earlier forms of this type of cell, being used merely for the purpose of keeping the mixture of manganese peroxide and broken gas carbon in contact with the carbon plate; and, to prevent the mixture being shaken out of the pot, it is closed at the top with pitch. A small hole is left in the pitch so that a little water or a little solution of sal ammoniac, may be poured in to start the action. The zinc is made in the form of a rod with a copper wire cast into the top of it, and the rod rests in a recess in the corner of the glass jar made to receive it.

The chemical action of the *Leclanché* cell is as follows :—

Before sending a current

$$k(C)+l(MnO_2)+m(NH_4Cl)+n(Zn),$$

After sending a current

$$k(C)+(l-2)(MnO_2)+(Mn_2O_3)+(OH_2)+2(NH_3)+(m-2)(NH_4Cl)+(ZnCl_2)+(n-1)(Zn).$$

Manganese peroxide is, therefore, reduced to *manganese sesqui-oxide*, Mn_2O_3, sal ammoniac and zinc are used up, water and *zinc chloride*, $ZnCl_2$, are formed, and *ammonia gas*, NH_3, is given off. Substituting the atomic weights, we see that for every 50 grains of zinc used up about 82 grains of sal ammoniac are consumed, and about 134 grains of the manganese peroxide are reduced to manganese sesqui-oxide. If, however, too little sal ammoniac be present, zinc oxide, or zinc oxy-chloride, is formed instead of zinc chloride, and the solution becomes milky; hence, when this happens, more sal ammoniac should be added.

Electric contact with the carbon plate is made by means of a *lead cap* cast on to it, firm connection being made between them by the lead running into two quarter-inch holes drilled sideways through the top of the rod, and thus riveting the cap on the plate. To prevent the liquid creeping up by capillary action between the top of the carbon plate and the lead cap, where it would form a salt of lead and introduce a high resistance between the two, the top of the carbon plate, after the holes have been drilled in it, is heated for one hour in paraffin wax at a temperature of 110°C., and thus rendered non-porous.

The E.M.F. of a Leclanché cell is about 1·5 volt, but in the case of the porous pot Leclanché cell (Fig. 201) the E.M.F. falls rapidly when the cell is used to send a strong current. It will, however, regain its value if the cell be left for some time unused, and it does not sensibly diminish when the cell is put on one side, even for some months. *Hence, while the Leclanché cell is much inferior to the Daniell's for the purpose of sending a steady current for an hour or two, it is much superior to the Daniell's cell for producing intermittent currents at any time during the course of a year or more—for example, such currents as are employed for the ringing of electric bells.*

The objections to this simple form of Leclanché cell,

in addition to its rapid polarisation, are—(1) the use of the porous pot, which increases the resistance of the cell; (2) the evaporation of the liquid indicated by the liquid filling only half the cell in Fig. 201; (3) the eating away of the zinc rod which occurs at the surface of the liquid, thus rendering the rod useless before the lower part is used up; and (4) the creeping of the salts, this latter defect being, however, partly counteracted by dipping the top of the porous pot and of the glass jar as well as the upper part of the carbon plate into melted ozokerite, or, best of all, into paraffin wax melted in warm oil. Various modifications of the Leclanché cell have been introduced to overcome the first two defects. M. Leclanché in 1871 dispensed with the porous pot by replacing the loose powder of manganese peroxide and gas carbon with a *solid agglomerate* composed of 40 parts of granulated manganese peroxide, 52 of granulated carbon, 5 of gum shellac, 3 of potassium sulphate, and a small quantity of sulphur. This mixture is heated to 100°C., and pressed into moulds under great pressure; the sulphur volatises and leaves the blocks in a porous condition, so that the liquid can soak into them. The negative plate is formed by binding a block of the agglomerate, *a*, on each side of the carbon plate with indiarubber bands (Fig. 202).

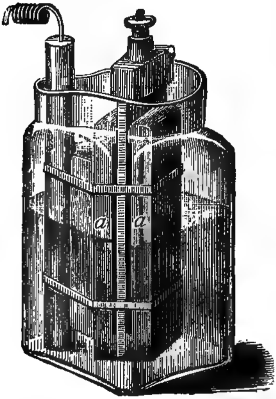
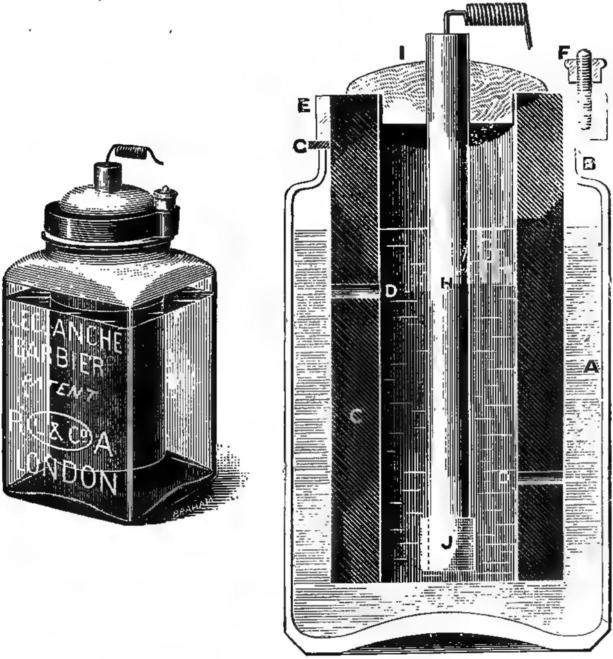


Fig. 202.—Leclanché Agglomerate Cell.

M. Barbier also dispensed with the carbon plate in the "*Leclanché-Barbier*" cell devised in 1886, which is seen in perspective and in section in Figs. 203 and 203*a*. The agglomerate block for this type of cell is in the

form of a cylinder, C (Fig. 203*a*), composed of manganese peroxide and plumbago, the plumbago being used in preference to bits of carbon in order to give the block greater conductivity. Round the top of this cylinder



Figs. 203 and 203*a*.—Leclanché-Barbier Cell.

there is cast a metal collar, E, and the binding screw F attached to this collar is the positive terminal of the cell. This metal collar also serves to support the agglomerate cylinder by resting on the edge of the glass jar, B, an india-rubber washer, G, being inserted between the collar and

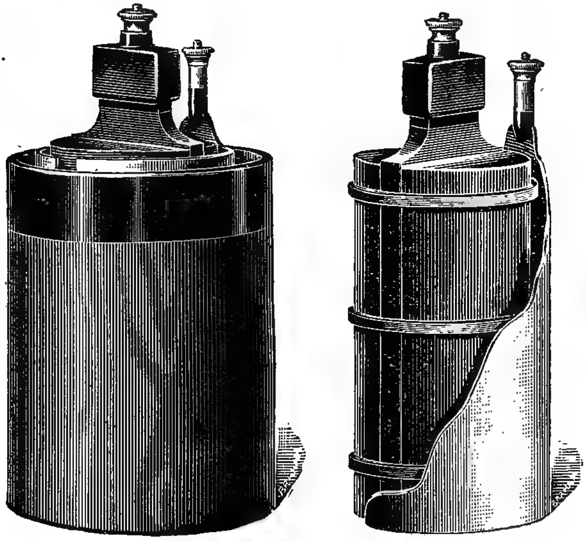
the ground edge of the jar B, to make a tight joint. The zinc rod H also does not rest on the bottom of the jar, but is supported by being tightly fitted in a disc of paraffined wood, I, which loosely closes the top of the agglomerate cylinder, and, to prevent contact between the zinc rod and the cylinder, should this wooden disc be displaced, a piece of indiarubber tube, J, is fitted on the bottom of the zinc rod. Small holes, D, D, are drilled through the carbon cylinder to facilitate the circulation of the liquid between the inside and outside of the cylinder.

While, then, the ammonia gas produced by the action of the cell can escape, there is practically no evaporation of the liquid with the *Leclanché-Barbier* cell; hence the creeping of the salt is avoided, and this type of cell can be left for long periods of time without any attention. A solution of sal ammoniac cannot, however, be used with this form of agglomerate, as it destroys the porosity, and to replace the sal ammoniac M. Leclanché has introduced a special salt, which also has the advantage of acting uniformly on the portion of the zinc rod immersed in the liquid, and not merely at the surface of the liquid. The composition of a specimen of this salt, as determined by an analysis carried out by Dr. Moody at the Central Technical College, is:—

Zinc chloride	20·00 per cent.
Ammonium chloride ...	75·81 ,,
Moisture that can be driven off at 150°C.	2·91 ,,
Sodium sulphate	0·48 ,,
Water which is not driven off at 150°C., and other impurities, including a trace of iron	0·8 ,,

Dr. Moody mentions that the solubility of this salt, compared with that of plain ammonium chloride, is as 1·36 to 1, but that it possesses greater creeping power than is shown by ammonium chloride. He adds that

the salt cannot be formed by simply mixing zinc chloride with ammonium chloride, as such a mixture would be deliquescent from the power that zinc chloride has to absorb moisture from the air, and, therefore, that it is a mixture of the double chloride of zinc and ammonium, $ZnCl_2 2NH_4Cl$, with ammonium chloride, formed by



Figs. 204 and 204*a*.—Aylmer-Leclanché Cell.

adding ammonium chloride to a solution of zinc chloride and allowing the salt to crystallise out by evaporation.

The zinc rod of the Leclanché cell certainly exposes too little surface to the liquid, and the author some years ago replaced this rod by a hollow zinc cylinder surrounding the porous pot. Still better results have been obtained by Mr. Aylmer, who has combined the agglomerate blocks without porous cell, already referred to, with the zinc

cylinder, as illustrated in Figs. 204 and 204*a*, which show respectively the complete "*Aylmer-Leclanché*" cell and a section of its interior.

The curves in Fig. 205 give the results of tests made on three types of Leclanché cell, when the outside resistance in each case was maintained constant at 10 ohms, the plan adopted by the Post Office for testing cells, and

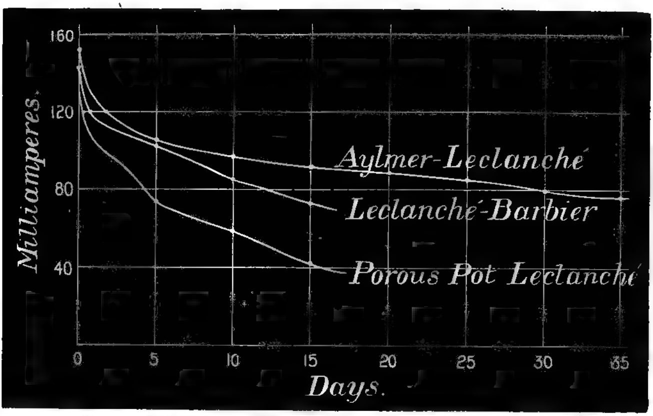


Fig. 205.—Curves showing the Time-fall of current with an external resistance of 10 ohms in each case.

we see that the current under these circumstances fell to half its value in six, seventeen, and forty-one days respectively with the ordinary porous pot *Leclanché* cell, the *Leclanché-Barbier* cell, and the *Aylmer-Leclanché* cell. It will be observed that in each case the polarisation is somewhat rapid at the beginning, and especially rapid in the case of the ordinary porous pot Leclanché cell.

Example 131.—If 2 lbs. of zinc have been consumed in a Leclanché battery, how much sal ammoniac has been utilised in the same time?

Answer.—About 3·3 lbs.

Example 132.—Compare the rates of using up manganese peroxide and sal ammoniac in a Leclanché cell.

Answer.—Approximately, as 163 to 100.

Example 133.—What is the cost of the material consumed in 6 Leclanché cells in series when developing a current of 0·1 ampere for three hours a day for 200 days, if 10 per cent. of the material used is wasted through local action? Take the price of zinc as $1\frac{3}{4}$d. per lb., of sal ammoniac 45s. per cwt., and of manganese peroxide as 14s. per cwt.

Answer.— Cost of zinc, about 2d.
" sal ammoniac, about $8\frac{1}{2}$d.
" manganese peroxide, about $4\frac{1}{2}$d.

Example 134.—With the prices of materials given in the preceding question, what would be the expense of supplying a Board of Trade unit to an external circuit by means of a battery of 6 Leclanché cells whose resistance was equal to that of the external circuit, and the mean value of whose E.M.F. during the time was 1·2 volt per cell?

Answer.—Cost of zinc, about 8d.
" sal ammoniac, about 2s. 10d.
" manganese peroxide, about 1s. $5\frac{1}{2}$d.

Hence, disregarding waste due to local action, the cost of supplying the Board of Trade unit would be about 5s.

Example 135.—If the resistance external to a Leclanché battery be very large compared with that of the battery itself, and if the average E.M.F. of each cell may be taken as 1·5 volt, what is the cost of developing a Board of Trade unit, with the prices of materials given in example 133?

Answer.—2s.

140. Dry Cells.—Many attempts have been made to construct a cell which could be turned upside down or used in any position without interfering with its action.* Volta constructed a battery of zinc and copper plates

* "Primary Batteries in Theory and Practice," W. R. Cooper, *Electrician*, vol. xxxi., 1893.

with pieces of moist cloth inserted between them. Zamboni used discs of paper covered on one side with tin and on the other with manganese peroxide; but batteries of this type, although they could produce a large E.M.F. when a sufficiently large number of elements was employed, were only able to furnish an extremely small current in consequence of their large internal resistance. Wolf, Keisen, and Schmidt tried to make a "*dry cell*" of moderate resistance by mixing sawdust with cellulose. Desruelles filled a Leclanché cell with asbestos fibre and spun glass; Pollak employed a gelatine glycerine; but the first to construct a *dry cell* which could be successfully used to produce an appreciable current was Gassner in 1888.

The "*Gassner's*" dry cell was a form of Leclanché cell, the plates being formed of carbon and zinc, the latter being made in the shape of a pot to contain the jelly which surrounded the carbon rod. This jelly was composed of sal ammoniac, zinc chloride and oxide, calcium sulphate, and water, the zinc oxide being possibly added to give porosity to the jelly. The E.M.F. was about 1·3 volt, the internal resistance of different cells of the same size was very different, and the resistance of any one cell varied in an irregular way during working. The cells polarised rapidly when used, and were also liable to short-circuit internally. Nevertheless, their compactness, portability, freedom from all creeping of the salts, and the fact that they did not dry up, led people to consider whether cells constructed somewhat on the principle of the *Gassner* dry cell might not be manufactured so as to be commercially useful.

Many experimenters attacked this problem; and of the various workers, two of the most successful were Hellesen, in Germany, and Burnley, in America.

141. Hellesen Dry Cell.—In the "*Hellesen*" dry cell, which was introduced into England by Messrs. Siemens Bros. about 1890, the carbon rod, C, is made hollow and packed with silicate cotton, S C (Fig. 206).

The rod is surrounded with a black paste, M, shown by analysis to be composed mainly of manganese peroxide, broken bits of carbon, water, the oxides of magnesium, silicon, and iron, together with a small quantity of calcium oxide. Outside this is a white paste, L, composed mainly of water, the oxides of calcium and zinc, ammonia, and a trace of magnesium oxide mixed with some gelatinous substance. Next comes the zinc, Z, in the form of a pot, the whole being contained in a millboard case, M C, packed inside with sawdust, S. The top is closed by pouring in melted pitch, P; and a water tube, W T, is inserted to carry off the gas that may be generated.

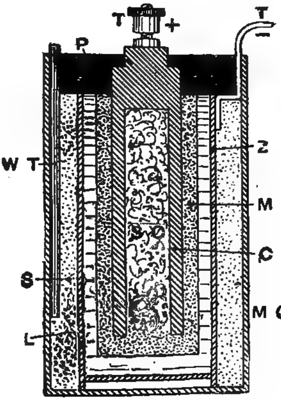


Fig. 206.—Hellesen Dry Cell.

The E.M.F. of a new Hellesen cell is about 1·45 volt, and for a cell weighing 1 lb. 7 oz. the resistance is about 1 ohm when 1 ampere is flowing. If a cell of this size be used to send a current of 0·1 ampere, the P.D. falls about 9 per cent. in the first hour, and in each subsequent hour about 3 per cent. more of its original value.

The oxide of iron is used in the Hellesen dry cell probably to assist the manganese peroxide as a depolariser, but several of the other oxides employed in this cell appear to have no use beyond giving porosity to the paste and perhaps absorbing the ammonia gas. The majority of these oxides have, therefore, been discarded in the Burnley dry cell as well as in the form of dry cell subsequently devised by Dr. Obach.

142. Burnley, or E.C.C., Dry Cell.—A simpler type of dry cell, devised by Burnley, is sold by the

General Electric Company, London, but is generally called the "*E.C.C.*" dry cell, because it was first brought out by the Electric Construction Corporation. It consists of a carbon rod, C (Fig. 207), surrounded with a black paste, M, composed of manganese peroxide, and powdered carbon, moistened with a solution of sal ammoniac and zinc chloride. This is surrounded with a white paste, L, composed of plaster-of-Paris and flour moistened with a solution of sal ammoniac and zinc chloride. The zinc case, Z, is in immediate contact with this paste, and is sealed at the top with a bituminous compound, P, proper arrangements being made for the escape of gas by the small tube, W T. The whole is then inserted in a mill-board case, I, which, in the later form of this type of cell, covers the whole of the zinc case, in order to produce good insulation, instead of merely serving as an insulating base, as in the figure.

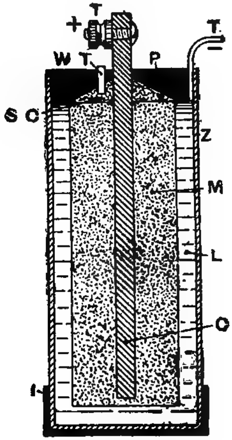


Fig. 207.—Burnley, or E.C.C., Dry Cell.

The E.M.F. of an E.C.C. cell is about 1·45 volt, and a cell weighing 2 lbs. 1 oz. will send a current of 0·1 ampere for 200 hours before the E.M.F. falls to 0·5 volt.

Tests made by Professor Jamieson* show that an *E.C.C.* cell polarises less rapidly when sending a given current than does a Hellesen cell of about the same weight, and therefore the E.C.C. cell gives out a larger amount of energy, for a given percentage fall in the terminal P.D.

143. Obach Dry Cell.—The Obach dry cell, which was introduced by Messrs. Siemens Bros. in 1894, is

* "Proceedings Phil. Soc. of Glasgow," 1892-93.

constructed as follows:—A zinc cylinder (Fig. 208) is cemented to an insulating base, B, composed of about 75 per cent. asphalt, $12\frac{1}{2}$ per cent. paper pulp, and $12\frac{1}{2}$ per cent. resin, the base being moulded when hot into the shape shown in the figure. The depolariser D is composed of a stiff paste containing about 55 per cent. manganese peroxide, 44 per cent. plumbago, and 1 per cent. gum tragacanth, which is shaped into the form of a hollow cylinder by being forced through an annular die. This cylinder is wrapped in porous paper, the carbon rod C put inside it, and the two are placed in position in the cell and centred by means of the shallow rim R of the base. In the space between this cylinder and the zinc there is poured a thin paste, E, consisting of about 85 per cent. plaster-of-Paris and 15 per cent. flour, moistened with a solution of sal ammoniac, which acts as the exciting fluid. The whole having been pressed down to bring the substances well into contact with one another, an annulus of paper, p, is placed on the top, above this a layer of rasped or ground cork, G, a second annulus of paper, p', and the whole is then sealed with a bituminous compound, K, a small tube, T, being inserted to afford an escape for the gas that is produced.

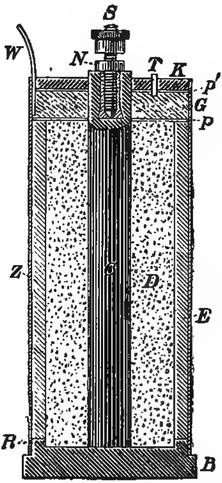


Fig. 208.—Obach Dry Cell.

A copper wire, W, soldered to the zinc cylinder, Z, forms one terminal, and the binding screw, S, forms the other. The screw-threaded pin S of this terminal is fastened to the carbon rod thus:—A round hole, larger than the pin, is bored into the top of the carbon, and by means of a V-shaped chisel a groove is cut down each side of the hole, the chisel being somewhat inclined so as to deepen

the grooves at the bottom. The pin s being held central in this hole, a fused alloy consisting of about 2 parts bismuth to 2 parts lead and 1 part tin, is poured into the hole. This alloy expands somewhat on solidifying, and so holds the screw-pin s tightly in position; but, in addition to this, a nut, N, is screwed down, the pin and the alloy being prevented from turning by the alloy having solidified in the grooves which are cut in the sides of the hole in the carbon rod. Finally the nut N, which is tinned on its lower side, is soldered in position.

The E.M.F. of the Obach dry cell, like that of other types of Leclanché cell, is about 1·5 volt when the cell is sending no current or but a very small current. With an Obach dry cell weighing 4 lb. 6 oz. the E.M.F. falls to about 1·39 volt when the current passing through the cell is $\frac{1}{5}$th of an ampere. The resistance of such a cell varies from about 0·4 ohm for a very small current to about 0·1 ohm when the current passing is $\frac{1}{5}$th of an ampere. If this current of $\frac{1}{5}$th of an ampere be sent continuously by a smaller type of Obach cell weighing 2 lb. 10 oz., the P.D. between its terminals falls from about 1·4 to about 1·27 volt in the first hour, but does not fall to lower than about 1·2 volt in the next six hours.

144. Edison-Lalande Cell.—Just as *hydrogen* deposited on the *copper* plate of a cell produces a back E.M.F. and checks the current, *oxygen* deposited on the *copper* plate sets up a forward E.M.F. and helps the action of the cell. Hence copper oxide can be used to serve both as the negative plate and as the depolariser. Cells on this principle were made by Lalande and Chaperon. The black copper oxide was contained in a cup of sheet-iron at the bottom of the cell, contact with this cup being made by an insulated copper wire riveted to it. A zinc plate was suspended from the top, and the cell was filled up with a solution of caustic potash, KHO, or caustic soda, NaHO, covered with a thin layer of heavy petroleum oil to prevent the alkali being converted into a carbonate by contact with the air. A single fluid

being used, no porous pot was necessary; therefore the resistance could be low, and hence such a copper oxide cell could furnish more energy than any other cell of the same weight.

The "*Lalande-Chaperon*" cell had, however, several defects. The outside of the iron cup was not protected by the copper oxide from being polarised by the hydrogen liberated from the solution, and the reduced copper did not make good contact with the iron cup. Hence Edison abandoned the iron altogether, and compressed finely-ground copper oxide powder into solid blocks, from which plates are cut. Such a plate is suspended from the top of an "*Edison-Lalande*" cell in a light copper framework, which also forms the positive electrode, and on each side of this plate there is fastened (Fig. 209) a plate of rolled zinc well amalgamated and shaped so as to taper down from the top, where it is thickest. This tapering is to compensate for the unequal eating away of the zinc, arising probably from local action set up by slight differences in the composition of the liquid at the different horizontal levels. The solution is formed by dissolving 1 lb. of caustic potash in 3 lbs. of water, a layer $\frac{3}{8}$ths of an inch thick of heavy paraffin oil being floated on the top.

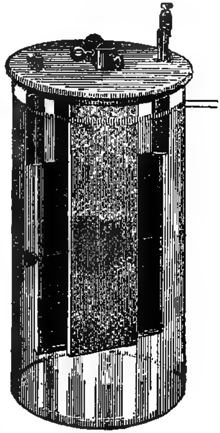


Fig. 209.—Edison-Lalande Cell.

The chemical action may be represented thus:—

Before sending a current

$$k(\mathrm{CuO})+l(\mathrm{KHO})+m(\mathrm{Zn}),$$

After sending a current

$$\longleftarrow$$

$$\mathrm{Cu}+(k-1)(\mathrm{CuO})+(\mathrm{OH_2})+(l-2)(\mathrm{KHO})+(\mathrm{K_2ZnO_2})+(m-1)\mathrm{Zn},$$

metallic copper being liberated, copper oxide and caustic potash being used up, while water and a double salt of potassium and zinc are formed.

The E.M.F. of the *Edison-Lalande* cell is low, being only about 0·75 volt; but its resistance is also low, that of a cell $11\frac{1}{4}$ inches high and $5\frac{3}{8}$ inches in diameter being about 0·025 ohm. Such a cell can send 14 amperes with not much polarisation for about twenty hours before the supply of material is exhausted, while on open circuit there is practically no local action; so that the cell can be left unused for a length of time without waste.

A test made on four such cells joined in series with an external resistance of 0·8 ohm placed in the circuit, gave the following results. The P.D. between the battery terminals commenced at about 2 volts, rose to about 2·3 volts in 44 hours, and then gradually fell to about 1·6 volt at the end of 108 hours, while the current commenced at about 2·5 amperes, rose to about 2·9 amperes in 48 hours, and then gradually fell to about 1·8 ampere at the end of the 108 hours, when the test was stopped.

The increase in the current up to the middle of the run was mainly due to the diminution in the resistance of the battery, for this resistance commenced at about 0·54 ohm, fell quickly for 20 hours, then more slowly until it finally reached a value of about 0·17 ohm in 44 hours. The battery resistance then rose very slowly for 84 hours, after which it increased rapidly to about 0·55 ohm at the end of the test at 108 hours. If, however, the only cause for the change in the current had been the variation in the internal resistance of the battery, the current and the P.D. would have reached their maxima simultaneously; hence the current reaching its maximum somewhat later than the P.D. showed that a small diminution in the external resistance also occurred as the current rose, possibly because part of the external resistance was made of carbon, and this diminished in resistance as it was more heated by the larger current.

The total quantity of zinc actually consumed was 1,450 grammes, which is almost exactly the amount that ought to have been consumed, for the mean value of the current was 2·76 amperes, the duration of the test 108 hours, and we know that about 0·000338 gramme of zinc is deposited per second per ampere in a single electrolytic cell. Hence the amount of zinc that ought to have been consumed was

$$2{\cdot}76 \times 108 \times 3{,}600 \times 0{\cdot}000338 \times 4, \text{ or } 1{,}450 \text{ grammes.}$$

Consequently, although the exact agreement between the number obtained by experiment and calculation must be regarded as accidental, this agreement shows that practically no zinc was wasted by local action.

From the equation given above, which represents the chemical action which takes place in an Edison-Lalande cell, it follows that a consumption of 1,450 grammes, or 3·196 lbs., of zinc corresponds with a consumption of about 3·88 lbs. of copper oxide and of about 5·48 lbs. of caustic potash. And since the total energy given by the battery to the external circuit was about 658 watt hours, it follows that when an Edison-Lalande battery gives a Board of Trade unit to an external resistance having about five times the resistance of the battery, there must be consumed about 4·86 lbs. of zinc, about 5·90 lbs. of copper oxide, and about 8·33 lbs. of caustic potash. Hence, taking zinc at only £16 per ton, copper oxide at only 6d. per lb., and caustic potash at only 30s. per cwt., the materials used up in generating a Board of Trade unit with an Edison-Lalande battery would cost about 8½d. for zinc, about 2s. 11½d. for copper oxide, and about 2s. 3d. for caustic potash, or about 6s. altogether if we neglect the value of the copper produced and of the solution of caustic potash with the double salt of potassium and zinc dissolved in it. (*See* also pp. 485, 487.)

It, therefore, costs more to develop a Board of Trade unit with an Edison-Lalande cell than with any of the other cells we have been previously considering. For further information on this subject, *see* § 148, page 482.

145. Clark's Cell.—This cell, devised in 1872 by Mr. Latimer Clark,* is not intended for producing currents, but for the purpose of furnishing a very accurate standard of E.M.F. The type of "*Clark's*" cell which was described in the earlier editions of this book is that known as Lord Rayleigh's H form, shown in Fig. 210. According to Lord Rayleigh,† it is constructed as follows:—One of the legs is partially filled with an amalgam of zinc, A, formed by putting some pure zinc into pure mercury which has been previously distilled in a *vacuum;* the other with pure mercury, M, which has been similarly distilled, covered with a layer of "*mercurous sulphate,*" M S. The whole is then filled up above the level of the cross tube with pure *saturated* zinc sulphate, Z, and a few crystals of zinc sulphate are added. Evaporation is prevented by the insertion of paraffined corks, C, and electrical contact is made with the amalgam and with the pure mercury by platinum wires, W, W, sealed into the glass. Marine glue may be employed instead of paraffin wax to make the corks C air-tight, or, best of all, the upper ends of the tubes may be hermetically sealed (*see* note, page 42).

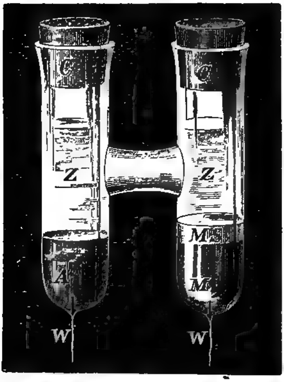


Fig. 210.—Clark's Cell, Lord Rayleigh's H form. About one-half of full size.

During the last few years, however, a large amount of experimental work has been carried out on *Clark* cells by Professor Carhart and Mr. Weston in America, by Dr. Kahle in Germany, and by Mr. Glazebrook, Dr.

* "Proc. Roy. Soc.," vol. xx., page 444.

† "Phil. Trans., Roy. Soc.," vol. xvii., p. 411. Part II., 1884.

Muirhead, and Mr. Skinner in England, to find out which was the best way to construct a Clark's cell, and to ascertain the exact causes why Clark cells made by different people in apparently the same way had occasionally somewhat different E.M.Fs. The Committee which was appointed in 1890 to advise the Board of Trade on Electrical Standards has aimed at drawing up a specification for the construction of a Clark's cell which should be sufficiently explicit and complete that any person, with a fair amount of chemical knowledge, constructing cells in exact accordance with this specification might feel sure that the E.M.F. of no one of them at 15°C. differed from 1·434 volt by more than 0·001 volt—that is, by more than about $\frac{1}{10}$th per cent. After much deliberation they drew up their first specification in 1891, and various people were asked to try to construct *Clark* cells in accordance with this specification, and to forward them to the Board of Trade Electrical Standardising Laboratory in Westminster to be tested. The result was not very satisfactory, so a second specification was issued, such modifications being introduced into it as were suggested by the faults which existed in the various batches of cells constructed in accordance with the first specification. From 1891 to 1894 five such specifications were circulated, and the last one, which forms part of the Order in Council signed by the Queen on August 23rd, 1894, is as follows :—

"SPECIFICATION B.

"ON THE PREPARATION OF THE CLARK CELL.

"*Definition of the Cell.*

"The cell consists of zinc, or an amalgam of zinc with mercury, and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess.

"*Preparation of the Materials.*

"1. *The Mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled *in vacuo.*

"2. *The Zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass-paper or a steel burnisher, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

"3. *The Mercurous Sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water, and repeat the process at least twice. After the last washing, drain off as much of the water as possible.

"4. *The Zinc Sulphate Solution.*—Prepare a *neutral* saturated solution of pure ('pure re-crystallised') zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent. by weight of the zinc sulphate crystals to neutralise any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30°C. Mercurous sulphate treated as described in 3 should be added in the proportion of about 12 per cent. by weight of the zinc sulphate crystals to neutralise any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

"5. *The Mercurous Sulphate and Zinc Sulphate Paste.*—Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the stock bottle to ensure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30°C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass; if this is not the case add more crystals from the stock bottle, and repeat the whole process.

"This method ensures the formation of a saturated solution of zinc and mercurous sulphates in water.

"*To set up the Cell.*

"The cell may conveniently be set up in a small test-tube of about 2 centimetres' diameter, and 4 or 5 centimetres deep. Place the mercury in the bottom of this tube, filling it to a depth of, say, ·5 centimetre. Cut a cork about ·5 centimetre thick to fit the tube; at one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork

cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water for some hours before use. Pass the zinc rod about 1 centimetre through the cork.

"Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the tube; one end forms the terminal, the other end and a portion of the glass tube dip into the mercury.

"Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red hot, and insert it in the mercury in the test-tube, taking care that the whole of the exposed platinum is covered.

"Shake up the paste and introduce it without contact with the upper part of the walls of the test-tube, filling the tube above the mercury to a depth of rather more than 1 centimetre.

"Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least twenty-four hours before sealing, which should be done as follows:—

"Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test-tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

"The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

"The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

"In using the cell sudden variations of temperature should as far as possible be avoided.

"The form of the vessel containing the cell may be varied. In the H form, the zinc is replaced by an amalgam of 10 parts by weight of zinc to 90 of mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one leg of the cell, and with the mercury in the other, by means of platinum wires sealed through the glass."

A Clark's cell constructed in accordance with the preceding specification is shown full size in Fig. 211, the

various materials employed and the space occupied by each of them being clearly indicated.

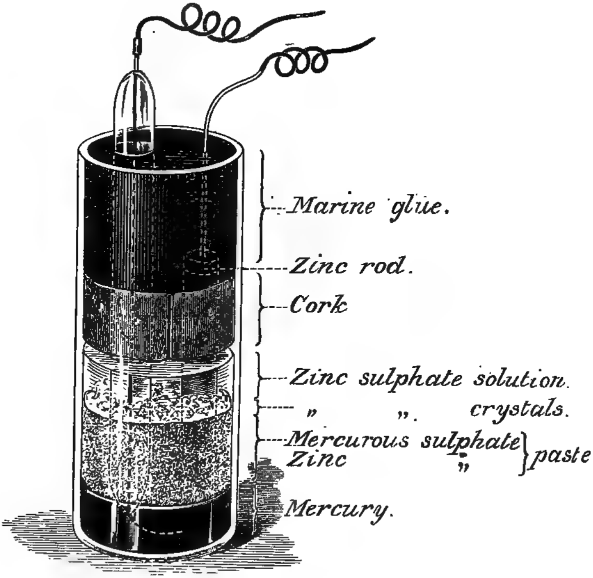


Fig. 211.—Clark's Cell, Board of Trade form (full size).

The following notes have been prepared by Mr Glazebrook to explain the object of each of the various steps which it is stated in the preceding "Specification B," should be followed in the construction of a Clark's cell, in order that it may have an E.M.F. of 1·434 volt at 15°C.

"Notes to the Specification on the Preparation of the Clark Cell.

"*The Mercurous Sulphate.*—The treatment of the mercurous sulphate has for its object the removal of any mercuric sulphate which is often present as an impurity.

"Mercuric sulphate decomposes in the presence of water into an acid and a basic sulphate. The latter is a yellow substance—turpeth mineral—practically insoluble in water; its presence, at any rate, in moderate quantities has no effect on the cell. If, however, it is formed, the acid sulphate is formed also. This is soluble in water, and the acid produced affects the electromotive force. The object of the washings is to dissolve and remove this acid sulphate, and for this purpose the three washings described in the Specification will in nearly all cases suffice. If, however, a great deal of the turpeth mineral is formed, it shows that there is a great deal of the acid sulphate present, and it will then be wiser to obtain a fresh sample of mercurous sulphate rather than to try by repeated washings to get rid of all the acid.

"The free mercury helps in the process of removing the acid, for the acid mercuric sulphate attacks it, forming mercurous sulphate and acid which is washed away.

"Pure mercurous sulphate, when quite free from acid, shows, on repeated washing, a faint primrose tinge which is due to the formation of a basic mercurous salt, and is distinct from the turpeth mineral or basic mercuric sulphate. The appearance of this primrose tint may be taken as an indication of the fact that all the acid has been removed, and the washing may, with advantage, be continued until this primrose tint appears. Should large quantities of this basic mercurous salt be formed, the sulphate should be treated as described in the instructions for setting up Clark's cells issued from the Physical Technical Institute of Berlin, *Zeitschrift für Instrumentenkunde*, 1893, Heft 5.

"*The Zinc Sulphate Solution.*—The object to be attained is the preparation of a neutral solution of pure zinc sulphate saturated with $ZnSO_4.7H_2O$.

"At temperatures above 30°C. the zinc sulphate may crystallise out in another form; to avoid this, 30°C. should be the upper limit of temperature. At this temperature water will dissolve about 1·9 times its weight of the crystals. If any of the crystals put in remain undissolved they will be removed by the filtration.

"The zinc sulphate should be free from iron and should be tested before use with sulphocyanide of potassium to ascertain that this condition is satisfied. If an appreciable amount of iron is present, it should be removed by the method given in the directions already quoted, *Zeitschrift für Instrumentenkunde*, 1893, Heft 5.

"The amount of zinc oxide required depends on the acidity of the solution, but 2 per cent. will, in all cases which will arise in practice with reasonably good zinc sulphate, be ample. Another rule would be to add the zinc oxide gradually until the solution become slightly milky. The solution, when put into the cell, should not contain any free zinc oxide; if it does, then, when mixed with the mercurous sulphate, zinc sulphate and mercurous

oxide are formed; the latter may be deposited on the zinc and affect the electromotive force of the cell. The difficulty is avoided by adding as described about 12 per cent. of mercurous sulphate before filtration; this is more than sufficient to combine with the whole of the zinc oxide originally put in, if it all remains free; the mercurous oxide formed, together with any undissolved mercurous sulphate, is removed by the filtration.

"*The Mercurous Sulphate and Zinc Sulphate Paste.*—Although, after the last washing of the mercurous sulphate, as much water as possible may have been drained off, sufficient water generally remains to necessitate the addition of a *very considerable* quantity of crystals of zinc sulphate from the stock bottle, in order to ensure saturation when the washed mercurous sulphate is added to the zinc sulphate solution as described in No. 4 of Specification B appended to the Order in Council.

"If the sides of the test-tube above the cork be soiled by the introduction of the paste, the marine glue does not adhere to the glass; the liquid in the cell rises by capillary action between the glue and the glass, and may damage the cell.

"The form of the vessel containing the cell may be varied. In the H form devised by Lord Rayleigh and modified by Dr. Kahle the zinc is replaced by an amalgam of zinc and mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one leg of the cell, and with the mercury in the other, by means of platinum wires sealed through the glass.

"The amalgam consists of about 90 parts of pure mercury mixed with 10 parts of pure redistilled zinc. These are heated in a porcelain crucible to about 100°C., and gently stirred until the zinc is completely dissolved in the mercury. The amalgam is liquid while warm, and must be poured into the cell before it becomes solid on cooling.

"The vessel containing the element consists of two vertica tubes. These, as shown in the figure [212], are closed below, and open above into a common neck, which can be closed by a ground stopper of glass. The two tubes should be 2 cm. in diameter and 3 cm. in length. The neck should be at least 1·5 cm. in diameter and 2 cm. long. A short length of platinum wire is sealed through the bottom of each tube.

"The end of the wire in one tube is covered by a small quantity of pure mercury; that in the other tube, by the zinc-mercury amalgam.

"Above the mercury a layer about 1 cm. thick of the mercurous sulphate paste is placed; above this, and also above the amalgam, a layer, also about 1 cm. in thickness, of zinc sulphate crystals, and the vessel is filled up with the saturated zinc sulphate solution.

"The zinc sulphate crystals are obtained by evaporating at a

temperature of less than 30°C. some of the zinc sulphate solution, prepared as in 4 of the Specification.

"The stopper is then inserted, leaving a small air-bubble above

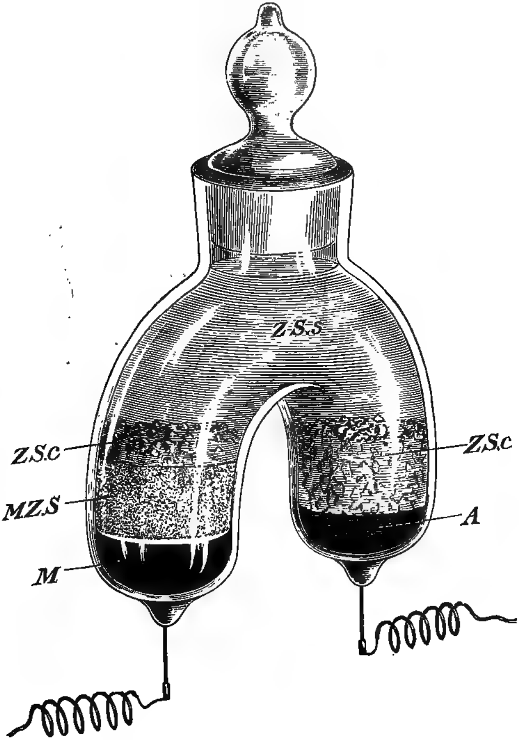


Fig. 212.—Kahle's Modification of the Rayleigh H form of Clark Cell (full size). *ZS.s*, zinc sulphate solution; *ZS.c*, zinc sulphate crystals; *MZ.S*, mercurous sulphate and zinc sulphate paste: *M*, mercury; *A*, amalgam of zinc and mercury.

the liquid, and sealed on the outside with shellac dissolved in alcohol.

"The ends of the platinum wires outside the cell form the two poles, and should be connected to suitable terminals."

The Americans and the Germans prefer the type of Clark cell shown in Fig. 212, partly because this form avoids a possible slight variation in the E.M.F., arising from the zinc rod in the sample-tube form (Fig. 211) being in contact with liquid at different depths, and,

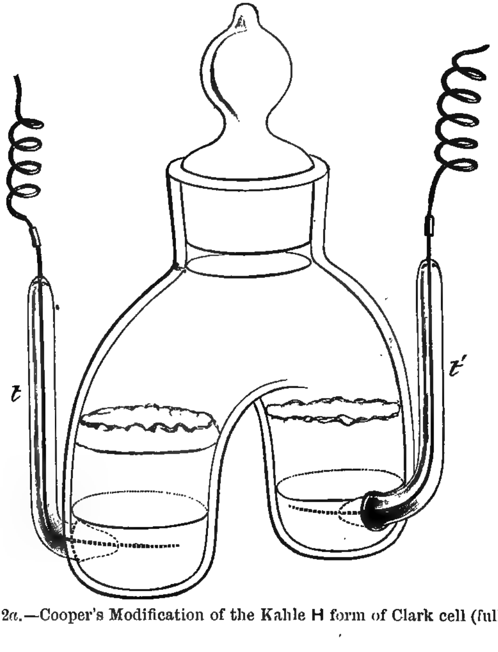


Fig. 212a.—Cooper's Modification of the Kahle H form of Clark cell (full size).

therefore, of different densities. Whereas, on the other hand, the greater ease with which the latter type can be immersed in a water-bath for ascertaining its temperature led the Board of Trade to give greater prominence in the Order in Council to this sample-tube form illustrated in Fig. 211.

By the addition, however, of two small glass tubes,

t, t' (Fig. 212a), sealed on to the main tubes to contain and insulate the platinum leading-in-wires, in accordance with the plan proposed by Mr. W. R. Cooper, the **H** type form of Clark's cell can be adapted for insertion in a water-bath. Considerable care, however, must be taken in annealing the joints between the small tubes and the main ones, otherwise a crack is very likely to occur at these places.

146. Temperature Variation of the E.M.F. of the Clark Cell.—Although the E.M.F. of a Clark cell made in accordance with the Board of Trade Specification will not, as a rule, differ by more than 0·001 volt from 1·434 volt if the cell be maintained *for some time* at a constant temperature of 15°C., this value of the E.M.F. will be considerably changed by raising or lowering the temperature of the cell through several degrees. By observing cells at different temperatures, which were maintained constant for two or three hours at each temperature before the observations were taken, Lord Rayleigh found that the E.M.F. was *diminished* by about 0·077 per cent. per 1°C. *rise* of temperature, and this result has been generally confirmed by other observers. In reality, the relation is not a simple linear one, as it involves a term depending on the square of the temperature. But, for all practical purposes, we may accept the above percentage variation of the E.M.F., which, however, is more conveniently expressed by saying that the E.M.F. of a Clark cell *falls* by 0·0011 volt for each degree *rise* of temperature above 15°C.

There are, however, one or two modifying causes that must be taken into account. For example, after a cell has been set up the mercurous sulphate settles down, leaving a layer of clear solution above it; now, if the zinc rod dips into this solution only, and not into the paste, it is found that the temperature coefficient is only about one-half the value given above. Again, if the zinc sulphate is not saturated at the temperature at which the cell is used, the temperature coefficient is also

diminished to about half the normal value, while the E.M.F. is always higher than that of a Clark cell containing a saturated solution of zinc sulphate at the same temperature.

As mentioned in the specification of the Board of Trade, rapid variations of temperature of a Clark cell should always be avoided. For it takes some time for the new temperature to be reached at all parts of the cell. Further, even if the cell has acquired the new temperature throughout, the variation in the amount of salt dissolved, which accompanies change of temperature, is effected slowly. Hence, while the E.M.F. of a Clark cell follows fairly quickly any variation that is made in the temperature, there is a certain amount of lag, and small changes in the E.M.F. may continue for a long time after the temperature has become steady. A large number of experiments* on the lag of the variation in the E.M.F. of a Clark cell behind the change in temperature have been carried out at the Central Technical College, under the superintendence of Mr. W. R. Cooper, and he finds that if a Clark cell of the dimensions illustrated in Fig. 211 be stood in water of constant temperature for half an hour before use, its E.M.F. will not differ by as much as 0·1 per cent. from its normal value for the particular temperature of the bath, even if the temperature of the cell differed previously from that of the water by 3° or 4°C. Changes in the E.M.F. will, no doubt, go on for a much longer time than half an hour, but they will be less than 0·1 per cent. of the E.M.F.

It sometimes happens that variations of temperature amounting to 1°C. or more take place during experimental work; in such a case Mr. Cooper's results show that the fall in the E.M.F. per 1°C. rise of temperature is not as great as 0·0011 volt, but instead, in consequence

* "Variations in the Electromotive Force of Clark's Cell with Temperature," W. E. Ayrton and W. R. Cooper. "Proc. Roy. Soc.," vol. 59, page 368, 1896.

of the variation of the E.M.F. lagging behind the change in temperature, the fall in the E.M.F. per 1°C. is more nearly represented by 0·001 volt.

If the terminals of a Clark's cell be joined by too small a resistance—that is, if the cell is allowed to send too large a current—a diminution in the E.M.F. is produced by polarisation, which lasts for some time after the current has been stopped. Mr. Cooper, however, has found that if the Board of Trade form of Clark's cell (Fig. 211) be used to send a current through a resistance of 2,000 ohms for 5 minutes, the E.M.F. regains its normal value to within 0·0002 volt one hour after the circuit has been broken.

Numerous other types of primary cells might be described, such as the Marié-Davy mercuric sulphate cell, the De la Rue silver chloride cell, &c., but such cells, after having been tried for several years, have been gradually abandoned as unsuitable for practical use.

147. Calculation of the E.M.F. of a Cell from the Energy Liberated by the Chemical Action.—In every cell through which a current is passing certain chemical changes are taking place; some of these changes, if they occurred outside the cell, would produce heat, while the others would produce cold. If, then, during the flow of the current the cell is simply warmed because it is a circuit possessing resistance, and is neither warmed nor cooled on account of the chemical changes that are taking place, the energy developed per second by the sum of these chemical changes must be equal to the rate at which the cell develops electric energy, and this, we have seen in Chapter V. (page 361), is equal in watts to the product of E, the electromotive force of the cell in volts, into A, the current passing in amperes.

Now, we know what are the chemical changes that occur in the ordinary galvanic cells; we also know what approximately are the amounts of the changes that are produced in an electrolytic cell per second per ampere

(*see* § 6, page 27). Further, the work of Thomsen and Berthelot enables us to form an idea of the heats of formation of the chemical products in question; therefore, we can calculate the power in watts which any particular cell must develop when a current of 1 ampere is flowing through it, and what must be its E.M.F. in volts.

For example, the chemical action in a Daniell's cell is simply a conversion of copper sulphate into zinc sulphate, the former being used up and the latter formed; and in § 6, page 27, we saw that when a current of 1 ampere passed through a solution of copper sulphate about 0·0003286 gramme of copper was deposited per second, also that when it passed through a solution of zinc sulphate about 0·0003386 gramme of zinc was deposited. If, therefore, a Daniell's cell be neither heated nor cooled by the chemical actions taking place, apart from the heating caused by the flow of the current, the electric energy developed by the cell per second per ampere must be the equivalent of the difference between the heat produced when 0·0003386 gramme of zinc is combined with sulphur and oxygen to form zinc sulphate and the heat absorbed when 0·0003286 gramme of copper is liberated by the separation of copper, sulphur, and oxygen composing copper sulphate.

Now, the combination of 65·38 grammes of zinc with sulphur and oxygen to form zinc sulphate produces 158,990 calories, according to Thomsen, and 242,000, according to Berthelot—let us say 200,000 as a rough mean; and the liberation of 63·44 grammes of copper by the separation of copper from sulphur and oxygen absorbs 111,490 calories, according to Thomsen, and 191,400 calories, according to Berthelot—let us take 150,000 as an approximate mean. Therefore, on the hypothesis stated above, the chemical energy that is converted per second per ampere into electric energy in a Daniell's cell equals roughly

$$\frac{0{\cdot}0003386}{65{\cdot}38} \times 200{,}000 - \frac{0{\cdot}0003286}{63{\cdot}44} \times 150{,}000 \text{ calories.}$$

The expressions $\frac{0{\cdot}0003386}{65{\cdot}38}$ and $\frac{0{\cdot}0003286}{63{\cdot}44}$ are each equal to *half* the weight of hydrogen gas liberated per second per ampere (*see* § 6, page 27): that is, to $\frac{0{\cdot}00001043}{2}$ (*see* § 6, page 33). We may, therefore, conclude that the chemical energy converted per second per ampere into electric energy in a Daniell's cell equals roughly

$$\frac{0{\cdot}00001043}{2} \times 50{,}000, \text{ or } 0{\cdot}2607 \text{ calorie.}$$

Next, it follows, from the equations in § 107, page 318, that one calorie is generated by a power of $\frac{1}{0{\cdot}2388}$ watt exerted for one second. Hence, when 1 ampere is flowing through a Daniell's cell, the power developed in the cell equals roughly

$$\frac{0{\cdot}2607}{0{\cdot}2388}, \text{ or } 1{\cdot}09 \text{ watt,}$$

so that the E.M.F. of this cell must be 1·09 volt.

The preceding method of calculating the E.M.F. of a cell from the thermal value of the chemical changes taking place in it was first used in 1851 by Lord Kelvin to ascertain what was the E.M.F. of a Daniell's cell in terms of the "*absolute electromagnetic unit*" of P.D., 10^8 of which were, several years afterwards—viz. in 1862—called 1 volt.* At the present time, the E.M.F. of any cell could be at once determined by applying a high-resistance voltmeter between its terminals; but in 1851 there were no voltmeters, no practical standard of E.M.F. like that of a Clark's cell, no ammeters, no resistance coils graduated in ohms—in fact, there were no practical standards either of current or of resistance.

* *See* Appendix III., page 575.

Hence, the result which Lord Kelvin arrived at for the E.M.F. of a Daniell's cell was not only of scientific but also of practical importance.

The number 50,000 in the preceding expression represents the chemical energy in calories that is converted into electric energy in a Daniell's cell when a current passes for a sufficient length of time to deposit 63·44 grammes of copper and use up 65·38 grammes of zinc. The same current, passed for the same time through a sulphuric acid voltameter, would liberate 2·0164 grammes of hydrogen and 16 grammes of oxygen. We may, therefore, say that the E.M.F. of any cell is approximately given in volts by multiplying $\frac{0 \cdot 00001043}{2 \times 0 \cdot 2388}$, that is, $\frac{1}{46,000}$ or 0·000022, by the energy, in calories, that is set free in the cell when a current of such a strength passes through it for such a time that if passed through a sulphuric acid voltameter it would liberate 2·0164 grammes of hydrogen.

If an *electropositive ion* in the cell is *bivalent*, like copper, this evolution of 2·0164 grammes of hydrogen corresponds with the liberation of the atomic weight of this ion in grammes; whereas if it be *univalent*, like sodium, it corresponds with the liberation of *twice* the atomic weight in grammes.

We may now apply this rule for calculating approximately what must be the E.M.F. of a Grove's cell. Referring to § 136, page 438, we see that when one equivalent of zinc sulphate, $ZnSO_4$, is formed in the Grove's cell two equivalents of water, $2(H_2O)$, and two of nitric peroxide, $2(NO_2)$, are also formed, while one equivalent of sulphuric acid, H_2SO_4, and two of nitric acid, $2(HNO_3)$, are used up. Zinc being bivalent, we have simply to substitute the atomic weights in grammes throughout, calculate the weights of the substances thus dealt with, and ascertain the heats of formation for these particular weights.

Now, according to Thomsen, the heats of formation of these weights of the various substances are—

161	grms. of	zinc sulphate, $ZnSO_4$,	about	159,000	calories
36	,,	water, $2(H_2O)$	,,	137,000	,,
92	,,	nitric peroxide, $2(NO_2)$	,,	39,000	,,
			,,	335,000	,,
98	,,	sulphuric acid, H_2SO_4	,,	122,000	,,
126	,,	nitric acid, $2(HNO_3)$	,,	126,000	,,
			,,	248,000	,,

∴ the E.M.F. of the Grove's cell

$$= 0{\cdot}000022 \times (335{,}000 - 248{,}000)$$
$$= 1{\cdot}9 \text{ volt approximately,}$$

which, again, is about the value found by direct measurement.

As already stated, the preceding method of calculation can only give the true value of the E.M.F. when the cell is neither warmed nor cooled by the chemical actions that take place in it, an assumption that is certainly not true in the case of certain cells. Further, the great difference between the heats of combustion as measured by Thomsen and by Berthelot leads us to suspect that our knowledge on this subject is at present not very exact. It is not, then, to be wondered at that the E.M.Fs. of certain cells, calculated in the way just described, do not agree very well with their actual E.M.Fs.

But, even if the heats of combustion had been determined with great accuracy, there would still probably be a difference between the E.M.Fs. of certain cells and their values calculated from the thermal actions, unless allowance was made for the energy represented by certain combinations taking place which do not constitute recognised chemical

actions. For example, in the Smee's cell hydrogen is liberated at the platinum plate, and sticks to this plate, producing a back E.M.F. The difficulty of freeing the platinum from the hydrogen without making the platinum red-hot shows that there is a genuine combination, although a mechanical, rather than a chemical, combination; perhaps it is such a combination as occurs when a salt is dissolved in water, or when sulphuric acid is diluted with water. And, in all probability, just as the solution of a salt produces cold and the dilution of sulphuric acid produces heat, the mechanical combination of the hydrogen with the platinum produces heat, and, therefore, must be considered in calculating the E.M.F. of the Smee's cell.

148. Cost of Producing Electric Energy with Galvanic Cells and with a Dynamo Compared.—From the examples given in the earlier part of this chapter we have seen that the cost of developing a Board of Trade unit in an external circuit by means of a battery depends on:—

(1) The materials that are consumed when a given current flows through the battery for a given time.

(2) The price per ton of the materials in question.

(3) The E.M.F. of the battery.

(4) The ratio of the external resistance to the sum of the external and internal resistances.

(5) The cost of the labour necessary for superintending the battery, &c.

To the preceding must be added interest on the capital initially expended, also some allowance must be made for depreciation, if we wish to estimate the *total* cost of giving a Board of Trade unit to an external circuit.

If the external resistance be very large compared

with that of a battery, then, excluding the chemical energy wasted in local electric currents, practically the whole of the chemical energy that is converted into electric energy will be usefully employed in the external circuit. But this conversion of chemical energy into useful electrical energy will proceed so slowly, in consequence of the smallness of the current, that ample time will elapse for a considerable amount of waste of the chemicals to take place through local action. Hence, it does not follow that a battery in practice will give energy most economically to an external circuit when the resistance of the circuit is extremely large compared with that of the battery itself.

For the purpose, however, of making a comparison between the cost of producing electric energy with primary cells and with a dynamo, we will give the battery the advantage of assuming that there is no local action whatever; further, that the *whole* of the energy which is represented by the chemicals consumed is usefully employed in the outside circuit. We will also, in the first instance, omit the important item of expenditure on the labour of superintending the battery, of removing the spent material, refilling the cells, &c.; and, lastly, we will suppose that the materials are bought in large quantities at the lowest *wholesale* prices quoted in December, 1895.

First, as to the amounts of the materials necessarily used up. In the following Table XII. are given the approximate weights of the materials consumed per hour per 1,000 amperes for some of the most important cells. These weights are only given *approximately*, since, as explained on pages 27 and 28, the amount of electrolytic action produced in a given time by a given current depends not only on the combining weights of the constitutents, but also on the temperature and on the "*current density*"—that is, on the ratio of the current to the cross-section of its path.

TABLE XII.

Approximate weight of the Materials Consumed per Hour per 1,000 Amperes.

Cell.	Material.	Approximate Weight in lbs. of Material consumed per Hour per 1,000 Amperes.
Daniell	Zinc	2·69
	Copper Sulphate (including water of crystallisation) ...	10·3
	[Copper produced.]	[2·61]
Grove	Zinc	2·69
	Sulphuric Acid (sp. gr. 1·8—*i.e.* containing about 90 per cent. of H_2SO_4)	4·43
	Nitric Acid (sp. gr. 1·4 —*i.e.* containing about 65 per cent. of HNO_3)	7·98
Leclanché	Zinc	2·69
	Sal ammoniac	4·41
	Manganese Dioxide	7·08
Potassium bichromate	Zinc	2·69
	Sulphuric Acid (sp. gr. 1·8) ...	10·32
	Potassium Bichromate... ...	4·04
Chromic acid	Zinc	2·69
	Sulphuric Acid (sp. gr. 1·8) ...	9·01
	Chromium Trioxide	2·89
Edison-Lalande	Zinc	2·69
	Caustic Potash (containing 70 per cent. of KHO)	3·30
	Copper Oxide	3·27
	[Copper produced.]	[2·61]

In the following list are given the wholesale prices quoted in December, 1895, for the materials used in the preceding batteries; and, to facilitate the calculation of the cost of using cells for generating a Board of Trade unit, the approximate prices per pound are given in a separate column. It is to be remembered, however, that these prices only apply when *very large* quantities are purchased, and that not only are they much lower than the prices at which single pounds of the materials can be bought, but, in some cases, they are even much less than the prices per pound charged by wholesale chemists when a hundredweight of the materials is purchased at once.

APPROXIMATE WHOLESALE PRICES, 1895.

Material.	———	At per lb. in pence.
Caustic potash (containing 70 per cent. of KHO)	£20 15 0 per ton ...	2¼
Copper	£47 10 0 ,, ...	5
Copper oxide	In very large quantities	12
Copper sulphate (containing about 36 per cent. of water of crystallisation)	£16 10 0 per ton ...	1¾
Chromium trioxide (containing 5 per cent. of water)	£ 3 6 0 per cwt. ...	7
Manganese dioxide ...	£ 0 12 0 ,, ...	1¼
Nitric acid (sp. gr. 1·4—*i.e.* containing about 65 per cent. of HNO_3)	In very large quantities	1¾
Potassium bichromate	,, ,, ...	4⅝
Sal ammoniac	£23 0 0 per ton ...	2¼
Sulphuric acid (sp. gr. 1·8—*i.e.* containing about 90 per cent. of H_2SO_4)	£ 1 15 0 ,, ...	¼
Zinc	£16 0 0 ,, ...	2½, including casting into battery plates and amalgamating.

We can now estimate the approximate weights of the different materials used up and formed, and the minimum cost of these materials, when a Board of Trade unit is developed by each one of the six cells. As already stated, we will give the cells the advantage of assuming that there is no local action whatever, and that the *whole* of the energy which is represented by the chemicals consumed is usefully employed in the outside circuit.

TABLE XIII.

APPROXIMATE WEIGHTS AND COST OF THE MATERIALS CONSUMED PER BOARD OF TRADE UNIT PRODUCED.

Cell.	Material.	Approximate Weight in lbs. used up per Board of Trade Unit.	Approximate *Lowest* Price of Material per Board of Trade Unit.
Daniell (E.M.F., 1·1 volt)	Zinc	2·44	$6\frac{1}{4}$d.
	Copper sulphate (including water of crystallisation	9·3	1s. $4\frac{1}{2}$d.
	[Copper produced]	[2·4]	[1s. 0d.]
			0s. 11d.
Grove (E.M.F., 1·9 volt)	Zinc	1·42	$3\frac{1}{2}$d.
	Sulphuric acid (sp. gr. 1·8 — *i.e.* containing about 90 per cent. of H_2SO_4)...	2·33	$\frac{3}{4}$d.
	Nitric acid (sp. gr. 1·4—*i.e.* containing about 65 per cent. of HNO_3) ...	4·2	$7\frac{1}{2}$d.
			1s. 0d.

TABLE XIII (*continued*).

Cell.	Materials.	Approximate Weight in lbs. used up per Board of Trade Unit.	Approximate *Lowest* Price of Material per Board of Trade Unit.
Leclanché (E.M.F., 1·48 volt)	Zinc	1·82	4½d.
	Sal ammoniac	2·98	6¾d.
	Manganese dioxide ...	4·78	6d.
			1s. 5d.
Potassium bichromate (E.M.F., 1·8 volt)	Zinc	1·49	3¾d.
	Sulphuric acid (sp. gr. 1·8 — *i.e.* containing about 90 per cent of H_2SO_4)...	5·76	1½d.
	Potassium bichromate ...	2·24	10¾d.
			1s. 3d.
Chromic acid (E.M.F., 1·8 volt)	Zinc	1·49	3¾d.
	Sulphuric acid (sp. gr. 1·8)	5·00	1¼d.
	Chromium trioxide ...	1·62	11½d.
			1s. 5d.
Edison-Lalande (E.M.F., 0·8 volt)	Zinc	3·36	8½d.
	Caustic Potash (containing 70 per cent. of KHO)	4·12	9¼d.
	Copper oxide	4·08	4s. 1d.
	[Copper produced]	[3·25]	[1s. 4¼d.]
			4s. 2d.

Excluding, then, the Edison-Lalande cell, which is the most costly of the six cells referred to above for producing electric energy, we may conclude that, *under the most favourable circumstances, the cost of producing a Board of Trade unit with cells cannot be less than from elevenpence to one-and-fivepence for the materials alone that are used up.* Further, that if we take into account local action and other causes of waste of material, and also if we include the value of the labour spent in keeping the cells in order, the cost of producing a Board of Trade unit will be even much larger than one-and-fivepence.

Contrasted with this, experience has shown that in a large London central station, using steam engines and dynamos, the cost of producing a Board of Trade unit, *inclusive of coal, water, oil, wages, and salaries in the generating station may be as low as* $1\frac{2}{3}d$. It might, therefore, be supposed that the question of employing batteries to compete with steam-engines and dynamos was one that had been definitely settled, and that any attempt to introduce galvanic cells for the electric lighting of towns would be regarded commercially as hopeless, as perpetual motion is now regarded scientifically as impossible.

This, however, is not the case; for at regular intervals a new type of primary cell is brought before the public with a flourish of trumpets, and for a time is able to command more or less attention, for three reasons—viz.:—

(1) It is stated that, if galvanic cells were used in houses, the cost of laying copper mains under the streets would be saved.

(2) It is claimed that the zinc oxide which is produced—in an Edison-Lalande cell, for example—has a commercial value, as it may be used for paint; also that it is not fair to such a primary cell to debit it with the whole cost of the caustic potash which is converted into the salt of potassium and zinc (K_2ZnO_2) during

the action of the cell, since the caustic potash can be used over and over again after the zinc oxide has been separated from the liquid.

(3) It is pointed out that the mechanical energy produced by even a good steam-engine does not exceed about one-tenth of the energy contained in the coal; and, therefore, allowing for the energy wasted in the shafting, in the dynamo, and in the copper mains under the streets, the electric energy delivered to a house does not exceed from 6 to 7 per cent. of the energy contained in the coal burnt in the central station. Whereas, with a good galvanic cell of low internal resistance, the electric energy delivered to the external circuit may be 95, or even more, per cent. of the whole energy represented by the consumption of battery material.

Advantage No. 1, which is claimed for using galvanic cells in houses for electric lighting, is based on the assumption that it would be cheaper for a company to maintain a staff of men to superintend batteries scattered throughout a town, and vehicles in which to convey the fresh supplies of battery material, than to expend capital in laying copper mains under the streets. But industrial experience is opposed to this, for it was found to be more convenient to distribute gas and water by means of pipes than to establish a little gasworks and to dig a well in every house. The carting of coal to work small steam-engines is already to some extent replaced by sending pressure-water, or compressed air, under the streets; even steam for warming groups of private houses is supplied from centres in American towns; and in that country kerosene oil for illuminating purposes is pumped through hundreds of miles of pipes.

Further, the cost of superintending the cells which it is proposed to place in the houses to be electrically lighted would probably be increased by the necessity of the number of cells in circuit in a house, or the resistance in each house circuit, having to be altered from time to time to compensate for the falling off in the E.M.F. of

the cells and for the increase in the internal resistance which always occurs when galvanic cells are used to produce a considerable current. For certain uses of the current, no doubt, small changes in its strength are of no importance, and no very careful regulation is necessary; but, since the light emitted by a glow lamp varies about 6 per cent. for each 1 per cent. change in the current, and about 7 per cent. for each 1 per cent. change in the pressure, the public will not tolerate any considerable fluctuation in the pressure maintained between the terminals of a glow lamp.

Now, when electric energy is supplied through mains from a central station, it is possible, by regulating at the central station, to keep the pressure fairly constant in any house; indeed, as stated on page 358, electric light companies in Great Britain are prevented by law from allowing the P.D. between the house-mains, where they enter the house, to vary by more than 4 per cent. from the standard pressure. And it is the endeavour of every well-regulated electric light company to keep well within this permissible limit of variation. If, however, a house were electrically lighted with galvanic cells on the premises, the regulation would have to be effected either by the householder, or by an inspector going from house to house, an arrangement which would not only be attended with some inconvenience, but, probably, with some expense.

As to the second advantage that is claimed, it does not seem wise to base calculations on the assumption of its truth until we have some practical proof that the cost of separating the zinc oxide from the spent liquid is more than compensated for by the selling value of the materials so obtained. This criticism does not, of course, apply to the copper deposited in a Daniell's cell, for it is deposited in a solid, very pure condition, and has certainly a definite money value, such as has been given to it in Table XIII. (page 486). But it does apply to a certain extent to the copper that is left behind after the reduction

of the copper oxide in the Edison-Lalande battery, for, since that is mixed with a certain amount of the unreduced oxide, its value may be less than that assigned to it in Table XIII.

The third advantage claimed for the galvanic cell—viz. that it is a very efficient transformer of chemical energy into electric energy—has not only a real existence, but is of the greatest importance. At present, however, this high efficiency of the cell has a scientific interest only, since, although *95 per cent. of the energy represented by the chemical action in a cell may be usefully employed,* we cannot with any known form of galvanic cell produce a Board of Trade unit with a consumption of less than about 1·4 lb. of zinc, which costs not less than 3½d., when allowance is made for casting and amalgamating; further, a sum of 8d., or so, has to be expended per Board of Trade unit for the oxidising and depolarising agents, if the waste product be regarded as practically valueless.

Hence, it is impossible at present for cells to compete with steam-engines and dynamos for the lighting of towns, since, although *95 per cent. of the energy contained in the coal may be entirely wasted* with our present methods of distributing it electrically, a Board of Trade unit delivered to a house represents a combination of at the most 3 lbs. of coal, which cost, say, 0·3d., with the oxygen of the air, which costs nothing. Hence, excluding labour, interest on capital, etc., the cost price of a Board of Trade unit delivered to a house now represents under 1d.; whereas, also excluding labour, interest on capital, &c., it would represent at the very least 1s. if supplied from cells in the house.

Should, however, a really economical method be devised of dealing with the waste product from some galvanic cell, or, better still, when we have found out how to cause some fuel of the value of coal, or of oil, to combine rapidly in a galvanic cell with some oxidiser as cheap as air or water, then the comparison of the cost

of producing electric energy with cells and with a dynamo will require careful reconsideration.

But it is to be remembered that the solution of this latter problem requires an entirely new discovery to be made, and that the mere cheapening of electric lighting is one of the smallest results that would follow from it. For a method of causing the energy liberated by the oxidation of *cheap* fuel to appear in an electric or mechanical form *without the employment of any heat-engine* would entirely revolutionise our existing methods of producing motive power, and might enrich the discoverer beyond the dreams of avarice. (*See* the note on page 563.)

Until, however, we are able to form some conception of that mechanism in man and in animals by the aid of which the energy of food is turned directly into muscular energy, or until we are in possession of new methods of dealing with such waste products as zinc sulphate, any scheme for using galvanic cells to electrically light towns should be received with considerable scepticism.

149. Measuring a Cell's Resistance when Very Small.—A method of measuring the resistance of a battery or cell by using a voltmeter and an ammeter was described on page 366, § 119; and, in the case of cells having a very low internal resistance, this method is very valuable. The resistance of an accumulator, for example, is frequently under 0·001 ohm; hence, until the current is as large as 20 amperes, the P.D. between the terminals of such a cell will not differ from its E.M.F. of 2 volts by as much as 1 per cent.—or, in other words, the fact that the cell has a resistance will not cause a diminution of as much as 1 per cent. in the value of the current for currents under 20 amperes. Hence, it follows that somewhat large currents must be employed in order to measure the resistance of such an accumulator with any degree of accuracy; and while, on the one hand, such currents are liable to change the resistance of a coil by warming it, and so prevent us

being quite sure of the resistance of a coil from previous tests, such currents are very suitable for being measured with an ordinary commercial ammeter.

Consequently, if a suitable voltmeter be also available, the method illustrated in Fig. 169, page 366, which does not require the resistance of the coil x to be known, may be very conveniently used for testing the resistance of a battery or cell intended to produce large currents.

Another reason why large currents must be used in testing the resistance of an accumulator is that this resistance is not a constant, but diminishes as the current increases. Hence, if it be desired to know the resistance for a current of 50 amperes, this current must be employed when making the test.

150. Measuring a Cell's Resistance when Not Very Small.—In the case of cells like the Minotto, which have a resistance of several ohms each, the method of measuring the resistance which is described in the preceding section would generally be unsuitable, as it would require the use of an ammeter graduated in hundredths of an ampere. Further, no difficulty is met with in using ordinary resistance coils, since, with such cells, the currents that can be produced are but small.

As with the preceding method, *two separate* tests must be made, since the E.M.F. of the cell as well as its resistance is generally unknown, and the tests employed must be of such a kind that the E.M.F. can be eliminated from the two equations which express the results obtained on carrying out the two tests in question. For example :—

Method No. 1 (*a*).—Let a cell of unknown resistance, c ohms, be used to send a current through a galvanometer of known resistance, g ohms — first, when a resistance of r ohms is inserted in the circuit in addition to c and g; second, when this added resistance is changed to r' ohms. Let C and C′ be the relative

strengths of the currents determined from the two deflections of the galvanometer and from its relative calibration curve. Let E be the unknown E.M.F. of the cell in volts, then

$$\frac{\mathrm{E}}{c+r+g} \div \frac{\mathrm{E}}{c+r'+g} = \frac{\mathrm{C}}{\mathrm{C}'},$$

$$\text{or } \frac{c+r'+g}{c+r+g} = \frac{\mathrm{C}}{\mathrm{C}'},$$

$$\therefore \quad c = \frac{\mathrm{C}'(r'+g) - \mathrm{C}(r+g)}{\mathrm{C}-\mathrm{C}'}.$$

No. 1 (*b*).—If r and r' be so chosen that C is twice C′, then

$$c = r' - 2r - g.$$

Method No. 2 (*a*).—Instead of varying the added resistance, let the galvanometer in the first test be unshunted, and, in the second, be shunted with a shunt of s ohms' resistance, and let C and D be the relative strengths of the currents produced, then

$$\frac{\mathrm{E}}{c+r+g} \div \frac{s}{g+s} \cdot \frac{\mathrm{E}}{c+r+\frac{g\,s}{g+s}} = \frac{\mathrm{C}}{\mathrm{D}},$$

$$\therefore \quad \frac{(g+s)(c+r)+g\,s}{s(c+r+g)} = \frac{\mathrm{C}}{\mathrm{D}},$$

$$\text{or } c = \frac{(\mathrm{C}-\mathrm{D})(r+g)\,s - \mathrm{D}\,r\,g}{\mathrm{D}\,g - (\mathrm{C}-\mathrm{D})\,s}.$$

No. 2 (*b*).—Hence, if r be nought, and g be large compared with s,

$$c = \frac{\mathrm{C}-\mathrm{D}\,s.}{\mathrm{D}}$$

Method No. 3 (*a*).—Let the galvanometer be shunted with a shunt of resistance s when making both tests, the former being made, however, with a resistance of r ohms in the main circuit, and the second with a resistance of r' ohms; then, if G and G′ be the relative strengths of the currents in the two cases,

$$\frac{s}{g+s} \cdot \frac{\mathrm{E}}{c+r+\dfrac{g\,s}{g+s}} \div \frac{s}{g+s} \cdot \frac{\mathrm{E}}{c+r'+\dfrac{g\,s}{g+s}} = \frac{\mathrm{G}}{\mathrm{G}'}$$

$$\therefore \quad \frac{(g+s)(c+r')+g\,s}{(g+s)(c+r)+g\,s} = \frac{\mathrm{G}}{\mathrm{G}'},$$

$$\text{or } c = \frac{\mathrm{G}'\,r' - \mathrm{G}\,r + (\mathrm{G}'-\mathrm{G})\dfrac{g\,s}{g+s}}{\mathrm{G}-\mathrm{G}'}.$$

No. 3 (*b*).—If r be made nought and the galvanometer be very sensitive, s may have to be made so small, to reduce the first deflection to a readable amount, that $\dfrac{g\,s}{g+s}$ will be inappreciably small compared with c. In that case

$$c = \frac{\mathrm{G}'\,r'}{\mathrm{G}-\mathrm{G}'};$$

No. 3 (*d*) therefore, if also r' be chosen so as to make G′ equal to half G,

$$c = r'.$$

Method No. 4 (*a*).—When making the first test, let a resistance of r ohms be in the main circuit, and let the galvanometer be unshunted; while, in making the second test, let r be charged to r' ohms, and let the galvanometer be shunted with a shunt of resistance s. Then, if

the relative strengths of the currents be C and F, we have

$$\frac{E}{c + r + g} \div \frac{s}{g + s} \cdot \frac{E}{c + r' + \frac{g\,s}{g+s}} = \frac{C}{F},$$

$$\therefore \frac{(g + s)(c + r') + g\,s}{s(c + r + g)} = \frac{C}{F},$$

$$\text{or } c = \frac{C\,(r + g)\,s - F\,\{(g + s)\,r' + g\,s\}}{F\,g - (C - F)\,s};$$

No. 4 (*b*) so that if s and r' be so chosen that F equals C,

$$c = \frac{s\,(r - r') - g\,r'}{g};$$

No. 4 (*d*) or, if r' be nought and s be so chosen that F equals C,

$$c = \frac{s\,r}{g}.$$

Method No. 5.—In making the first test let the galvanometer be shunted with a shunt of s ohms, and let a resistance of r ohms be in the main circuit, while, in making the second test, let the shunt be changed to one of s' ohms and the added resistance to r' ohms.

Then, if G and G″ be the relative strengths of the currents, we have

$$\frac{s}{g + s} \cdot \frac{E}{c + r + \frac{g\,s}{g+s}} \div \frac{s'}{g+s'} \cdot \frac{E}{c + r' + \frac{g\,s'}{g+s'}} = \frac{G}{G''},$$

$$\therefore \frac{s}{s'} \times \frac{(g + s')(c + r') + g\,s'}{(g + s)(c + r) + g\,s} = \frac{G}{G''},$$

$$\text{or } c = \frac{G\,s'\,\{(g + s)\,r + g\,s\} - G''\,s\,\{(g + s')\,r' + g\,s'\}}{G''\,s\,(g + s') - G\,s'\,(g + s)}.$$

This last case includes all those previously given in this section, for they may be obtained from this last case by simply giving different values to r, r', s and s', as seen from the following table:—

	FIRST TEST.		SECOND TEST.	
	Value of Added Resistance.	Value of Shunt.	Value of Added Resistance.	Value of Shunt.
Method No. 1 (*a*)	r	∞	r'	∞
,, ,, (*b*)	r	∞	r' chosen so as to halve the current	∞
Method No. 2 (*a*)	r	∞	r	s
,, ,, (*b*)	0	∞	0	s small compared with g
Method No. 3 (*a*)	r	s	r'	s
,, ,, (*b*)	0	s small compared with g	r'	s small compared with g
,, ,, (*d*)	0	s small compared with g	r' chosen so as to halve the current	s small compared with g.
Method No. 4 (*a*)	r	∞	r'	s
,, ,, (*b*)	r	∞	r' chosen so as to keep current constant	s chosen so as to keep current constant
,, ,, (*d*)	r	∞	0	Ditto
Method No. 5 .	r	s	r'	s'

151. Remarks on the Preceding Methods of Measuring the Resistance of Cells.—*All* the preceding methods of measuring the resistance of a cell are based on the assumption that its E.M.F. remains *exactly the same* during the carrying out of the *pair* of tests that have to be made, whichever method of measurement be employed. In consequence of the time, however, during which each of the two currents has to be allowed to flow while the galvanometer deflection is becoming steady, and also in consequence of its being necessary for the carrying out of the measurement that these two currents passing through the cell should differ considerably in strength, the assumption that the E.M.F. of the cell remained wholly unchanged would be quite wrong in the case of cells like the Leclanché, which polarise rapidly. On this account the "*condenser method of measuring the resistance of cells*," to be subsequently described in Volume II., is preferable for use with such cells.

In employing any of the preceding methods, care must be taken *not* to use resistances that are very large compared with the resistance of the cell to be measured; for not only would the introduction of such large resistances render the test very unsensitive, but, in addition, any small percentage error which might exist in this large resistance would itself be comparable with the resistance to be measured, and so would introduce a large error into the answer.

For example, to weigh a few grains of some powder in a weighing-machine used for weighing luggage at a railway station would certainly lead to hopeless inaccuracy; but beginners are apt to forget that, although a coil of 10,000 ohms and another of $\frac{1}{100}$th of an ohm may be put into boxes of about the same size, there is the same sort of difference between these resistances as there is between the weight of 30 tons and of 1 ounce, and therefore that giving r and r' in the preceding tests values of thousands of ohms must invariably lead to

error when the resistance of the cell to be tested is 2 or 3 ohms, or less.

If, then, the galvanometer used for measuring the resistance of a cell be very sensitive and of high resistance, methods Nos. 1, 2, and 4 would be quite unsuitable to be employed, since, to reduce the galvanometer deflection to readable limits, it would be necessary to give r a large value. Methods Nos. 3 or 5, on the other hand, could be successfully used, since the galvanometer being shunted in both the tests, the deflection could be made to have the desired value in each case without making r very large. And if the cell were one that was not seriously polarised when the external resistance was small, method No. 3 (*d*) would be a very convenient one to employ.

If, on the other hand, the galvanometer, although having a high resistance, be not very sensitive, so that when the cell to be tested is applied direct to its terminals the deflection is not unreadably large, then method No. 2 (*b*) may be employed with advantage; for it is to be observed that the formula in this case which gives the resistance of the cell does not involve the high resistance of the galvanometer, but only that of the shunt, and this may be made comparable with that of the cell.

Since r has the value nought in this method No. 2 (*b*), and the high resistance galvanometer is therefore applied directly between the terminals of the cell, this method may be regarded as measuring, first, a current which is proportional to the E.M.F. of the cell; second, a current proportional to the P.D. between its terminals when the galvanometer and the cell are shunted with a resistance of s ohms—that is, when the cell is allowed to send a current through an external resistance of s ohms. But this is practically the method described in § 119, page 366; and the s in method No. 2 (*b*) is, therefore, simply the value of x in Fig. 169, page 366.

Hence, all the equations given on pages 364, 365,

and 366 apply also in this case, only if E is the E.M.F. in volts of the cell, and if V is the P.D. in volts between its terminals when it is sending a current of A amperes through the shunt or external resistance of s ohms, it must be remembered, first, that in method No. 2 (b) we do not actually measure the current with an ammeter; second, that the currents C and D do not indicate the values of E and of V in volts, but only are proportional to E and V—that is to say,

$$\frac{C}{D} = \frac{E}{V}.$$

From this, however, it follows that the equation

$$c = \frac{C-D}{D} s,$$

which is given under the method No. 2 (b), is simply the equation

$$b = \frac{E-V}{V} x,$$

a form in which the last equation but two on page 364 can be at once written. Consequently, the equation used in method No. 2 (b) is simply a statement of the fact that the P.D. employed in sending the current through the cell bears to the P.D. employed in sending it through the external circuit the ratio that the resistance of the cell does to the external resistance.

In § 149, page 492, it was explained that the method given in § 119 for measuring the resistance of a cell was specially suitable when the cell's resistance was very small; and the reason that in method No. 2 (b) we have been again led to the equations given in § 119 is that by assuming in method No. 2 (b) that r was nought, that g was large relatively to s, and that s was comparable in value with c, we have in reality made the assumption that c was small.

Example 136.—A Daniell's battery produces a deflection of 38° on a tangent galvanometer when a resistance of 27 ohms is inserted in the circuit, and a deflection of 46° when this resistance is reduced to 12 ohms. What is the resistance of the battery if that of the galvanometer be $2\frac{1}{2}$ ohms?

Inserting these values in the equation, we have

$$b = \frac{\tan. 38° \times (27 + 2\frac{1}{2}) - \tan. 46° \times (12 + 2\frac{1}{2})}{\tan. 46° - \tan. 38°.}$$

Answer.—$31\frac{1}{2}$ ohms about.

Example 137.—With a galvanometer having a resistance of half an ohm, and constructed so that the angular deflection is directly proportional to the current, a battery of 20 Grove's cells in series produces a deflection of 28 divisions when a resistance of 2 ohms is inserted, and 14 divisions when a resistance of 8 ohms is inserted. What is the resistance of the battery?

If b be the resistance of the entire battery,

$$b = 8 - 2 \times 2 - \tfrac{1}{2}.$$

Answer.—$3\frac{1}{2}$ ohms.

Example 138.—A cell attached directly to a low resistance ammeter produces a current of 1·24 ampere, which is reduced to 0·62 ampere when a resistance of 1·35 ohm is inserted in the circuit. What is the resistance of the cell?

Answer.—1·35 ohm.

Example 139.—A cell produces a current of $1\frac{3}{4}$ ampere when joined in series with a resistance of $\frac{1}{2}$ an ohm and an ammeter having 1 ohm resistance. When, however, the ammeter is shunted with a resistance of $\frac{1}{4}$ ohm the reading is 0·49 ampere. What is the resistance of the cell?

Answer.—0·15 ohm.

Example 140.—When 4 ohms are introduced into the circuit of a Leclanché cell and a sine galvanometer

having 6 ohms' resistance, a deflection is produced corresponding with a necessary rotation of the sine galvanometer through 22°. When, however, the sine galvanometer is shunted with 2 ohms, the rotation required is only 8°. What is the resistance of the Leclanché cell?

Substituting the values in the equation, we have

$$b = \frac{\sin. 22° \times (4+6) \times 2 - \sin. 8° \times \{(2+6) \times 4 + 2 \times 6\}}{\sin. 8° \times (2+6) - \sin. 22° \times 2.}.$$

Answer.— 4 ohms about.

Example 141.—The same deflection is produced on a galvanometer of $2\frac{1}{2}$ ohms' resistance, when 8 ohms are in circuit, as when only 2 ohms are in circuit, and the galvanometer is shunted with 2 ohms. What is the resistance of the current generator?

$$b = \frac{2 \times (8-2) - 2\frac{1}{2} \times 2}{2\frac{1}{2}},$$

Answer.—$2\frac{4}{5}$ of an ohm.

152. Comparing the Electromotive Forces of Cells.—Although it requires two tests to be made to determine the resistance of a cell, unless its E.M.F. at the moment of making the test is known, one measurement with a suitable voltmeter enables the E.M.F. of a cell in volts to be ascertained. To compare, however, the E.M.Fs. of two cells requires two tests; they may, for example, be both measured with a voltmeter, or, if only their *relative* values and not the absolute value of either be required, one or other of the following methods may be employed:—

Method No. 1.—When a galvanoscope of known resistance, g ohms, but of unknown calibration, is available as well as a box of resistance coils, attach each cell in succession to the galvanometer, with the resistance box in circuit, and employ resistances r and r' ohms in the two cases respectively so as to make the current the

same. Then, if c and c' are the resistances of the cells respectively in ohms, and E and E$'$ are their unknown E.M.Fs., in volts,

$$\frac{\mathrm{E}}{c+r+g} = \frac{\mathrm{E}'}{c'+r'+g};$$

$$\therefore \frac{\mathrm{E}}{\mathrm{E}'} = \frac{c+r+g}{c'+r'+g}.$$

If the galvanoscope is of high resistance and sensitive, c and c' will be negligible compared with $r+g$ and $r'+g$; therefore, in that case,

$$\frac{\mathrm{E}}{\mathrm{E}'} = \frac{r+g}{r'+g}.$$

Or, if the galvanoscope be *very* sensitive, $c+g$ may be small compared with r, and $c'+g$ small compared with r'. In that case we have

$$\frac{\mathrm{E}}{\mathrm{E}'} = \frac{r}{r'} \text{ approximately.}$$

Method No. 2.—If the relative calibration of the galvanometer as well as its resistance, g ohms, be known, and a few resistance coils be available, but not necessarily enough to enable the resistance to be changed within wide limits, attach each cell in succession to the galvanometer with one or more coils in the circuit, so as to obtain readable deflections. Let C and C$'$ be the relative strengths of the currents when the resistances r and r' ohms are inserted with the two cells respectively, then

$$\frac{\mathrm{E}}{c+r+g} \div \frac{\mathrm{E}'}{c'+r'+g} = \frac{\mathrm{C}}{\mathrm{C}'},$$

$$\therefore \frac{\mathrm{E}}{\mathrm{E}'} = \frac{c+r+g}{c'+r'+g} \times \frac{\mathrm{C}}{\mathrm{C}'}.$$

If the E.M.Fs. and the resistances of the two cells be

not very different, a single resistance coil of r ohms may be used in both cases, and then the equation becomes

$$\frac{\mathrm{E}}{\mathrm{E}'} = \frac{c + r + g}{c' + r + g} \times \frac{\mathrm{C}}{\mathrm{C}'}.$$

Hence, if the galvanometer be of high resistance and sensitive, both c and c' will be small compared with $r+g$, and the ratio becomes

$$\frac{\mathrm{E}}{\mathrm{E}'} = \frac{\mathrm{C}}{\mathrm{C}'};$$

or, in other words, the E.M.Fs. of two cells are proportional to the currents they send respectively through a circuit consisting of a galvanometer in series with a high resistance. But this result also follows at once from the fact that such a combination constitutes an instrument for measuring the relative value of P.Ds. (§§ 47, 48, pages 176 to 179); and that when the resistance external to a cell is large, the P.D. between its terminals is equal to its E.M.F. (§ 119, page 365).

Method No. 3.—If we know neither the resistance of the galvanometer nor of the cells, nor of any coil of wire, the ratio of their E.M.Fs. can be found thus:—First, join the cells so that they help one another, and place them in series with the galvanometer and any convenient coil of wire of unknown resistance; second, reverse the direction of one of the cells so that they now oppose one another. Then, if G and G′ be the relative strengths of the currents,

$$\frac{\mathrm{E} + \mathrm{E}'}{p} \div \frac{\mathrm{E} - \mathrm{E}'}{p} = \frac{\mathrm{G}}{\mathrm{G}'},$$

where p is the unknown, but constant, resistance of the circuit;

$$\therefore \quad \frac{\mathrm{E} + \mathrm{E}'}{\mathrm{E} - \mathrm{E}'} = \frac{\mathrm{G}}{\mathrm{G}'},$$

$$\text{or} \quad \frac{\mathrm{E}}{\mathrm{E}'} = \frac{\mathrm{G} + \mathrm{G}'}{\mathrm{G} - \mathrm{G}'}$$

This method has, however, the serious disadvantage that reversing the direction of the current through one of the cells may temporarily alter its E.M.F., so that the E′ in the second term of the first equation may differ somewhat from the E′ in the first term; and, therefore, the third equation given above cannot be accurately deduced from the second.

Example 142.—Two batteries having internal resistances of 15 and 10 ohms respectively produce the same deflection on a galvanometer of 40 ohms' resistance when 305 and 250 ohms are respectively introduced into the circuit. What is the ratio of their E.M.Fs.?

Answer.—1·2.

Example 143.—The same two batteries produce the same deflection on a much more delicate galvanometer, having 100 ohms' resistance, when 6,031 and 5,000 ohms are respectively introduced into the circuit. What is the ratio of their E.M.Fs.? *Answer.*—1·2027.

Example 144.—What result will be obtained in the last question if the resistances of the cells and of the galvanometer be neglected?

Answer. $\frac{6031}{5000}$, or 1·206. Hence, by neglecting the resistances of the cells and of the galvanometer in question 143, nearly as accurate an answer is obtained as by taking these resistances into account in question 142, where the galvanometer was much less sensitive.

Example 145.—A current-generator of 2 ohms' resistance produces a deflection of 60° on a tangent galvanometer of 3 ohms' resistance when a resistance of 4,500 ohms is introduced into the circuit. A Daniell's cell, on the other hand, having an E.M.F. of 1·07 volt and 5 ohms' internal resistance, produces a deflection of 45° when a resistance of 55 ohms is placed in the circuit. What is the E.M.F. of the current-generator?

Answer.—

$$1{\cdot}07 \times \frac{2 + 4500 + 3}{5 + 55 + 3} \times \frac{\tan 60^\circ}{\tan 45^\circ}, \text{ or } 132{\cdot}5, \text{ volts.}$$

Example 146.—What is the E.M.F. of a Grove's cell if, when connected so as to assist a Daniell's cell having an E.M.F. of 1·1 volt, a rotation of 38° of a sine galvanometer is necessary to bring the needle to a fixed mark, whereas, when the Grove's cell is reversed, a rotation of $8\frac{1}{3}°$ in the opposite direction is necessary?

Answer.—1·78 volt.

Example 147.—What must be the relationship between the resistance of the galvanometer, the added resistance, and the resistances of two cells whose E.M.Fs. are to be compared by method No. 1, page 502, in order that neglecting the resistance of the galvanometer and of the cells may not cause an error of more than 1 per cent. in the answer?

Answer.—If c and c' ohms be the resistances of two cells having E.M.Fs. of E and E′ volts, and r and r' ohms be the added resistances when the same deflection is produced by the two cells on a galvanometer of resistance g ohms, we have

$$\frac{\mathrm{E}}{\mathrm{E}'} = \frac{c+g+r}{c'+g+r'},$$

and we require that

$$\frac{c+g+r}{c'+g+r'} - \frac{r}{r'} \text{ shall numerically not exceed } \frac{1}{100}\frac{c+g+r}{c'+g+r'};$$

also that

$$\frac{c'+g+r'}{c+g+r} - \frac{r'}{r} \text{ shall numerically not exceed } \frac{1}{100}\frac{c'+g+r'}{c+g+r}.$$

Hence the limiting values will be found from the equations

$$\frac{c+g+r}{c'+g+r'} - \frac{r}{r'} = \pm\frac{1}{100}\frac{c+g+r}{c'+g+r'},$$

and
$$\frac{c'+g+r'}{c+g+r} - \frac{r'}{r} = \pm\frac{1}{100}\frac{c'+g+r'}{c+g+r},$$

which give the four answers

$$99\,\frac{c+g}{r} - 100\,\frac{c'+g}{r'} = 1,$$

$$99\,\frac{c'+g}{r'} - 100\,\frac{c+g}{r} = 1,$$

$$100\,\frac{c+g}{r} - 101\,\frac{c'+g}{r'} = 1,$$

$$100\,\frac{c'+g}{r'} - 101\,\frac{c+g}{r} = 1.$$

Hence, 100 times the ratio of the neglected to the added resistance for either cell must not differ from 99 times, or from 101 times, the ratio of the neglected to the added resistance for the other cell, by more than unity.

Example 148.—A battery sends a current through a resistance box and a milammeter of 57 ohms' resistance. When the resistance in the box is 400 ohms the current is 35·5 milliamperes, which increases to 56 milliamperes when the resistance in the box is reduced to 180 ohms. What are the E.M.F. and resistance of the battery?

Answer.—17·2 volts and 32·6 ohms.

153. Poggendorff's Method of Comparing Electromotive Forces.—With many types of cells the E.M.F. is fairly constant, even for wide variations in the current passing through them, and in such a case any of the previous methods can be employed for comparing their electromotive forces. But with other types—for example, the Clark's cell—a very small current passing through the cell is sufficient to alter the E.M.F. In such a case the following method, due originally to Poggendorff, may be employed:—

From what has preceded we know that if a current of A amperes, produced by any convenient current generator, be made to flow along a wire, J K (Fig. 213), the P.D. in volts between any two points, L, M, is equal

to the product of A into r, the resistance of the wire, in ohms, between the points L and M. Hence if L and M be joined with another circuit containing a cell of E.M.F. equal to E volts, and a galvanoscope, G, and, if one, or both, of the ends of this second circuit be moved along the wire J K composing the first circuit until no current passes through the galvanoscope G, we know that E is equal and opposite to the P.D. between L and M, or

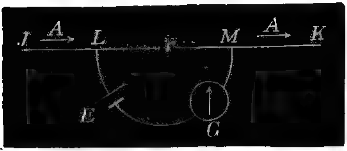


Fig. 213.

$$E = A\,r.$$

If, now, a second cell of E.M.F. equal to E′ volts, and a second galvanoscope, G′, be attached to two other points, U, V, of the wire J K (Fig. 214), the points U and V being so selected by trial that no current passes through this galvanoscope, and if r' be the resistance of the wire U V in ohms, then

$$E' = A\,r',$$

$$\therefore \frac{E}{E'} = \frac{r}{r'};$$

and hence the two E.M.Fs. can be compared without our knowing the value of the current flowing through

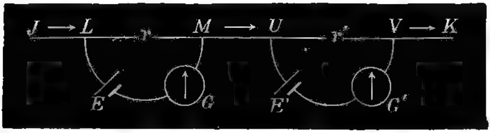


Fig. 214.—Diagram illustrating Poggendorff's Method for comparing E.M.Fs.

the wire J K. If the wire J K is everywhere uniform in material, cross-section, and temperature, the resistances

r and r' are simply proportioned to the lengths L M and U V, so that the E.M.Fs. of the batteries are simply proportioned to the lengths of L M and U V.

Of course, care must be taken to attach the cells whose E.M.Fs. we desire to compare in such a way that their E.M.Fs. tend to oppose the potential differences between L and M and between U and V respectively, since, if either of the cells be attached in the opposite way, no two points can, of course, be found such that the current passing through the galvanoscope attached to that cell is nought.

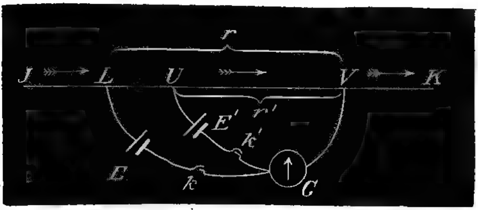


Fig. 215.—Diagram illustrating Poggendorff's Method for comparing E.M.Fs.

If the generator used to produce the current through the wire J K be such that the current is maintained *quite constant*, there is no occasion to use two galvanoscopes, as the points L and M can be ascertained with the first cell, and then the points U, V with the second, such that in each case no current passes through the galvanoscope. Further, it is not necessary that the lengths L M and U V should be taken from wholly different parts of the wire J K, as in Fig. 214; for M and V may be the same point, as in Fig. 215. The key k is depressed and the contact point, L, is altered until no current passes through the galvanoscope G; then the key k' is depressed and the contact point U is altered, until again no current passes through the galvanoscope, when

$$\frac{E}{E'} = \frac{\text{length of wire LV}}{\text{length of wire UV}}.$$

The accuracy of the method will be the greater the longer are the wires L V and U V, because any given small error in the position of one of the sliders corresponding with, say, a millimetre in the length of the wire, will represent a less proportional error in the length, and so r and r' can the more accurately be compared. Hence, it is desirable to make the wire J K *as long as possible*, and to send through it a steady current, of such a strength that the P.D., between its *extreme ends*, is just equal to the larger of the two E.M.Fs. to be compared.

This arrangement, devised by Poggendorff for the comparison of P.Ds., is sometimes called a "*potentiometer.*" It may conveniently be used for the calibration of a voltmeter, but it is desirable to arrange the apparatus differently, depending on whether we desire, as in § 154, to determine the absolute value in volts which correspond with fixed points on the voltmeter scale, or whether, as in § 155, it is the deflections corresponding with fixed P.Ds. in volts that we wish to ascertain.

154. Potentiometer Method of Testing the Accuracy of a Voltmeter Scale.—Fig. 216 shows a method, which is based on that devised by Poggendorff, for testing the accuracy of a scale of a voltmeter V in terms of the E.M.F. of a standard cell, S. The illustration is intentionally somewhat distorted in order that the details of the key C H L may be clearly seen; in reality, however, the base-board and the wire J K, which is stretched on it zig-zag fashion, are much longer than they appear to be in Fig. 216. This wire is made of *platinum-iridium*, as the hardness of that alloy tends to prevent its surface being worn by the pressure of the contact-maker C of the key, and the wire is drawn by the manufacturer so as to have, as nearly as possible, the same diameter everywhere; but, as it is practically impossible to draw a wire several metres long quite uniform in diameter throughout, in consequence of the slight wearing of the draw-plate, the relative resistance

of each inch of the wire should be carefully tested by some method such as that given in the next section, and recorded for future reference.

B is a battery of cells that will send through the resistance r_1 and the wire J K a fairly constant current, and produce between the points J and K a P.D. as large as the voltmeter V is intended to be calibrated for. In circuit with the standard cell, S, preferably of the Clark

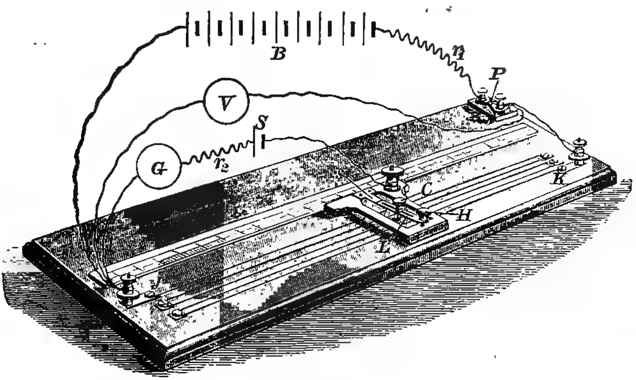


Fig. 216.—Potentiometer Method for Testing a Voltmeter.

type, and the delicate galvanometer G, there is inserted a resistance r_2 sufficiently large to keep the current passing through the cell small even when the point of contact of the key with the wire J K is at some distance away from the position that gives no current through the cell and galvanometer. If S is a Clark's cell r_2 should be not less than about 2,000 ohms.

To make a measurement the plug P is inserted, and the resistance r_1 is adjusted until the P.D. between the ends of the wire J K produces the required deflection of the voltmeter. The handle H of the key is then depressed for an instant when, if there be any deflection of the galvanometer, the handle H is liberated and the key

slid in the proper direction along the wire J K, and contact again made, and so on, until a point M is found such that no current passes through the galvanometer on depressing H. Then, if E be the E.M.F. of the standard cell in volts at the temperature at which the test is made, the true value of the P.D. which is producing the voltmeter deflection is given by the expression

$$\frac{\text{resistance of the whole wire J K}}{\text{resistance of the portion J M}} \times \text{E volts.}$$

The resistance r_1 is now altered so as to produce another deflection of the voltmeter, and the position of the key again adjusted until no current flows through the galvanometer on depressing H, &c.

In order to enable the contact-maker C to touch any one of the five wires composing J K, C can be slid along the slot in the lever L of the key at right angles to the wires; and, to prevent the platinised knife-edge, attached to the lower part of C, being pressed too hard against the stretched wire, and damaging it, the flat spring *s* is made rather weak. Hence, on depressing H, first the knife-edge attached to C comes into contact with the wire, and, on still further depressing H, the lever turns about the knife-edge until it comes against the stop placed underneath it.

As the accuracy of the preceding method is quite independent of the current passing through the voltmeter, the method can be employed either with a voltmeter of high resistance or with one of low resistance, and therefore passing an appreciable current. Further, the method may be made very sensitive, since the *whole* of the wire J K is employed in determining the value in volts of *each* point of the voltmeter scale. But it possesses the disadvantage that a fresh adjustment of the resistance r_1, as well as a fresh adjustment of the position of the key C H L, have to be made for each point on the voltmeter scale whose absolute value we desire to ascertain. While, therefore, the method may very suitably be employed when the relative calibration of a

voltmeter is already known, and the absolute value of one point only has to be determined, it is somewhat tedious to use when many points of the scale have to be calibrated absolutely.

155. Foster's Method of Subdividing a Wire into Lengths having Equal Resistances.—The stretched wire of a potentiometer, or of a Wheatstone's bridge, can be easily calibrated by using the following method, due to Prof. G. C. Foster. Let what are called the "unknown" and "known" resistances in Fig. 127, page 248, have values of a and b ohms respectively, these values not differing much from one another; and let coils, having resistances of x and y ohms respectively which also do not differ much from one another, be inserted in place of the short-circuiting pieces, S_1 and S_2, in Fig. 127. Move the sliding key, K, until balance is obtained, at some point, P say, and let the resistances of the two portions of the whole wire, $w\,w$, on the two sides of the point P be l and m ohms respectively, then

$$\frac{a}{b} = \frac{x+l}{y+m}.$$

Next interchange the coils having resistances x and y ohms, but not the other pair, and move the sliding key, K, to a new point, P′ say, at which balance is obtained, and let the resistances of the two portions into which the wire is divided be now l' and m' ohms, then

$$\frac{a}{b} = \frac{y+l'}{x+m'},$$

$$\therefore \frac{x+l}{y+m} = \frac{y+l'}{x+m'};$$

$$\text{hence, } \frac{x-y+l-m}{x+y+l+m} = \frac{y-x+l'-m'}{x+y+l'+m'}.$$

$$\text{Now} \quad l+m = l'+m',$$

since each expression is the resistance of the whole wire, therefore also

$$l-l' = m'-m.$$

But each of the latter expressions is the resistance of the wire between the points P and P′, at which the sliding key was placed in the two tests to obtain balance; consequently,

$$2 \times \text{ resistance of wire } \mathrm{P\,P'} = 2\,(x - y),$$
$$\text{or,} \quad \text{resistance of wire } \mathrm{P\,P'} = x - y.$$

Next, vary slightly the value of a, or of b, by shunting, for example, the coil marked "unknown," or the coil marked "known," in Fig. 127, and repeat the preceding. Then two new points Q and Q′ in the wire will be found such that

$$\text{resistance of wire } \mathrm{Q\,Q'} = x - y.$$

Consequently,

$$\text{resistance of wire } \mathrm{Q\,Q'} = \text{resistance of wire } \mathrm{P\,P'}.$$

In this way, then, the wire may be subdivided into any desired number of portions, all having exactly the same resistance—viz. that equal to the difference between the resistances of the coils which are interchanged in the test.

The resistances of the metal strips connecting the parts of the bridge together, as well as the resistances of the contacts at the ends of the coils, and at the ends of the wire, $w\,w$, are included in the resistances, a, b, x, y, l, m, l', m'. But the actual values of these contact resistances do not affect the answer, since this only involves the resistance of a piece of the wire $w\,w$ and the difference between the resistances of two coils. It is very important, however, that no change occurs in any one of the contact resistances while the tests are being made; for if such a variation occurred comparable in magnitude with the values of the resistances of the coils, &c., some one or more of the equations given above would be incorrect, and, therefore, we could not conclude that the resistance of the piece of the wire between the points Q and Q′ was equal to that of the piece between the points P and P′.

It would not, therefore, be a good plan to attach the ends of the interchangeable coils to the rest of the bridge, by means of the screws and nuts that are used to hold the short-circuiting pieces, s_1, s_2, in Fig. 127; for such a mode of attachment might introduce an alteration in the contact resistances when the coils were interchanged in position. A better plan would be to employ four metal mercury cups, each being fitted with a lug suitable for being held by one of the screws and nuts, and to construct the terminals of the movable coils in the form of thick rods, like the terminal rods, W, W, of the standard coil (Fig. 134, page 276). Then, if the mercury cups, as well as the ends, E, E, of the rods, were well amalgamated, and if the ends not merely dipped into the mercury in the cups, but also pressed on the metallic bottoms of the cups, the contact resistances would be so small that any small percentage change in them would be quite negligible.

In the preceding the wire to be calibrated is supposed to form part of a Wheatstone's bridge; but if that is not the case—if, for example, the stretched wire belongs to a potentiometer, as in Fig. 216, page 511—then it must be joined up to a Wheatstone's bridge so as to take the place of the bridge wire. But this can be done with two pieces of connecting wire attached to the binding screws at the points J and K of the potentiometer, and to binding screws at the ends of the bridge, since the resistance of such connecting wires, *if constant*, will not affect the calibration of the potentiometer wire with Foster's method.

With such a combination, then, there is no necessity to spread out the metal strips in line as shown at the back of Fig. 127, page 248, and the arrangement can be made much more compact by fixing five metal bars *ab*, *cde*, *fjk*, *lmn*, and *pq side by side*. Binding screws are soldered to the strips at the points *a*, *d*, *j*, *m*, and *q*, the binding screws at *a* and *q* being for the purpose of connecting the bridge arrangement with the potentiometer, at *d* and *m* for the battery, and at *j* for the

wire coming from one end of the galvanometer. At the points b, c, e, f, k, l, n, and p mercury cups are soldered to the metal strips at such distances apart that the cups are suitable for receiving the ends E, E, of the terminal rods W, W, of the four resistance coils respectively which are used with the Foster's method, and which may all be constructed as shown in Fig. 134, page 276.

156. Potentiometer Method of Graduating a Voltmeter in terms of the E.M.F. of a Clark's Cell.—Let

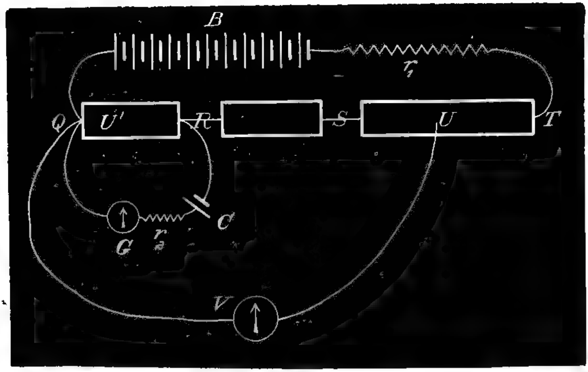


Fig. 217.—Potentiometer Method of Graduating a Voltmeter with a Clark's Cell.

Q R (Fig. 217) be a resistance box with which any resistance can be employed up to, say, 150 ohms subdivided to 0·1 ohm; and let R S be a second box with which any resistance can be employed up to, say, 60 ohms subdivided to 0·1 ohm; and let S T be a third box containing coils of equal resistance, say, of 100 ohms each. As a shunt to the resistance box Q R, place the Clark's cell C in series with the galvanometer G and any suitable resistance r_2; then if the E.M.F. of the cell be, say, 1·442 volt at the temperature of the experiment, unplug 144·2 ohms in box Q R and 200 - 144·2, or 55·8 ohms, in box R S. Next

adjust the number of cells in the battery B, or the resistance r_1, so that no current passes through the galvanometer G; then the P.D. between the points Q and S will be exactly 2 volts, and the P.D. between the terminals of any one of the 100-ohm coils in the box, S T, will be exactly 1 volt. Hence, P.Ds. increasing by exactly 1 volt can be applied to the voltmeter, and the corresponding deflections which they produce on the instrument noted and recorded. For example, suppose the terminals of the voltmeter V be joined to the points Q and U, where S U contains 9 of the 100-ohm coils, then the P.D. between the voltmeter terminals will be exactly 11 volts.

And it is to be noticed that the preceding is true even if the voltmeter, V, allows a considerable current to pass through it, for, although in that case the current through the resistance U T will be greater than that through the resistances Q R, R S, and S U, the current through each of the latter will be the same. Hence, no matter what current flows through the voltmeter, V, the P.D. between the points Q and U will bear to the P.D. between the points Q and R the ratio that the resistance Q U bears to the resistance Q R. In fact, the only effect produced by the resistance of the voltmeter not being very large compared with that of the part of the potentiometer it is placed across, will be that, as one of the voltmeter terminals is shifted along the resistance box, it will be necessary to alter the resistance r_1 in order to keep the galvanometer current zero. Whereas, if the resistance of the voltmeter be high compared with that of Q T, the point of contact U may be shifted between the points R and T without disturbing the balance.

Further, if the resistance of the voltmeter be so large that the current passing through it is inappreciable compared with that flowing through the potentiometer itself, the point of contact U may be placed actually between the points Q and R without affecting the accuracy of the test. For example, the terminals of such a high resistance voltmeter may be joined with the points Q and U′,

and, if the resistance of Q U′ be 10 ohms, for example, the P.D. maintained between the voltmeter terminals will be 0·1 volt.

From the preceding it would appear that the calibration of a voltmeter would be facilitated by using smaller resistances throughout in the potentiometer—for example, by making each of the coils in the box S T of 10 ohms instead of 100 ohms, and by unplugging resistances of 14·42 and 5·58 ohms in the boxes Q R and R S instead of 144·2 and 55·8 ohms respectively. The employment of such smaller resistances would, however, be attended with the disadvantage that for the same P.D. between the points Q and T the potentiometer would have to carry far larger currents, and, therefore, there would be a much greater risk of variation of the resistances through heating; secondly, that, if an accuracy of 1 in 1000—that is, of one-tenth per cent.—were desired, the coils would have to be adjusted to about 0·01 ohm, if the unit 10 ohms were employed instead of that of 100 ohms, and the boxes Q R and R S would have to be subdivided to 0·01 ohm, if the unit 10 ohms were employed; whereas with the unit of 100 ohms an accuracy of one-tenth per cent. can be attained with coils adjusted to 0·1 ohm, and with the boxes Q R and R S subdivided to 0·1 ohm.

If, however, an accuracy of only about 1 per cent. were sufficient, the unit of 10 ohms might be employed, and resistances of 14·4 and 5·6 ohms unplugged in the boxes Q R and R S respectively.

With the following modification of the preceding method, an accuracy of about one-tenth per cent. can be obtained without using coils having a resistance as high as 100 ohms, or as low as 0·01 ohm. The box S T (Fig. 217) is composed of a number of 10-ohm coils, each correct to 0·01 ohm; the boxes Q R and R S contain about 150 ohms and 60 ohms respectively, subdivided to 1 ohm, and correct to 0·01 ohm; and the boxes Q R and R S, instead of being joined directly together as in Fig. 217, are connected by a length of platinum-iridium wire having

1 ohm resistance and divided accurately into 100 parts having equal resistances. If, then, the E.M.F. of the Clark's cell, at the temperature of the experiment, were, say, 1·439 volt, 14 ohms would be employed in the box Q R, 5 ohms in the box R S, and the lead attached to the Clark's cell C would be connected with a point in the platinum-iridium wire distant from the end of the box Q R by 0·39 of the length of this wire; or, more strictly, the point in this platinum-iridium wire which divided it into two parts having resistances of 0·39 ohm and 0·61 ohm respectively would be connected with the Clark's cell.

157. Use of a Clark's Cell and a Known Resistance as a Standard of Current.—In § 53, page 188, we saw that a voltmeter in parallel with a fixed resistance could be used as an ammeter. If, now, the voltmeter V of Fig. 92, page 189, be replaced with a Clark's cell and a galvanometer G, so that the strip X Y of Fig. 92 becomes the resistance L M of Fig. 213, page 508; and, if the current passing through the strip be adjusted until no current passes through the galvanometer G, then we know that the current flowing through the strip, and through any apparatus in series with the strip, equals $\frac{E}{r}$ amperes, where E is the E.M.F. of the Clark's cell in volts, at the temperature of the experiment, and r is the resistance in ohms of the strip between the points X and Y. For example, suppose that the length, width and thickness of the strip have been so selected that the resistance between the points X and Y is 1·434 ohm, and that the strip is composed of manganin or platinoid, so that its resistance is practically constant at all temperatures, and let the current passing through the strip be adjusted so that there is no current through the Clark's cell and galvanometer, then the current flowing through the strip equals

$$1{\cdot}434 \{1 - 0{\cdot}00077 (t - 15^\circ)\} \frac{1}{1{\cdot}434} \text{ amperes.}$$

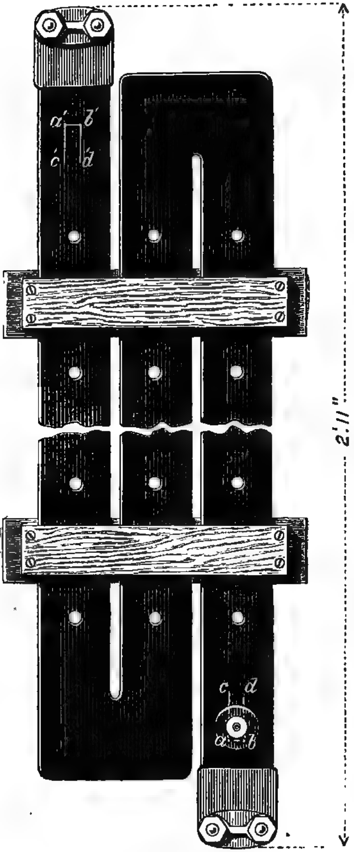


Fig. 218.—Crompton Standard Resistance used in measuring a Current.

Hence, at 15°C. the current flowing through the strip, when the galvanometer deflection is zero, equals 1 ampere.

An ingenious device has been carried out by Mr. R. E. Crompton for making the resistance between two points of such a strip have exactly the desired value. First, two points are sought for in the strip between which the resistance is too large — that is to say, when a given current is sent through the strip the P.D., as measured with a high resistance voltmeter, in volts, is larger than the product of the current in amperes into the required resistance in ohms — then short saw-cuts, *a b*, *a′ b′* (Fig. 218), are made at these points at right angles to the length of the strip, and next saw-cuts *a c*, *b d*, *a′c′*, *b′d′*, parallel to the length of the strip so as to form two little peninsulas *c a b d*, *c′a′b′d′*. As these

little peninsulas only make electric connection with the strip along the lines *c d* and *c′ d′*, the potential of *any* point of either of the peninsulas will be the same as that of the strip at the line of junction. Hence on making the longitudinal saw-cuts *a c*, *b d*, *a′c′*, *b′d′*, longer and longer until, on passing a known steady current through the strip, the P.D. between the little peninsulas *c a b d*, *c′a′b′d′*, equals the product of the current in amperes into the required resistance in ohms, we arrive at two lines *c d*, *c′ d′*, in the strip between which the resistance has exactly the required value.

For the subsequent attachment of the voltmeter wires a small hole is drilled anywhere in each of the peninsular pieces, and the screw of a brass terminal passed through the hole and clamped with a nut on the other side of the strip. To prevent this nut, or the metallic shoulder of the terminal, electrically connecting the peninsula with the strip ebonite washers are inserted, as indicated at the lower right-hand corner of Fig. 218.

158. Calibrating a Galvanometer by using Known Resistances and a Cell of Constant E.M.F.—There are four convenient methods of calibrating a galvanometer by using known resistances. The first consists in employing the arrangement illustrated in Fig. 91, page 186, for calibrating a voltmeter; in the second, the same method is employed, only the ammeter A in Fig. 91 is dispensed with, and the constancy of the current flowing through the circuit $B_1 B_2 B_3 B_4$ is ensured, and, if desired, its value is ascertained by employing a Clark's cell and a galvanometer to test zero current as in Fig. 217, page 516; the third plan is based on employing known resistances and a constant P.D., as in § 95, page 287; while in the fourth, the device giving a constant P.D. is replaced by a cell having an E.M.F. which does not change when the cell is used to produce a current.

Let the value of this constant E.M.F. be E volts, and let *c* ohms be the resistance of the cell; then if $d_1°$, $d_2°$, $d_3°$,

&c., be the deflections on the galvanometer, when r_1, r_2, r_3, &c., ohms are the resistances respectively in the resistance box, R (Fig. 140, page 288), we know that the currents producing these deflections are respectively

$$\frac{E}{c+g+r_1}, \quad \frac{E}{c+g+r_2}, \quad \frac{E}{c+g+r_3}, \text{ \&c., amperes,}$$

so that an absolute calibration curve can be drawn for this galvanometer.

If the E.M.F. of the cell is not known in volts, but if we are sure that it is constant, we can draw the relative calibration curve, although not the absolute one.

In order to see quickly the kind of law connecting deflection and current for any particular galvanometer, it is convenient in making this experiment to unplug resistances in the box R, such that $c + g + r_2$ equals $\frac{1}{2}(c + g + r_1)$, $c + g + r_3$ equals $\frac{1}{3}(c + g + r_1)$, &c., since in that case the second current is double the first, the third thrice the first, &c. Of course, r_1 should be chosen so that the deflection corresponding with this resistance is a conveniently small one, for example, about 10° with an ordinary galvanometer having a scale reading up to 90°.

159. Constant P.D. and Constant E.M.F.—In Chapters IV. and V. a constant P.D. was frequently spoken of, and it was mentioned in § 95, page 287, that a galvanic cell or battery of low internal resistance maintained automatically a constant P.D. between its terminals. This arises from the fact that, if a battery have an E.M.F. of E volts and a resistance of b ohms, and if V be the P.D. in volts between its terminals when the battery is sending a current through an external resistance of x ohms,

$$V = \frac{x}{x+b} E,$$

as shown in § 119, page 364. Consequently, as long as b is small compared with x, V is practically equal to E,

and is, therefore, constant for a cell whose E.M.F. is practically independent of the current flowing.

When, however, b is not negligible compared with x, V will not be constant even although E is. But, as seen from Fig. 173, page 377, the current and the distribution of potential *outside* a battery will not be altered if we assume that the entire E.M.F. of the battery is produced at one point G, and has a value G W, instead of being made up of G H, I J, K L, the E.M.Fs. of the separate cells. So that if G T, which represents b the resistance of the battery, be regarded as part of the external resistances, we may say, as regards all points outside the battery, that the combination is the same as if a cell of infinitely small resistance, and possessing an E.M.F. of E volts, were placed between the terminals of an external circuit having a resistance of $b+x$ ohms. Or, as far as points outside the battery are concerned, the combination acts as if a fixed P.D. of E volts were maintained between the terminals of a resistance of $b+x$ ohms.

Hence, the method of calibrating a galvanometer described in § 95, page 287, is exactly the same as that contained in § 158, page 522, if each resistance $g+r$ in the former is replaced by $c+g+r$ in the latter when c is the resistance of the cell employed. Similarly, all the formulæ connected with the "Increase of the Main Current Produced by Applying a Shunt" in § 101, page 298, which were obtained on the assumption that a constant P.D. of V volts was maintained between the terminals of a circuit consisting of a resistance of m ohms in series with the galvanometer, can be converted into formulæ to be employed with a battery having a constant E.M.F. of E volts, and a constant resistance of b ohms, by replacing the V by E and the m by $b+m$.

Example 149.—If the resistance of a galvanometer be 1,000 ohms, what must be the resistance of a shunt to diminish the current passing through the galvan-

ometer to one-half, first, when the resistance of the battery is 30 ohms and that of the rest of the circuit is 99,970 ohms; secondly, when the battery has the same resistance, but the resistance of the rest of the circuit is only 70 ohms?

Answer.—The formula given on page 299,

$$\frac{G_2}{G_1} = \frac{s\,(m + g)}{m\,(g + s) + g\,s},$$

becomes in this case

$$\frac{1}{2} = \frac{s\,(30 + 99{,}970 + 1{,}000)}{(30 + 99{,}970)\,(1{,}000 + s) + 1{,}000\,s}$$

and $$\frac{1}{2} = \frac{s\,(30 + 70 + 1{,}000)}{(30 + 70)\,(1{,}000 + s) + 1{,}000\,s}$$

in the two divisions of the question respectively. Therefore, the two values of s are respectively 990·1 and 90·9 ohms, the former being a little less than the resistance of the galvanometer, while the latter is not one-tenth as large.

Example 150.—What must be the resistance of a galvanometer relatively to that of the rest of the circuit, including the battery, so that shunting the galvanometer with a quarter of its own resistance may halve the current passing through it?

Answer.—Here the formula of page 399 becomes

$$\frac{1}{2} = \frac{\frac{g}{4}\,(b + m + g)}{(b + m) \times \frac{5}{4}\,g + \frac{g^2}{4}};$$

consequently, $$g = 3\,(b + m).$$

Example 151.—A galvanometer having a resistance of 240 ohms is in series with a coil of resistance 1,427 ohms and a cell of resistance 11 ohms. If the galvanometer be now shunted with a resistance of 165 ohms,

by how much per cent. will the current passing through the galvanometer be diminished?

Answer.—

$$\frac{G_2}{G_1} = \frac{165\,(11 + 1{,}427 + 240)}{(11 + 1{,}427)\,(240 + 165) + 240 \times 165}$$
$$= 0{\cdot}4451\,;$$

therefore, the current through the galvanometer will be diminished by about 55 per cent.

160. Independence of Currents in Parallel Circuits.—In §§ 98, 99, and 101, pages 291 to 302, it was seen that, if a constant P.D. were maintained between the ends of a circuit consisting of a resistance m in series with a group of resistances a, c, d, &c., in parallel with one another (Fig. 142, page 293), the current through any one of these parallel resistances would be independent of the currents flowing through the remainder provided that m was small compared with

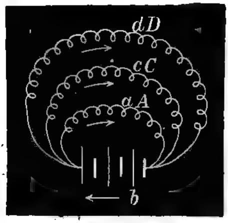


Fig. 210.

$$\frac{1}{\frac{1}{a} + \frac{1}{c} + \frac{1}{d} + \&c.}.$$

Hence, if instead of a constant P.D. being maintained between the ends of a circuit, a group of resistances be joined in parallel with a battery of resistance b (Fig. 219), the current through any one of the resistances will be independent of the current passing through the remainder provided that b be small compared with

$$\frac{1}{\frac{1}{a} + \frac{1}{c} + \frac{1}{d} + \&c.},$$

and that the E.M.F. of the battery is constant. For example, if E be the value in volts of this constant E.M.F., and b be small, the current through any one of the resistances d is practically equal to

$$\frac{E}{d},$$

whatever may be the values of a, c, &c.

In Fig. 149, page 301, it was assumed that the P.D. was kept practically constant at the junction of the street mains S with the house mains H, and it was seen that if the resistance of the house mains H and of the leads l was small compared with the resistance of the glow-lamp filaments, the current through any one lamp was practically unaffected by turning other lamps on or off.

It would, however, be impossible for the Electric Supply Company to keep the pressure absolutely constant at the junction of the street mains S with the house mains H for every house in a district. For that would mean either no current at all flowing through the street mains if the pressure supplied to every house were kept the same—say, at 100 volts—or, if the pressure supplied to different houses had not the same value, although maintained always the same for any particular house—in other words, if the pressure at *any point* in the street mains was kept *constant*, although *not uniform*, throughout the *whole length* of the street mains—this constancy of pressure at the junction of S and H would mean that the current flowing through any one section of the street mains was never allowed to vary, a condition obviously inconsistent with the freedom of each householder to turn on and off lamps in his own house.

The pressure, therefore, between the mains is kept constant at one or more points by means of "*feeders*," or auxiliary mains, which are used not to supply current to houses attached to the feeders, but to convey current to points in the street mains, and thus to raise the

pressure between the street mains at these points. Then, by using very thick copper conductors for the street mains themselves, the pressure at any point in them is prevented from differing much from the value it has where the feeders join them. The street mains S (Fig. 149, page 301), from which the house mains H branch off, may in certain districts be likened to the rim of a wheel, the *feeders* to the spokes, and the hub on which the ends of the spokes are collected to the Electric Light Central Station. Whereas the street mains belonging to other supply companies terminate in the central station, and the feeders are then employed to keep up the electric pressure at the distant parts of the mains.

For some hours during the night, as well as during the day, the demand for artificial light in shops and houses is so small that it is not necessary to run the steam-engines and dynamos when cells are available at the central station for supplying the current required. The cells, however, for the reason already explained, must have a small resistance; and, since the resistance of an accumulator, a secondary cell, or "*storage cell,*" as it is indifferently called, is frequently less that 0·001 ohm, this type of cell is much used in connection with electric lighting.

The consideration of the independence of currents in parallel circuits also explains why Grove's cells, which, as stated in § 135, page 440, have a small resistance compared with that of Daniell's, Minotto's, and other well-known cells, were employed in the early days in telegraph offices, when the *different* messages used to be sent along *several* telegraph wires *with one battery*. The trouble and expense, however, involved in keeping the Grove's cells in order caused the plan of working several telegraph wires with one battery to be abandoned in favour of having a separate battery of much higher resistance to work each line *independently*. But the invention of accumulators by Planté, and the improvements that have been effected in them by Faure, Swan,

Epstein, and others, during the last few years, are leading to a return to the old plan of several telegraph wires being worked with one battery.

Example 152.—How many lamps in parallel, each requiring 80 volts, and 0·6 of an ampere, can be supplied with current from 42 accumulators in series, each having 2 volts E.M.F. on discharging, and 0·0048 ohms' internal resistance?

Let n be the number of lamps; then, since the total current will be $n \times 0{\cdot}6$, we have

$$42 \times 2 - n \times 0{\cdot}6 \times 42 \times 0{\cdot}0048 = 80.$$

Answer.—33.

Example 153.—If the number of accumulators in the last question be increased by one, by how many may the number of lamps be increased?

Answer.—The number of lamps may now be 48·4, that is, may be 49 all a trifle too dull, or 48 somewhat too bright, unless a small resistance be introduced. The addition, therefore, of one accumulator in this case increases the number of lamps that can be supplied with current by one-half.

Example 154.—If 20 of the lamps in Example 152 be turned out, what will be the P.D. between the terminals of the remaining 13?

The resistance of each lamp may be taken approximately as $\frac{80}{0{\cdot}6}$, or 133·3, ohms, and of the 13 lamps as 10·25 ohms, although the resistance of a glow lamp really varies with the current passing through it, and, therefore, with the P.D. maintained between its terminals. Hence the current through this group of lamps will be about $\frac{84}{42 \times 0{\cdot}005 + 10{\cdot}25}$, or 8·03, amperes. Consequently the P.D. between the terminals of the 13 lamps will be about $8{\cdot}03 \times 10{\cdot}25$, or 82·3, volts.

Answer.— 82·3 volts.

This question may also be solved approximately thus :—Let us assume that the current through each of the lamps is exactly 0·6 ampere, even when the P.D. between the terminals of the group is not exactly 80 volts, then the required P.D. equals

$$84 - 13 \times 0{\cdot}6 \times 42 \times 0{\cdot}005, \text{ or } 82{\cdot}36, \text{ volts.}$$

This result is larger than 80 volts, therefore the current through each lamp is larger than 0·6 ampere, and by employing a current of only 0·6 in the negative term in the latter equation we have over-estimated the answer. Hence, the true P.D. is somewhat less than 82·36 volts.

On the other hand, if the answer had come out less than 80 volts, this approximate method of calculating the required P.D. would have given an answer somewhat too small.

Example 155.—If a battery of 53 accumulators in series be used to supply current to glow lamps in parallel, each cell, when discharging, having an E.M.F. of 1·95 volt and a resistance of 0·0054 ohm, how many glow lamps can be turned on if each lamp requires a P.D. of 100 volts between its terminals and a current of 0·4 ampere passing through it? *Answer.*—29 lamps.

Example 156.—If the number of accumulators in the last question be increased by one, by how many may the number of lamps be increased?

Answer.—The number of lamps may now be 45·4; that is, may be 46 all a trifle too dull, or 45 a trifle too bright, unless a small resistance be introduced. The addition, therefore, of one accumulator increases the number of lamps that can be used by about 55 per cent. in this particular case.

Example 157.—If there be 52 accumulators in series, each having an E.M.F. of 1·98 volt, and a resistance of 0·001 ohm, and if 150 lamps in parallel receive current from them, each lamp requiring 100 volts at its terminals, and 0·6 of an ampere passing through it when properly glowing, how much per cent. will the current passing through the lamps be too great or too small?

The resistance of each lamp is approximately $\frac{100}{0{\cdot}6}$, or 166·7 ohms; hence, the resistance of all the lamps will be $\frac{166{\cdot}7}{150}$, or 1·111, ohm; consequently, the current passing through them will be about

$$\frac{52 \times 1{\cdot}98}{52 \times 0{\cdot}001 + 1{\cdot}111}, \text{ or } 88{\cdot}5, \text{ amperes.}$$

The current that ought to pass through the lamps is 150 × 0·6, or 90, amperes. Hence, the current is about 1·7 per cent. too small.

Answer.—About 1·7 per cent. too small.

Example 158.—How many of the 150 lamps in the last question must be turned off so that the current through each of the remainder shall be exactly 0·6 ampere?

When the current through each of these lamps is exactly 0·6 ampere, the P.D. between the lamp terminals is by hypothesis exactly 100 volts. Hence, if n is the number of lamps to be left turned on, so that $150-n$ is the number to be turned off, we have

$$52 \times 1{\cdot}98 - n \times 0{\cdot}6 \times 52 \times 0{\cdot}001 = 100$$
$$\therefore n = 94.$$

Answer.—56 lamps must be turned off.

Example 159.—By how much per cent. is the P.D. maintained between the terminals of 70 glow lamps in parallel too small if a lamp requires a P.D. of 110 volts and a current of 0·3 ampere to glow properly, and 55 accumulators are employed each having an E.M.F. of 2 volts and a resistance of 0·001 ohm?

If V is the P.D. in volts actually maintained between the lamp terminals,

$$\begin{aligned} V &= 110 - 70 \times 0{\cdot}3 \times 55 \times 0{\cdot}001 \text{ approximately,} \\ &= 108{\cdot}95. \end{aligned}$$

Answer.—The P.D. is about 1 per cent. too low.

Example 160.—A battery of 51 accumulators in series, each having an E.M.F. of 2 volts and a resistance of 0·0005 ohm, is joined across the ends of a pair of street mains consisting of two conductors made of copper of 98 per cent. conductivity, each conductor having a cross-section of 0·1 square inch and the temperature being 20°C. At 50 and at 100 yards from the central station there is a house with 100 glow lamps in it, the lamps having a resistance of 180 ohms apiece. If all the lamps in a house are, for simplicity, assumed to be arranged in a group together at the end of a pair of house mains having a joint resistance of 0·02 ohm, calculate first the current that will flow through a lamp in each house when all the lamps are turned on; secondly, the current that will flow through a lamp in the nearer house when 10 lamps only are turned on in this house and no lamps at all turned on in the farther house.

Answer.—The resistance of each 50 yards of going and return street mains equals about

$$\frac{0{\cdot}63 \times 36 \times 100}{10^6 \times 0{\cdot}1} \times \frac{100}{98} \times (1 + 20 \times 0{\cdot}004),$$

or 0·02499, ohm,

and the resistance of each house circuit when all the 100 lamps are turned on equals

$$0{\cdot}02 + \frac{180}{100}, \text{ or } 1{\cdot}82, \text{ ohm.}$$

Therefore, the resistance of the whole circuit when all the lamps are turned on in both houses equals

$$51 \times 0{\cdot}0005 + 0{\cdot}02499 + \frac{1{\cdot}82\,(0{\cdot}02499 + 1{\cdot}82)}{1{\cdot}82 + 0{\cdot}02499 + 1{\cdot}82},$$

or 0·9667, ohm;

so that the current flowing through the battery is

$$\frac{102}{0{\cdot}9667}, \text{ or } 105{\cdot}5, \text{ amperes.}$$

Of this current, $\frac{1\cdot845}{3\cdot665} \times 105\cdot5$, or 53·1, amperes, flows through the lamps in the nearer house, and 52·4 amperes through those in the farther one. Hence, a current of 0·531 ampere flows through each lamp in the nearer house, and 0·524 through each lamp in the farther, the difference being about 1·3 per cent.

If now all the lamps in the farther house be turned off, as well as 90 of those in the nearer one, the current through the remaining 10 will be

$$\frac{102}{51 \times 0\cdot0005 + 0\cdot02499 + 0\cdot02 + \frac{180}{10}},$$

or 56·5, amperes ;

so that the current through each of the 10 lamps in this nearer house is 0·565 ampere, which is about 6·4 per cent. larger than the current passing through these lamps in the previous case.

Example 161.—If three telegraph wires, having resistances of 200, 250, and 300 ohms respectively, including in each case the resistance of the "*receiving instrument*," or the instrument by means of which the messages are received, be worked by *one* battery having a resistance of 20 ohms, by how much per cent. will the current which passes along the first line, when no current is passing along either the second or the third lines, be altered : 1st, by a current being sent along the second also ; 2nd, by a current being sent along both the second and the third lines, in addition to the one sent along the first ?

If E be the E.M.F. of the battery in volts, then the current A_1, flowing along the first line when no current is flowing along either the second or the third, is

$$\frac{E}{20 + 200} \text{ amperes.}$$

If a current is also being sent along the second wire, the total current flowing through the battery is

$$\frac{E}{20 + \dfrac{200 \times 250}{200 + 250}} \text{ amperes,}$$

and of this current A_2, flowing along the first line, is

$$\frac{250}{200 + 250} \times \frac{E}{20 + \dfrac{200 \times 250}{200 + 250}},$$

or
$$\frac{250\ E}{20\ (200 + 250) + 200 \times 250}, \text{ amperes.}$$

Similarly, if a current is also being sent along the third line, the current A_3, flowing along the first line, is

$$\frac{\dfrac{1}{200}}{\dfrac{1}{200} + \dfrac{1}{250} + \dfrac{1}{300}} \times \frac{E}{20 + \dfrac{1}{\dfrac{1}{200} + \dfrac{1}{250} + \dfrac{1}{300}}} \text{ amperes.}$$

Therefore,
$$A_1 = \frac{E}{220} \text{ amperes,}$$
$$A_2 = \frac{E}{236} \quad ,,$$
and
$$A_3 = \frac{E}{249{\cdot}4} \quad ,,$$

Answer.—Hence, the current flowing through the first line will be diminished by about 6·8 per cent. by allowing a current to flow along the second line, and by about 11·7 per cent. by allowing a current to flow along both the second and the third lines.

Example 162.—If two telegraph lines each have a resistance of 500 ohms, including the resistance of the receiving instrument, what may be the greatest resistance of the battery employed to send the current along

both, so that the current flowing along either shall not be diminished by more than 1 per cent. by sending a current also along the other?

Let E be the E.M.F. in volts, and b the resistance of the battery in ohms, then the current flowing along either line, when no current is being sent along the other, is

$$\frac{E}{b + 500} \text{ amperes;}$$

and the current flowing along either line, when a current is also being sent along the other, is

$$\frac{1}{2} \times \frac{E}{b + 250} \text{ amperes.}$$

Now we want b to be of such a value that

$$\frac{E}{b + 500} - \frac{1}{2} \times \frac{E}{b + 250} \text{ is not greater than } \frac{1}{100} \times \frac{E}{b + 500}.$$

Consequently, the largest permissible value of b will be found by making

$$\frac{E}{b + 500} - \frac{1}{2} \times \frac{E}{b + 250} = \frac{1}{100} \times \frac{E}{b + 500},$$

$$\text{or} \quad \frac{99}{100} \times \frac{1}{b + 500} = \frac{1}{2} \times \frac{1}{b + 250}.$$

Answer.—5·1 ohms.

Example 163.—There are two telegraph lines, one having a resistance of 400 ohms, and the other of 500 ohms, including the resistance of the receiving instruments. The receiving instrument on the first line is so arranged that it will work without adjustment, with currents varying between 5 and 5·2 thousandths of an ampere. What must be the E.M.F. and resistance of the common battery for the two lines, so that the current flowing along the first line may be always between these

limits, whether or not a current is being sent along the second line?

If E be the E.M.F. in volts, and b the resistance in ohms, of the battery, the maximum current flowing along the first line will be

$$\frac{E}{b + 400} \text{ amperes,}$$

and the minimum current

$$\frac{500}{400 + 500} \times \frac{E}{b + \dfrac{400 \times 500}{400 + 500}}$$

$$\text{or} \quad \frac{500\,E}{900\,b + 200{,}000}, \text{ amperes.}$$

The first current must not exceed 5·2 thousandths of an ampere, and the second must not be less than 5 thousandths of an ampere. Taking, therefore, the limiting values, we may say that

$$\frac{E}{b + 400} = \frac{52}{10{,}000},$$

$$\text{and } \frac{5\,E}{9\,b + 2{,}000} = \frac{5}{1{,}000}.$$

Solving these two equations for E and b, we find that

$$E = 2{\cdot}19 \text{ volts about,}$$
$$\text{and } b = 21 \text{ ohms} \quad ,,$$

Answer.—About 2·19 volts, and 21 ohms.

In practice, larger E.M.Fs. than this must be used to allow for leakage along the line, in consequence of which only a portion of the current that leaves the sending or signalling end arrives at the receiving end.

Example 164.—If 50 or more glow lamps in parallel, each requiring 0·6 ampere and 100 volts to glow properly, be supplied with current from 52 accumulators in series, each having an E.M.F. of 1·98 volt when discharging,

what must be the resistance of each accumulator, and what is the maximum number of lamps that can be lighted, so that the P.D. between their terminals never exceeds 102, and is never less than 98 volts?

The resistance of each lamp may be taken as $\frac{100}{0{\cdot}6}$, or 166·7, ohms. Hence, considering the case of the least number of lamps, 50, which will correspond with the highest number of volts, 102, we have, if a be the resistance of one accumulator in ohms,

$$52 \times 1{\cdot}98 - \frac{102}{\frac{166{\cdot}7}{50}} \times 52 \times a = 102$$

$$\therefore a = 0{\cdot}0006034.$$

Answer.—0·0006034 ohm.

Next, let n be the number of lamps which reduces the P.D. between their terminals to 98 volts, then

$$52 \times 1{\cdot}98 - \frac{102}{\frac{166{\cdot}7}{n}} \times 52 \times 0{\cdot}0006034 = 98$$

$$\therefore n = 258{\cdot}3$$

Answer.—258 lamps.

161. Arrangements of Cells.—A battery may be formed of cells joined up in a variety of ways. All the cells may be "in series," as in Fig. 220, or they may be joined up all "in parallel," as in Fig. 221, or "partly in series and partly in parallel," as in Fig. 222. These three arrangements are symbolically shown in A, B, C (Fig. 223), where the long thin lines stand for the plates in the battery from which the positive electricity flows; or, with the definition of direction of current we have adopted, the current flows

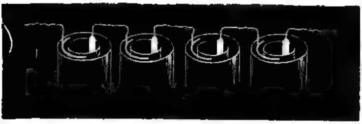


Fig. 220.—Four Cells joined in Series.

in the circuit outside the battery from the plate represented by the long thin line to that represented by the short thick line, while in the battery itself the current flows from the short thick line to the long thin one. In the Daniell's cell, then, the thin lines stand for the copper plates, and in the Grove's for the platinum plates, while in the Bunsen's, Potassium Bichromate, and Leclanché cells the thin lines stand for the carbon plates. Also in all these cells the thick lines indicate the zinc plates.

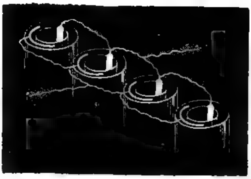


Fig. 221.—Four Cells joined in Parallel.

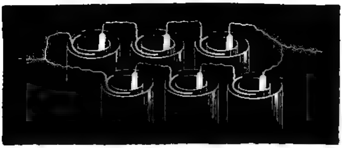


Fig. 222.—Six Cells joined Three in Series and Two in Parallel.

When all the cells are in series, the total current produced by the battery passes through each cell; therefore, as explained in § 118, page 361, the E.M.F. of the battery is equal to the sum of the E.M.Fs. of each of the cells. If, on the other hand, the cells are joined up all in

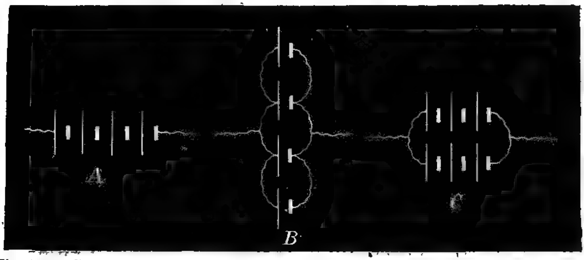


Fig. 223.—Symbolical Representation of A, Four Cells in Series; B, Four Cells in Parallel; C, Three Cells in Series and Two in Parallel.

parallel, the current divides itself between the cells; and if the cells are all made with the same materials, but not necessarily of the same size or of the same internal resistance, the total chemical action, and therefore the total amount of fuel burnt per second, is exactly the same as if the entire current went through one of the cells. Hence, the E.M.F. of the battery is simply that of any one of the component cells. The resistance, however, of the battery will be less than that of one cell, as the road for the current through the battery is made wider by putting cells in parallel; and if the cells have each the same resistance of c ohms, and if there be p of them in parallel, the resistance of the battery is $\frac{c}{p}$ ohms. If the cells be partly in series and partly in parallel, we must combine the last two sets of conclusions, so that if the E.M.F. of each cell be e volts, and if there be s cells in series, and p in parallel, the total E.M.F. of the battery E, and the total resistance b will be given by

$$E = s\,e \text{ volts,}$$

$$b = \frac{s\,c}{p} \text{ ohms;}$$

so that, if A be the current in amperes which the battery sends through an external resistance x,

$$A = \frac{s\,e}{x + \frac{s\,c}{p}}.$$

In order to experimentally test the accuracy of these results, a number of similar cells, freshly put together, and having their corresponding plates of the same size, the plates in the different cells at the same distance apart, and the amount of liquid in each cell the same, should be joined up in a variety of ways, and the resistances of the combinations measured, as well as the E.M.Fs. of the batteries compared with the E.M.F. of

a single cell, by one or other of the methods of testing previously given. The cells should be of such a type that the E.M.F. of each cell is a constant, a condition very satisfactorily fulfilled with Daniell's cells; and to avoid the cell used as the standard having a higher, or a lower, E.M.F. than the average E.M.F. of the cells employed, different cells may be selected from the combination to serve as the standard cell in the different experiments.

162. Mercury Switch-board for Batteries.—In Fig. 85, page 170, a mercury switch-board is shown in use

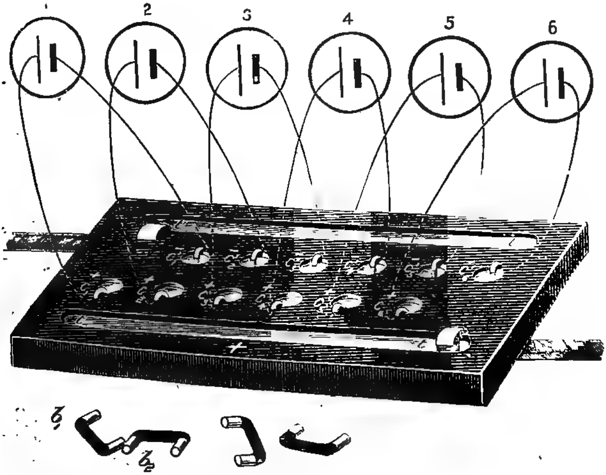


Fig. 224.—Mercury Switch-board for Altering the Arrangements of a Group of Cells.

with a battery of cells, and by means of this device the arrangement of a group of cells can be quickly varied. A slab of ebonite, or of paraffined wood (Fig. 224), has two troughs $t\ t$, $t't'$, and two sets of shallow cups c_1, c_2, c_3, c_4, &c., c_1', c_2', c_3', c_4', &c., made in it, the

distances between c_1 and c_2, c_2 and c_3, &c., being equal to one another, and also to the distances between c_2 and c_1', c_3 and c_2', &c., but less than the distance between diagonal holes c_1 and c_1', c_2 and c_2'. The positive and negative terminals of the first cell are joined to the cups c_1 and c_1' respectively, of the second cell to c_2 and c_2', &c., by means of wires which are passed up through the slab and have their ends bent over into the cups, as seen in Fig. 224. In this figure 6 cells are indicated symbolically, and wires connecting these cells with the mercury cups are shown coming into the cups from above. In reality, however, as just stated, the wires are led under the slab, and come up through holes in it, made near the sides of the mercury cups, and the wires, used to connect the switch-board with the rest of the apparatus in circuit, have their ends connected in a similar way with the troughs $t\,t$ and $t't'$ respectively. The troughs and cups are now filled with mercury.

It might appear that a simpler plan of connecting the ends of the various wires with their respective mercury cups would be to bring the ends of the wires up through holes in the bottoms of the cups; but this method has the objection that it is very difficult to close up such holes through which the wires pass in such a way as to prevent the mercury leaking out between the edge of the wire and the edge of the hole in consequence of the slow amalgamation that occurs with the copper wire.

Bridge-pieces b_1, b_2, &c., bent out of stiff copper wire are made of such a length that any one of them can join two adjacent mercury cups such as c_2 and c_3, or two mercury cups immediately opposite one another, as c_3 and c_2', but cannot be made to connect two diagonal mercury cups such as c_1 and c_1'. Hence, by connecting the terminals of a cell, *not* to *opposite* but to *diagonal* holes, in accordance with the plan adopted with all the mercury switch-boards at the Central Technical College, a cell cannot be short-circuited, no matter how the bridge-pieces be inserted in the mercury cups. The

middles of these bridge-pieces may, for convenience of handling, be covered with insulating material.

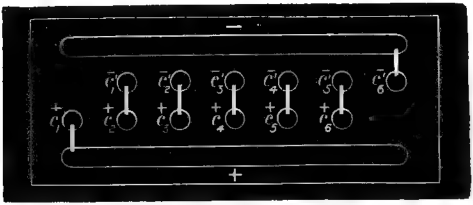


Fig. 224*a*.—Bridge-Pieces inserted so as to join all the Cells in Series.

To join all the cells in series the bridge-pieces are placed as in Fig. 224*a*, while to join all the cells in

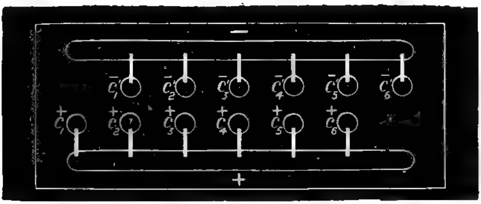


Fig. 224*b*.—Bridge-Pieces inserted so as to join all the Cells in Parallel.

parallel the connectors are inserted as in Fig. 224*b*; next, if they are placed as in Fig. 224*c*, the cells will be

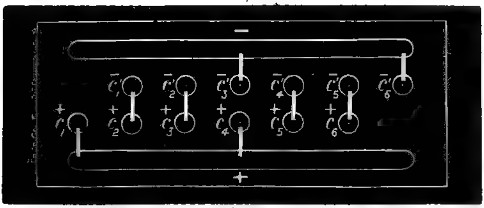


Fig. 224*c*.—Bridge-Pieces inserted so as to join Three Cells in Series and Two in Parallel.

joined up 3 in series and 2 in parallel; and lastly, with the arrangement indicated in Fig 224*d* there are 2 cells in series and 3 in parallel, &c.

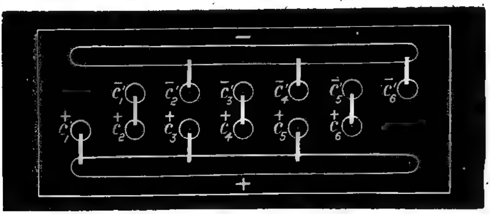


Fig. 224*d*.—Bridge-Pieces inserted so as to join Two Cells in Series and Three in Parallel.

Example 165.—40 exactly similar cells, each having an internal resistance of $\frac{3}{4}$ ohm, when joined in series send a current of 0·5 ampere through a glow lamp of 80 ohms' resistance: how many cells in series would be required to produce the same current through each of 2 such lamps arranged in parallel?

Let e be the E.M.F. of 1 cell in volts, then

$$\frac{40 \times e}{80 + 40 \times 0{\cdot}75} = \tfrac{1}{2},$$

$$\text{or} \quad e = 1{\cdot}375 \text{ volt};$$

therefore, since the resistance of the 2 lamps in parallel will be $\frac{80}{2}$ ohms, and they will require together 1 ampere, we have, if n be the required number of cells,

$$\frac{n \times 1{\cdot}375}{\frac{80}{2} + n \times 0{\cdot}75} = 1;$$

$$\therefore \quad n = 64.$$

Answer.—64 cells.

Example 166.—Find the current that 12 Daniell's cells, each having a resistance of 0·6 ohm and an E.M.F. of 1·1 volt, can send through an external resistance of 5 ohms if the cells be placed four in series and 3 in parallel.

$$A = \frac{4 \times 1{\cdot}1}{5 + \frac{4 \times 0{\cdot}6}{3}}.$$

Answer.—0·76 ampere.

Example 167.—How many such Daniell's cells must be used in series to send a current of 1 ampere through an external resistance of 8 ohms, if one line of cells in series only be employed?

Let n be the required number of cells, then

$$1 = \frac{n \times 1{\cdot}1}{8 + n \times 0{\cdot}6};$$

$$\therefore \quad n = 16.$$

Answer.—16 cells.

Example 168.—If in the last question the current be 2 amperes instead of 1, then how many cells will be required?

$$2 = \frac{n \times 1{\cdot}1}{8 + n \times 0{\cdot}6};$$

$$\therefore \quad n = -160.$$

Therefore, no number of such cells put in one line in series could send this current. In fact, if one cell be short-circuited with a piece of thick wire, the current it will send will be $\frac{1{\cdot}1}{0{\cdot}6}$, or 1·83, ampere, and this is the maximum current that one, or any number, of the cells, arranged simply in series, can send. For if there be n of them arranged in series, and the whole be short-circuited, the current will be $\frac{n \times 1{\cdot}1}{n \times 0{\cdot}6}$, or 1·83 ampere, that is, simply the current sent by one cell when short-circuited.

Hence, if there be any external resistance, the current sent by one row of these cells in series, no matter how many there may be in the row, will be less than 1·83 ampere.

But by arranging two rows of such Daniell's cells in parallel it is possible to send a current of 2 amperes through an external resistance of 8 ohms. Let s be the number of cells required to be placed in series in each row, then

$$2 = \frac{s \times 1{\cdot}1}{8 + \frac{s \times 0{\cdot}6}{2}},$$

$$\text{or} \quad s = 32\,;$$

and as there are two such rows, the total number of cells required is 64.

Answer.—64 cells.

Example 169.—A battery consisting of 4 Daniell's cells in series and 2 in parallel is employed in sending a current through a simple conductor having a resistance of 2 ohms. If the E.M.F. of each cell be 1·07 volt and the resistance 0·8 ohm, how many watts are developed by the battery, how many are employed in heating the external resistance, and how many are wasted in heating the battery?

Answer.—The current produced $= \dfrac{4 \times 1{\cdot}07}{2 + \frac{4 \times 0{\cdot}8}{2}}$

$= 1{\cdot}189$ ampere;

the power developed by the battery $= 1{\cdot}189 \times 4{\cdot}28$
$= 5{\cdot}089$ watts;

$$\left.\begin{array}{l}\text{the watts employed in heating the}\\ \text{external resistance}\end{array}\right\} = \frac{2}{3{\cdot}6} \times 5{\cdot}089$$
$$= 2{\cdot}827 \text{ watts};$$

$$\left.\begin{array}{l}\text{the watts employed in heating the}\\ \text{battery}\end{array}\right\} = 5{\cdot}089 - 2{\cdot}827$$
$$= 2{\cdot}262 \text{ watts.}$$

163. Arrangement of a Given Number of Cells to produce the Maximum Current through a given External Resistance.—If n be the total number of cells employed in a battery, p being arranged in parallel, and s in series,

$$n = p\,s,$$

and the formula in § 161, page 538, may be written

$$A = \frac{s\,e}{x + \dfrac{s^2\,c}{n}}.$$

If, therefore, we desire to ascertain what arrangement of a definite number of cells, each having a *fixed* E.M.F. of e volts, and a *fixed* internal resistance of c ohms, will send the greatest current through a *fixed external resistance of x ohms*, we must ascertain what value of s will make the last expression a maximum.

This problem may be easily solved by the graphical method that was employed in § 124, page 386; for give numerical values to e, x, and $\frac{c}{n}$, let them, for example, be 2, 3, and 4 respectively, then the expression becomes

$$\frac{2\,s}{3 + 4s^2};$$

next draw the curve (Fig. 225) having the values of s for the abscissæ, and the corresponding value of the expression for the ordinates, and we see at once that the value of s which makes A a maximum is about 0·87. But this is approximately the value of s which makes

$$\frac{s^2\,c}{n} = x,$$

or, in other words, *the proper arrangement of a given number of cells to send the maximum current through a given external resistance is that which makes the resistance of the battery equal to the external resistance.*

S

Whether s is made greater or less than about 0·87, the value of A is less than the maximum, but, on examination of the curve in Fig. 225, it is seen that the value

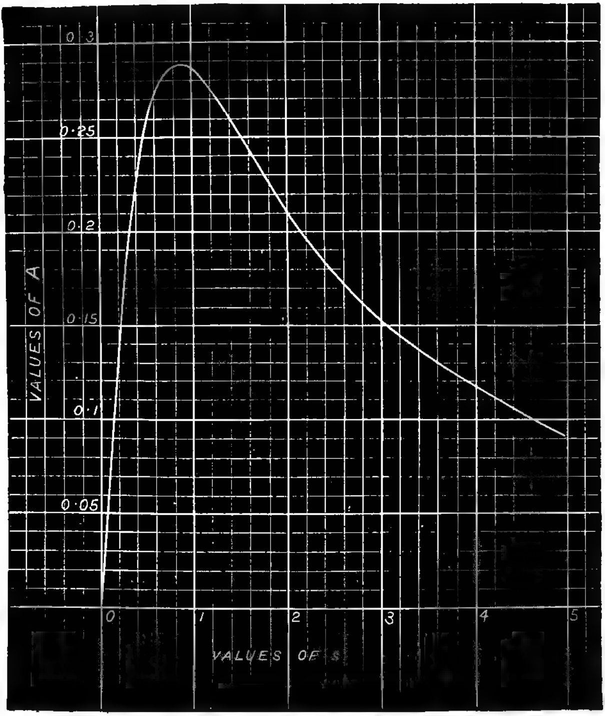


Fig. 225.—Curve connecting the Current and the number of Cells in Series when the total number of Cells and the External Resistance are Fixed.

of A falls off much less rapidly as s is *increased* than when it is *diminished*. This tells us that the current will be not so much lessened by making s too large as it will

be by making it too small; hence, if the number of cells and the resistance of each are such that it is impossible to arrange the battery so that its internal resistance is equal to the fixed external resistance, it is better, when the external resistance is midway between the resistances which the battery has when arranged in these two ways, to select the arrangement that puts rather too many cells *in series* than the one that puts rather too many *in parallel.* For example, suppose we have 12 cells, each having a resistance of 3 ohms, and we desire to arrange them so that they may send a maximum current through an external resistance of $3\frac{1}{8}$ ohms, then if we arrange them 3 in series and 4 in parallel, the resistance of the battery will be

$$\frac{3 \times 3}{4} \text{ or } 2\tfrac{1}{4} \text{ ohms};$$

whereas, on the other hand, if we put them 4 in series and 3 in parallel, the resistance will be

$$\frac{4 \times 3}{3} \text{ or } 4 \text{ ohms};$$

and the given external resistance of $3\frac{1}{8}$ ohms is exactly half-way between $2\frac{1}{4}$ and 4. Let us consider the currents produced by these two arrangements of the cells. With the first,

$$A = \frac{3\,e}{3\frac{1}{8} + 2\frac{1}{4}} \text{ amperes,}$$

if e be the E.M.F. of each cell in volts, while with the second arrangement,

$$A = \frac{4\,e}{3\frac{1}{8} + 4} \text{ amperes.}$$

The first reduces to $\dfrac{24}{43}e$ and the second to $\dfrac{32}{57}e$ ampere, and of these the second is the greater by $\dfrac{8}{2451}e$ of an ampere.

Since the power in watts furnished to a circuit of given resistance equals the square of the current into the resistance, it follows that the arrangement of a given number of cells that sends the greatest current through a fixed external resistance is also the one that supplies the greatest amount of power to it. Hence, *the arrangement of a given number of cells that gives the largest amount of power to a fixed external circuit is the one that makes the resistance of the battery equal to that of the external circuit, and, therefore, is the one that causes half the power developed by the battery to be wasted in heating the battery.*

Comparing the preceding result with that obtained in § 124, page 389, we see that wasting half the power developed—*i.e.* working with 50 per cent. efficiency, or using a P.D. between the battery terminals, which equals half its E.M.F.—is the condition of maximum power developed in an external circuit, either when the generator is fixed and the resistance of the external circuit is the variable, or when the external circuit is fixed and the arrangement of a given number of cells of a fixed type is the variable.

Example 170.—What arrangement of 24 cells, each having a resistance of 0·47 ohm, will send the maximum current through an external resistance of 1·2 ohm?

$$\text{We have} \quad s\,p = 24$$
$$\text{and} \quad c = 0{\cdot}47,$$

also, when the cells are arranged to send the greatest current through the external circuit,

$$\frac{s\,c}{p} = 1{\cdot}2,$$
$$\text{hence} \quad s = 7{\cdot}83,$$
$$\text{and} \quad p = 3{\cdot}06.$$

Answer.—8 cells should be placed in series and 3 in parallel.

Example 171.—What is the maximum current that can be sent by 100 cells, each of 1·4 volt E.M.F. and 3 ohms' resistance, through an external resistance of 20 ohms?

Answer.—0·904 ampere.

Example 172.—What is the maximum current that can be sent by 80 such cells through the same resistance?

Answer.—0·800 ampere.

Example 173.—Would it be possible to arrange 48 Grove's cells, each having an E.M.F. of 1·87 volt, and a resistance of 0·14 ohm, so as to develop ½ a horse-power in an external resistance of 0·1 ohm?

$$\text{We have} \qquad s\,p = 48,$$
$$\text{and} \qquad c = 0{\cdot}14;$$

also, when the cells are arranged to give the greatest power to the external circuit,

$$\frac{s\,c}{p} = 0{\cdot}1,$$
$$\therefore \qquad s = 6$$
$$\text{and} \qquad p = 8.$$

With this arrangement of cells the current will be 54·7 amperes; consequently, the power developed in the external circuit will be about 299 watts, which is about 0·4 of a horse-power.

Answer.—It is not possible to develop ½ a horse-power in the external circuit in question with any arrangement of the particular cells; but if they be placed 6 in series and 8 in parallel, the power given to the external circuit will be about 0·4 of a horse.

164. Minimum Number of Cells Required to Produce a Given Current and P.D.—In this case the type of cell to be used is fixed, but not the number, and it is desired to find what arrangement of cells will necessitate the use of the smallest number to produce a given P.D. of, say, V volts between the battery terminals when a given current of, say, A amperes is flowing.

This is obviously the same problem as finding the smallest number of cells of a given type that will send a given current of A amperes through a fixed resistance of $\frac{V}{A}$ ohms; and a consideration of the result arrived at in the last section leads us to conclude that the least number of cells required in the present instance must fulfil the following condition—viz. that when the cells are joined up so that the resistance of the battery is equal to $\frac{V}{A}$ ohms the current produced through an external circuit having the same resistance must equal A amperes. For if the required current were produced with any other arrangement of cells, a current greater than was required could be produced by arranging the same number of cells in the manner just described, and, therefore, we should not be using the smallest number of cells.

Consequently, *the least number of cells of a given type required to produce a P.D. of V volts when a current of A amperes is flowing can be found thus:—*

(1) *Arrange a sufficient number of the cells in series to make the E.M.F. of the battery equal to 2 V;*

(2) *Then add a sufficient number of rows of cells in parallel to make the resistance of the battery equal to* $\frac{V}{A}$.

The first of the preceding rules must, however, only be used when the second can also be applied. For example, suppose it is desired to arrange Grove's cells, each having an E.M.F. of 1·85 volt and a resistance of 0·1 ohm, so that the P.D. between the battery terminals shall be 50 volts when a current of 2 amperes is flowing. The first rule would tell us to put $\frac{2 \times 50}{1{\cdot}85}$, or about 54 cells, in series. But it is clearly unnecessary to use as

many as 54 Grove's cells to produce a P.D. of 50 volts when a current of 2 amperes is flowing, seeing that s, the number in series required, as given by the equation (page 364)

$$s \times 1{\cdot}85 - 2 \times s \times 0{\cdot}1 = 50,$$
$$\text{is} \quad s = 30{\cdot}4.$$

Hence, one row of 31 cells in series is all that is necessary. The second rule, however, tells us that when 54 cells are used in series, p, the number of rows to use in parallel, can be found thus :—

$$\frac{54 \times 0{\cdot}1}{p} = \frac{50}{2},$$
$$\therefore \quad p = 0{\cdot}216.$$

Hence, 54 cells should be used in series and 0·216 of a cell in parallel, in order that the total number may be a minimum. It is, however, of course impossible to use a fraction of a cell in parallel, so that in this particular case the two rules cannot help us to determine the minimum number of cells necessary. And this must always be the case as long as the number of cells in series given by the first rule, viz. $\frac{2\,V}{e}$, multiplied by c, the resistance of a cell, is less than the external resistance $\frac{V}{A}$; *i.e.* as long as

$$\frac{2\,V}{e} \times c < \frac{V}{A}.$$

Consequently, as long as

$$\frac{e}{2\,c} > A,$$

that is, as long as half the current a cell can produce on short circuit (assuming that neither its E.M.F. nor resistance are altered by the short-circuiting) is greater

than the current the battery is required to produce, all the cells employed must simply be joined up in series.

The rules given on page 550 state that the least number of cells will be employed when

$$s\,e = 2\,\mathrm{V}$$

$$\text{and} \quad \frac{s\,c}{p} = \frac{\mathrm{V}}{\mathrm{A}};$$

$$\textit{i.e.}\text{ when} \quad s\,e = 2\,\mathrm{V}$$

$$\text{and} \quad p\,e = 2\,\mathrm{A}\,c,$$

$$\text{or when} \quad s\,e = 2\,\mathrm{V}$$

$$\text{and} \quad \frac{e}{2\,c} = \frac{\mathrm{A}}{p}.$$

Hence, *the least number of cells of a given type will be required to produce a P.D. of V volts, when a current of A amperes is flowing, by arranging*—

(1) *Sufficient cells in series to make the E.M.F. of the battery equal to 2 V;*

(2) *Sufficient rows in parallel to make the current passing through each cell equal to half the current it would produce on short circuit, supposing that neither its E.M.F. nor resistance were altered by the short-circuiting.*

Example 174.—It is required, by means of cells each having an E.M.F. of 1·8 volt and a resistance of 0·3 ohm, to maintain a terminal P.D. of 12 volts when a current of 8 amperes is flowing. What is the least number of cells that must be used, and how should they be arranged?

$$s = \frac{2 \times 12}{1\cdot 8} = 13\cdot 3,$$

$$\text{and } p = \frac{13\cdot 3 \times 0\cdot 3}{\frac{12}{8}} = 2\cdot 67.$$

Take, therefore, p equal to 3, and recalculate s from the equation

$$s\,e - \frac{A\,s\,c}{p} = 12,$$

which gives $$s = 12.$$

Answer.—36 cells, 12 in series and 3 in parallel.

Example 175.—What is the least number of Daniell's cells, each having an E.M.F. of 1·1 volt and a resistance of 0·5 ohm, that will send a current of 4 amperes through a resistance of 1 ohm?

Answer.—27 cells, arranged 9 in series and 3 in parallel, will send a current of 3·96 amperes through the external circuit; while 28 cells, arranged 7 in series and 4 in parallel, will send a current of 4·1 amperes through the external circuit.

165. Minimum Number of Cells required to give a Fixed amount of Power to a given External Circuit.—Since both V and A were fixed in the problem dealt with in the last section, the power received by the external circuit was fixed. Further, when the *least* number of cells of a particular type is arranged to give a definite amount of power to a circuit, each cell must be contributing a larger share of the power than if more were employed. Hence, just as we saw at the end of § 163, page 548, that a fixed number of cells gave the maximum power to an external circuit when the cells were so arranged that half the power developed by the battery was wasted in heating itself, so now we see that *when a battery composed of a given type of cell is arranged to give a fixed amount of power to a definite external circuit, the least number of cells will be used, and each cell will, therefore, be contributing the greatest share of the power received by the external circuit when half the power produced in each cell is wasted in heating itself.*

The preceding rule really follows at once from the conclusion given in italics at the top of page 389; and

from this it follows that the greatest amount of power each cell in a battery can contribute towards the total power given to any external circuit is one-quarter of the power which a cell could develop if short-circuited. Hence, *the minimum number of cells required to give a fixed amount of power to a given external circuit will be used when the cells are so arranged that each cell develops one-quarter of the power it would on short circuit (supposing that neither its E.M.F. nor resistance were altered by the short-circuiting).*

Example 176.—It is required to expend 30 watts in a resistance of 1·2 ohm by using Grove's cells, having each an E.M.F. of 1·9 volt and a resistance of 0·15 ohm. How many cells are required, and how should they be arranged?

If A is the current in amperes required,

$$1{\cdot}2\ \mathrm{A}^2 = 30$$
$$\therefore \mathrm{A} = 5.$$

Consequently, the P.D. to be maintained between the terminals of the external resistance must equal 5 × 1·2, or 6, volts; therefore, using the rule developed in § 164,

$$s = \frac{2 \times 6}{1{\cdot}9} = 6{\cdot}32,$$

$$\text{and } p = \frac{6{\cdot}32 \times 0{\cdot}15}{1{\cdot}2} = 0{\cdot}79.$$

Now, as already explained in § 164, p coming out less than unity means that all the cells must be arranged in series, and that the total number required is less than that given by the value of s as calculated above. In fact, it is easy to see that 6 cells are sufficient to send a current of 5·4 amperes round the circuit, which is a somewhat larger current than necessary to develop the power of 30 watts.

Answer.—6 cells in series, with a small resistance added.

Example 177.—A circuit is to receive 250 watts at a pressure of 20 volts from cells having an E.M.F. of 1·5 volt each, and a resistance of 0·1 ohm. What is the least number of cells required, and how should they be arranged?

The power developed by one cell, if short-circuited, would be $\frac{1{\cdot}5^2}{0{\cdot}1}$, or 22·5, watts. Hence, when the least number of cells is used, each cell will give $\frac{22{\cdot}5}{4}$, or 5·625, watts to the external circuit; and, therefore, at least $\frac{250}{5{\cdot}625}$, or 44·44 cells, are necessary.

The number that must be placed in series equals $\frac{2 \times 20}{1{\cdot}5}$, or 26·7; practically, then, 23 cells in series and 2 in parallel is what is necessary.

Answer.—46 cells, 23 in series and 2 in parallel.

166. Arrangement of Circuit requiring the Minimum Number of Cells to give a Fixed Amount of Power to the External Portion.—In this case neither the resistance of the external circuit, nor the current required, nor the number of cells to be used, is fixed, but only the type of cell and the power to be given by the battery to the external circuit; and it is desired to ascertain what is the arrangement of the external circuit and of the cells with which the minimum number of cells will have to be used.

From the conclusions arrived at in the last section, it follows that if a power of W watts has to be given to any external circuit by cells having each an E.M.F. of e volts and a resistance of c ohms, the least number of such cells that can possibly be used is

$$\frac{4\,c\,W}{e^2};$$

further, that to enable the number of cells actually required to be as small as is given by the expression, it is

necessary that we shall be able to arrange the cells and the external circuit so that the resistances of the battery and of the external circuit are equal to one another.

Now, if p be the number of rows of cells in parallel, the minimum number of cells in series will be

$$\frac{1}{p} \times \frac{4\,c\,W}{e^2};$$

hence, any arrangement of the external circuit producing a resistance of x ohms will receive the power of W watts from the *minimum* number of cells, viz.

$$\frac{4\,c\,W}{e^2},$$

provided that

$$x = \frac{1}{p} \times \frac{4\,c\,W}{e^2} \cdot \frac{c}{p}$$

$$= \frac{4\,c^2\,W}{p^2 \, e^2},$$

where *p is a whole number.*

For example, if 40 lamps, each requiring 20 volts and 0·5 ampere to glow properly, are to be used with cells each having an E.M.F. of 2 volts and a resistance of 0·1 ohm, we have

$$\begin{aligned} c &= 0{\cdot}1 \\ W &= 40 \times 20 \times 0{\cdot}5 \\ &= 400\,; \end{aligned}$$

therefore, in order that the minimum number of cells, viz.

$$\frac{4 \times 0{\cdot}1 \times 400}{4}, \text{ or } 40,$$

can be used, the lamps must be arranged so that their resistance equals

$$\frac{4 \times 0{\cdot}1^2 \times 400}{p^2 \times 4},$$

where p is a whole number.

Consequently, taking p equal to 1, 2, 3, &c., the arrangements of the lamps that make the resistance of the group equal to 4, 1, $\frac{4}{9}$, &c., ohms respectively, can all receive the 400 watts from 40 cells. Now, the resistance of one lamp is $\frac{20}{0 \cdot 5}$, or 40, ohms; hence, putting the 40 lamps 2 in series and 20 in parallel will make the group have a resistance of 4 ohms, corresponding with p equal to 1—that is, with all the 40 cells in series. Again, putting all the 40 lamps in parallel will cause the group to have a resistance of 1 ohm, corresponding with p equal to 2—that is, with 20 cells in series and 2 in parallel.

No arrangement of the 40 lamps can have a less resistance than 1 ohm, the resistance of the group when they are all in parallel; hence, in this case there are only two arrangements of the lamps which require the minimum number of cells—40; and all other arrangements of the lamps, such as 4 in series and 10 in parallel, or 10 in series and 4 in parallel, will require a larger number of cells to enable the lamps to receive a power of 400 watts.

The solution may be stated simply thus:—If N be the number of lamps to be lighted, w the power in watts taken by each lamp, and l the resistance of each lamp in ohms when glowing properly, then the number of cells will be the least, and equal to

$$\frac{4\,c\,w}{e^2}\,\mathrm{N},$$

when p' rows of lamps and p rows of cells are placed in parallel, and when p' and p, being whole numbers, satisfy the equation

$$\frac{\mathrm{N}\,l}{p'^2} = \frac{4\,c^2\,\mathrm{N}\,w}{p^2\,e^2};$$

that is, when

$$p' = \frac{e}{2\,c}\sqrt{\frac{l}{w}}\cdot p.$$

Example 178.—18 glow lamps, each requiring 5 volts and 1 ampere to glow properly, are to be used with cells each having an E.M.F. of 2 volts and a resistance of 0·2 ohm. Calculate the minimum number of cells required, and the arrangements of lamps and cells that may be employed with about that number of cells.

Answer.—We have $c = 0{\cdot}2$, $w = 5$, $N = 18$, $e = 2$, and $l = 5$;

$$\text{hence} \qquad \frac{4\,c\,w}{e^2} = 1,$$

$$\text{and} \qquad \frac{4\,c\,w}{e^2}\,18 = 18,$$

or 18 is the smallest number of cells necessary.

$$\text{Also,} \qquad p' = 5\,p\,;$$

and, as there are 18 lamps, p' may have the values 18, 9, 6, 3, 2, and 1; therefore, in order that the minimum number of cells—18—may be used, p should have the values respectively of 3·6, 1·8, 1·2, 0·6, 0·4, and 0·2. The nearest whole numbers to these are 4, 2, and 1; and as we have calculated that 18 is the least number of cells possible, it is clear that the practicable arrangements of lamps and cells requiring a number of cells not differing much from the minimum are :—

Number of Lamps.		Number of Cells.		Total Number of Cells.
In Parallel.	In Series.	In Parallel.	In Series.	
18	1	4	5	20
9	2	2	9	18
6	3	1	19	19

Example 179.—It is desired to expend 100 watts for heating purposes in a coil of wire, the current being supplied with cells having each an E.M.F. of 1·7 volt and a resistance of 0·3 ohm. What is the least number of cells that must be employed, and what are the various resistances that can be given to the coil so that the required amount of power can be developed in it with the least number of cells?

Answer.—The minimum number of cells equals $\frac{4\,c\,\mathrm{W}}{e^2}$, that is, $\frac{4 \times 0{\cdot}3 \times 100}{1{\cdot}7^2}$, or 41·5; so that practically 42 cells must be used. These 42 cells may be arranged either 42, 21, 14, 7, 6, 3, 2, or 1 in parallel, and the corresponding resistances of the coil must equal $\frac{4 \times 0{\cdot}3^2 \times 100}{p^2 \times 1{\cdot}7^2}$, where p has the values just given.

Hence, we have—

Number of Cells in Parallel.	Resistance of Coil in Ohms.
42	0·007065
21	0·02826
14	0·06357
7	0·2543
6	0·3460
3	1·385
2	3·115
1	12·46

167. Modifications introduced in the Previous Results by a Safety Limitation of the Maximum Current a Cell may produce.—The maximum current that can be produced with certain types of cells, such as accumulators, is not limited by the values of the E.M.F. and the resistance per cell, but by the disintegration of the plates which would be caused if the current per

square foot of the plates were allowed to exceed a certain value. Such a limitation of the permissible current density leads to a modification in the solution of the problems considered in the preceding sections when the maximum current which a cell may produce is less than $\frac{e}{2c}$.

For example, the arrangement of a given number of cells n to produce the maximum current through a given external resistance must in such a case be ascertained as follows :—Let a be the largest current that may pass through a cell, then, if A is the current through the given external resistance x, A must not exceed $p\,a$, and the value of p which causes A to have its largest permissible value is given by

$$\frac{s\,e}{x+\frac{s\,c}{p}} = \frac{\frac{n}{p}\,e}{x+\frac{n\,c}{p^2}} = p\,a.$$

$$\therefore p = \sqrt{\frac{n}{x}\left(\frac{e}{a} - c\right)}.$$

Also, to determine the minimum number of cells required to produce a given current of A amperes, and a given terminal P.D. of V volts, when a, the maximum current a cell may produce is less than $\frac{e}{2\,c}$, we have

$$s\,e - \mathrm{A}\frac{s\,c}{p} = \mathrm{V},$$

$$\text{and } p = \frac{\mathrm{A}}{a}$$

$$\therefore s = \frac{\mathrm{V}}{e - c\,a}.$$

Lastly, if with the limiting condition that a cell must not produce a greater current than a amperes, we desire

to arrange the circuit so that a given amount of power, W watts, may be given to the external circuit with the least number of cells, then the conditions connecting a, W, s, p, n, e, and c are

$$s\,p = \frac{W}{a\,e - a^2\,c}$$

$$p^2\,a^2\,x = W.$$

Example 180.—What arrangement of 20 cells, each having an E.M.F. of 1·1 volt, and a resistance of 0·5 ohm, will send the largest current through an external resistance of 4 ohms, if no cell is to produce a larger current than 1 ampere? What is the value of this maximum current?

$$\textit{Answer.—}p = \sqrt{\frac{20}{4}\left(\frac{1{\cdot}1}{1} - 0{\cdot}5\right)} = \sqrt{3}\,;$$

therefore, the cells must be arranged 2 in parallel and 10 in series. The current will be 1·69 ampere.

Example 181.—With the cells referred to in the last question, and with the same condition as to the maximum current a cell may produce, what is the least number of cells that will maintain a P.D. of 10 volts between the terminals of an external circuit when sending a current of 3 amperes through it?

$$\textit{Answer.—}p = 3$$

$$s = \frac{10}{1{\cdot}1 - 0{\cdot}5} = 16{\cdot}\dot{6}\,;$$

therefore, 48 or 51 cells must be employed, the former maintaining a P.D. of rather less than 10 volts, and the latter a P.D. of more than 10 volts, when producing a current of 3 amperes.

Example 182.—It is desired to give a power of 125 watts to an external circuit by means of storage cells, each having an E.M.F. of 1·9 volt and a resistance of 0·01 ohm, on the condition, however, that a cell may not produce a larger current than 10 amperes. What is

the least number of cells required, how should they be arranged, and what should be the resistance of the outside circuit?

Answer.—$s\,p = \dfrac{125}{10 \times 1{\cdot}9 - 100 \times 0{\cdot}01} = 6{\cdot}94$;

therefore, 7 cells must be used, and since this is an odd number the cells must all be placed in series. There will consequently be only one value of x, viz. that given by the equation

$$\frac{7 \times 1{\cdot}9}{7 \times 0{\cdot}01 + x}\, x = 125.$$

Hence, the external circuit must have a resistance of 1·271 ohm.

Example 183.—12 glow lamps, each requiring a P.D. of 20 volts and a current of 3 amperes to glow properly, are to be run with accumulators, each cell having a E.M.F. of 2 volts and a resistance of 0·01 ohm. If a cell may not produce a greater current than 15 amperes, what is the least number of cells necessary, and how should the cells and the glow lamps be arranged in order that the minimum number of cells can be used?

Answer.—$s\,p = \dfrac{12 \times 20 \times 3}{15 \times 2 - 225 \times 0{\cdot}01} = 25{\cdot}94$, so

that 26 cells of the given type is the smallest number that can furnish a power of 720 watts if no cell may produce a current of more than 15 amperes. A larger number than 26, however, will be required unless the cells and lamps allow of such a combination that each cell is producing a current of 15 amperes; unless, in fact, the equation

$$p^2\, a^2\, x = \mathrm{W},$$

where a is 15 and W is 720, leads to, at any rate, one pair of values of p and x, which are possible for the cells and lamps.

Now, with 26 cells p may be 1, 2, 13, or 26, and, since the resistance of each lamp is $\dfrac{20}{3}$, or 6·667, ohms,

and there are 12 of them, x may be 80·00, 20·00, 8·889, 5, 2·222, and 0·5556. The value of $\frac{W}{a^2}$ is 3·2, and it is clear that this value cannot be produced exactly by multiplying the square of any of the values of p by the corresponding value of x. Also values of p and x must not be selected which make p^2x less than 3·2, for that would necessitate the cells producing a current of more than 15 amperes apiece, which is inadmissible.

Hence, we must take values which make the product somewhat greater than 3·2; for example, p equals 1 and x equals 5, the latter being obtained by putting 3 lamps in series and 4 in parallel. And it is easy to show that with this value of x, and the cells in one line in series, 32 cells must be employed.

Note to page 492.—Attempts have been made to solve the problem of economically converting the energy of fuel into electric energy by using thermo-piles. But the action of a thermo-pile depends on an E.M.F. being set up when there is a *difference* of temperature between two junctions. For example, if a bar of antimony have each of its ends joined respectively with those of a bar of bismuth and the junctions be at *different* temperatures, an electric current will flow from antimony to bismuth through the cold junction and from bismuth to antimony through the hot one. Heat will undoubtedly be converted into electric energy at the hot junction, but a nearly equal amount of electric energy will be converted back again into heat at the cold junction unless the temperatures of the junctions differ very much. There will also be a transformation of electric energy into heat in the bars since they possess resistance, and the ordinary heat conduction along them will cause a further transference of heat from the hot to the cold junction. On the whole then, the efficiency of a thermo-pile and electromotor used as a combination for converting the energy of fuel into mechanical energy is much smaller than that of a good steam-engine. (*See* Volume II. for further information regarding the construction and behaviour of thermo-piles.)

APPENDICES.

I.—MAGNETIC LINES OF FORCE DIAGRAMS.

WROUGHT-iron, or steel, filings are the best to employ for preparing lines of force diagrams, cast-iron ones being generally too small and too dirty. The filings should be free from non-magnetic substances, and should be neither too large nor too small. Those that are too large can be separated by sifting ordinary workshop filings through a 30-mesh sieve—that is, one having about thirty meshes to the inch length—and those that are too small, by a second sifting through a 50-mesh sieve.

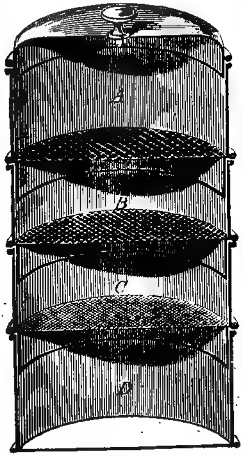


Fig. 226.—Box of Wire Sieves for preparing Iron Filings for Magnetic Lines of Force Diagrams.

The filings that pass through the 30-mesh, but not through the 50-mesh, sieve, are very suitable for tracing magnetic lines of force to be seen at no great distance; but if it is desired to prepare diagrams for lecture purposes, then it is better to use filings that will pass through a 10-mesh sieve but not through a 30-mesh one.

Filings of the right size can be kept separated and ready for use by means of a box (Fig. 226) consisting of four compartments, or trays, A, B, C, D, fitting into one another.

The bottoms of A, B, and C are made respectively of 10-, 30-, and 50-mesh wire gauge, while that of D has no holes in it. Fairly clean iron, or steel, filings, such as can be obtained from a workshop, are placed in the top compartment, A, the lid put on, and the box well shaken; then filings suitable for class diagrams will be found in B; while those in C are of the right size for diagrams intended for close inspection. The bottom compartment, D, receives the fine dust which has passed through the three sieves, and prevents it falling about.

When the filings are to be fixed by the employment of waxed paper, as described on page 74, it is desirable that the paper should have a smooth surface, that it should be left in the melted wax until the heat has driven all the air out of the pores of the paper, otherwise it will present a spotted appearance when the wax hardens; and, lastly, as much of the superfluous wax as possible should be drained off the paper to prevent the wax from forming inequalities, and, by thus rendering the surface of the paper uneven, hindering the filings from taking up their right positions.

II.—THE PREPARATION AND USE OF SILK FOR GALVANOMETER SUSPENSIONS.

THE silk-worm spins two separate fibres at the same time, but, as they are covered with *silk-glue*, they stick together; indeed, if a hank of *raw* silk becomes damp, a great many fibres adhere. Raw silk may also contain calcareous and mineral substances.

The latter are first removed by rinsing the silk in tepid dilute hydrochloric acid, then the *ungumming* is effected by suspending the hank on a smooth wooden rod and moving it about in a bath heated to 90° or 95°C. containing a solution prepared by dissolving 30 to 35 per cent. of soap in *soft* water. After the silk has been kept for about twenty minutes in this bath, it is well to place it in a fresh one, and again, about

twenty minutes later, in a third soap-bath, for the glue dissolves but slowly off the silk when there is much already in solution.

This treatment of the silk enables the fibres to be easily separated with a pair of tweezers, which should be used instead of the fingers, to avoid the fibres becoming greasy or dirty. If the silk be placed on a smooth black surface, like that of a slab of ebonite, a fibre of the required thickness can be more easily selected. For sensitive galvanometers, in which a very light suspended system is employed, the thinnest fibre will generally be strong enough. In all cases it will be found convenient to cut off a piece of fibre about two inches longer than that of the actual suspension required.

A fibre, just before it is used, may be freed from the last traces of grease, as well as from any twist, in the following way: By means of cap cement, bicycle cement, or other cement that softens with heat, fasten a piece of wire of non-magnetic material and having about the same weight as the magnetic needle, pointer, &c., to one end of the fibre, and hang up the fibre by clamping its other end in a support. To facilitate the clamping, the upper end of the fibre may also be stuck to a piece of wire, or, best of all, to the suspension head, when this can be easily removed from the galvanometer as in the case of the pin P (Fig. 25, page 49). Dip a camel's-hair brush in clean absolute alcohol (not methylated spirit), and wash the fibre several times, with an *upward* motion, to avoid breaking the fibre by increasing the downward stress. Cover the support and suspended fibre with a glass shade, and leave it for ten minutes, or so, to untwist.

In attaching the end of the fibre to the needle, only the merest touch of cement should be used, and on no account should the hot cement be allowed to come into contact with the part of the fibre that it is intended shall hang free, otherwise the effective length of the fibre will be shortened, and a slow variation in the

value of the deflection may arise from a viscous yielding of the cement.

The risk of the fibre becoming detached from the hook at the end of the pin P (Fig. 25, page 49) can be removed by tying the fibre to the hook. This can be easily done, if the end of the fibre be stiffened, by touching it with cement so that it can be used as a needle, a plan which also greatly facilitates the end of the fibre being passed through a hole in the roller R (A, Fig. 25).

III.—SHORT HISTORY OF THE ABSOLUTE UNIT OF RESISTANCE,* AND OF THE ELECTRICAL STANDARDS OF THE BOARD OF TRADE.

In 1821 Sir Humphry Davy published the results of his experiments proving that metals varied in their power of conducting electricity, and that this conducting power diminished as the temperature rose; but the idea of resistance being a property of a conductor was due to Ohm, who published the mathematical proof of his famous law in 1827. The writers, however, who immediately followed Ohm did not employ a unit of resistance, but contented themselves with reducing, by calculation, the resistance of all parts of a heterogeneous circuit to a given length of some part of that circuit, so that Lenz, for example, in his paper of 1833, calls the resistance of a conductor its "*reduced length.*"

The next step, when comparing different circuits, was naturally to refer these "reduced lengths" to the length of some one standard wire, although the wire might not form part of any of the circuits under test,

* The earlier part of this History is abstracted from a "Report to the Royal Society on the New Unit of Electrical Resistance, &c.," by the late Prof. Fleeming Jenkin, which, together with the Reports from 1862 to 1869 of the British Association Committee on Standards of Electrical Resistance, and with his Cantor Lectures, were issued by him in 1873 in the form of a very useful book.

and to consider the resistance of unit length of this standard wire as the unit resistance: thus, we find Lenz, in 1838, stating that one foot of No. 11 copper wire was his "unit" of resistance—a unit, however, which he appeared to have selected at random, and without any idea of suggesting that it should be used by others.

In 1840 Wheatstone constructed the first instrument by which definite multiples of a resistance unit could at will be added to, or substracted from, a circuit. And in 1843 he proposed that the resistance of one foot of copper wire weighing 100 grains, which was selected with reference to the British standards of length and weight, should constitute the standard of resistance. Later on other wires were proposed as units of resistance; and, to avoid the inconvenience arising from the multiplicity of standards, Jacobi, in 1848, sent a certain copper wire to Poggendorff and to others, requesting that copies might be taken of it. For Jacobi pointed out that the mere definition of a standard of resistance in terms of the length and weight of a wire of some material was not sufficiently definite, and that good copies of a standard, even if that standard had been originally chosen at random, would be more exact.

Until about 1850, measurements of resistance were confined, with few exceptions, to the laboratory, but about that time underground wires, followed shortly afterwards by submarine cables, began to be employed; and, since it was impossible to ascertain the position of a defect in such a telegraph line by inspection, electrical methods of "*localising the position of a fault*" by measuring the resistance of the wire between the testing station and the faulty spot had to be developed. As early as 1847 C. F. Varley is said to have used a rough method of distance testing, while in 1850 Werner Siemens published two methods, and in 1852 Charles Bright patented a plan for determining the position of a fault by the use of resistance coils.

The first effect of this commercial use of resistance

was to turn the "foot" of the laboratory into the "mile": thus, the unit of resistance in England became that of a mile of No. 16 copper wire; in Germany, of a German mile of No. 8 iron wire; and in France, of a kilometre of iron wire 4 millimetres in diameter. Next, Marié Davy and De la Rue pointed out that, as it was possible by chemical cleaning and subsequent distilling to remove practically all impurities from mercury, this metal was specially suitable for selection as a standard substance; and in 1860 Werner Siemens constructed standards in which his unit was the resistance of a column of chemically pure mercury 1 metre long, 1 square millimetre in cross-section at a temperature of 0°C.

The definition of the "*Siemens unit*" of resistance was a very simple one; and, since mercury in a nearly pure state is not very difficult to obtain, it might be thought that the unit proposed by Siemens would have been finally adopted. The simplest way, however, to obtain a column of mercury of uniform cross-section is to place mercury in a tube of uniform bore, and the cross-section of the bore of such a tube can be most accurately determined by finding the weight of mercury that is contained in a given length, and deducing the volume from a knowledge of the specific gravity of mercury. Although, then, the definition of Siemens unit is apparently based simply on length, cross-section, and temperature, it really depends on weight, specific gravity, and temperature.

In the specimens of this unit originally issued there was an error of 2 per cent., and even in later issues an error of over one-quarter per cent. was introduced up to 1873, through Werner Siemens having adopted 13·557 as the specific gravity of mercury instead of 13·596. The labour, however, bestowed by the late Werner von Siemens on perfecting electrical measurements merits special recognition, as it materially helped in introducing strict accuracy.

All the preceding units of resistance are based on the

more or less arbitrary size and weight of some more or less suitable material; but measurements of resistance can be conceived and carried out entirely without reference to the special qualities of any particular material. In 1849 Kirchhoff effected a measurement of this kind; but it is to W. Weber that we owe the first distinct proposal, made in 1851, of a system of electrical and magnetic measurement in which an electrical resistance would be expressed as an absolute velocity, were "*magnetic permeability*" a simple numeric.

Previous to this, Gauss, desiring to make precise measurements of the distribution of terrestrial magnetism, found it necessary at the outset to decide on a unit of force which, unlike the weight of a given mass, should not be affected by the position of the place at which the experiment was made, and on a magnetic pole whose strength should be independent of any molecular change in steel. He therefore devised what have since become well known as Gauss's "*absolute unit of force*," and the "*unit magnetic pole*," the former being defined as the force which, acting on unit mass, generates unit acceleration, and the latter as the pole which repels an exactly similar pole at unit distance with unit force.

Following Gauss's nomenclature, Weber called the two systems of units to which he was led the "*absolute electromagnetic*" and the "*absolute electrostatic*" systems; but the name "*derived*" would have conveyed the meaning better than "absolute," since the essence of Weber's system consisted in the various electrical and magnetic units being *derived* from those of length, mass, and time.

As soon as the proposal of Weber appeared, W. Thomson (now Lord Kelvin) accepted and extended it by showing that the absolute unit of work formed part of the same system. And ten years later, at the meeting of the British Association in 1861, W. Thomson proposed that a Committee of that Association should be formed to determine the best standard of electrical resistance.

This Committee, which consisted of only six names at the outset, gradually increased its numbers as it enlarged the scope of its work. A few of the members of thirty years ago are still taking an active part in the labours of the Committee on Electrical Standards of to-day, but the Committee has lost by death Clerk Maxwell, Cromwell Varley, Fleeming Jenkin, Joule, Matthiessen, and others whose names are distinguished for the active part they took in the development of electrical science.

The principle of the method employed by the British Association Committee in 1863 for the determination of the unit of resistance was, briefly, as follows: If a coil like that of a tangent galvanometer—for example, c c (Fig. 52, page 96)—be spun in a uniform magnetic field round a vertical axis passing through the centres of the coil and of the needle, an E.M.F. is induced in the coil, this E.M.F. reaching its maximum when the plane of the coil is parallel to the lines of force, and becoming zero when it is perpendicular to the lines of force. If, then, the coil be short-circuited a current will be induced in it, and, although the E.M.F. reverses its direction each time the plane of the coil is perpendicular to the lines of force, and although, therefore, as regards the coil the current flows in opposite directions during the two halves of its revolution, it flows in the same direction as regards the needle. Hence, for a uniform speed of rotation of the coil there will be a constant mean value of the deflecting force exerted on the suspended needle, and, therefore, if the time taken by the coil to make one revolution is small compared with the time of vibration of the needle, the needle will remain steadily deflected as if it were acted on by a perfectly constant deflecting force.

Further, since for a given angular velocity of the coil the average value of the induced current is directly proportional to the strength of the uniform magnetic field, while the controlling force exerted on the needle

is also directly proportional to the strength of this magnetic field, it follows that the magnitude of the deflection is independent of the field. And, as proved on page 91, the deflection is also independent of the strength of the needle. In fact, when the equations connecting the various electric and magnetic magnitudes are written in their simplest forms, *without the introduction of useless coefficients*, it can be shown that, to the first degree of approximation,

$$\tan.\ d = \frac{\pi^2\, a\, n^2\, \omega}{r};$$

where d is the angular deflection produced by a coil of mean radius a, wound with n convolutions of wire, and having a resistance r, when spun with a uniform angular velocity ω, in a medium the magnetic permeability of which is taken as the simple numeric unity, without dimensions.

The product $a\,\omega$ in the last equation equals v, the linear velocity of a point, C (Fig. 52, page 96), at the end of a horizontal diameter; hence,

$$r = \frac{\pi^2\, n^2}{\tan.\ d}\, v;$$

or the resistance of the coil equals a number multiplied into the linear velocity of a point at the end of a horizontal diameter.

If v be measured in centimetres per second, then r will be expressed in *absolute electromagnetic units of resistance.* This unit would, however, be inconveniently small for practical purposes, since, for example, 1 mile of copper wire, $\frac{1}{16}$th of an inch in diameter, has a resistance of about fourteen thousand million, 14×10^9, of such units, and a Siemens mercury unit equals about 98×10^7 absolute electromagnetic units. It was, therefore, decided to call 10^9 of these new units 1 B.A. unit; and, in order to familiarise people with its use, Sir Charles Bright and Mr. Latimer Clark proposed the distinctive name of "*ohmad*," which, in its abbreviated form of *ohm*, was finally adopted.

Twenty-six coils having nearly the form shown in Fig. 134 (page 276), wound with platinum silver wire, and adjusted so as to have 1 B.A. unit of resistance, were distributed gratuitously in 1865 to the directors of public telegraphs in various countries, and to other important people. Also the Committee announced that they would furnish similar coils at the price of £2 10s. apiece, and would undertake "to verify at a small charge any coils made by opticians, as is done for thermometers and barometers at Kew." The expression "opticians" is interesting as indicating that the electrical instrument maker—of which so many exist to-day—was unknown in 1865.

A platinum-silver alloy containing 33 per cent. of platinum and 66 per cent. of silver was used for the wire of these copies of the standard, in consequence of the results obtained by Matthiessen in his excellent work "On the Variation of the Electrical Resistance of Alloys due to Change of Temperature,* On the Electrical Permanency of Metals and Alloys," &c. &c., accounts of which formed part of the Committee's Reports for 1862, 1863, 1864, and 1865.

The Report for 1863 was also remarkable in containing a most valuable article by Clerk Maxwell and Fleeming Jenkin "On the Elementary Relations between Electrical Measurements." To that article the author is indebted for nearly all his early ideas on the subject of exact electrical measurement, for at the time that it appeared there existed no one of the hundred text-books of the present day dealing with the *quantitative* science of electricity, as distinct from the *qualitative* effects obtainable with glass-legged stools and electrified heads of hair. Indeed, ten years later Fleeming Jenkin said, in the preface to his book on "Electricity and Magnetism," published in 1873:—"In England at the present time it may almost be said that there are two sciences of Electricity—one that is taught in ordinary text-books,

* *See* § 86, page 266.

and the other a sort of floating science known more or less perfectly to practical electricians, and expressed in a fragmentary manner in papers by Faraday, Thomson, Maxwell, Joule, Siemens, Matthiessen, Clark, Varley, Culley, and others. . . A student might have mastered De la Rive's large and valuable treatise, and yet feel as if in an unknown country and listening to an unknown tongue in the company of practical men. It is also not a little curious that the science known to the practical men was, so to speak, far more scientific than the science of the text-books."

In the 1863 Report of the B.A. Committee the "*absolute electromagnetic unit of current*" is defined as the current which, flowing through unit length placed along the circumference of a circle of unit radius, exerts a unit of magnetic force at the centre. Methods are also described for measuring a current in absolute units by employing Weber's "*electro-dynamometer*," an instrument in which the torque is measured that is exerted between two coils conveying the current in question. The weight of water that is decomposed per second, as well as the weight of zinc that is deposited per second by this absolute unit of current are stated when the centimetre, gramme, and second are taken as the fundamental units of length, mass, and time. But of so little practical importance was a unit of current at that time that no multiple of the absolute unit was chosen for commercial purposes, and, therefore, no name corresponding with that of the ohm was given to a unit of current.

Defining, however, the "*absolute electromagnetic unit of difference of potentials*" as that which sends the absolute electromagnetic unit of current through the absolute electromagnetic unit of resistance, then Lord Kelvin had shown in 1857 (*see* § 146, page 479) that the E.M.F. of a Daniell's cell was about 10^8 absolute electromagnetic units of P.D. in the centimetre, gramme, second system, a result that was subsequently confirmed by Bosscha. There was, therefore, a good reason for

giving a distinctive name to 10^8 absolute electromagnetic units of P.D.; and in 1862, at the suggestion of Sir Charles Bright and Mr. Latimer Clark, the name *volt* was adopted for this purpose.

Passing over the Reports for the next three years, we come to that for 1867, which, in addition to containing an account of Fleeming Jenkin's first determination of the capacity of a condenser, and a most interesting description of various electrometers constructed by Lord Kelvin, is remarkable in that there is given in it the results obtained by Joule of a very accurate determination of the mechanical equivalent of heat carried out electrically. Joule remarks, in connection with these results: "The equivalents obtained in the two foregoing series of experiments are as much as one-fiftieth in excess of the equivalent I obtained in 1849 by agitating water."

The significance of the preceding remark was not appreciated at the time, for it was supposed that the discrepancy arose from the inherent difficulties met with in such experiments, and it was not even suspected that the difference between the value of the mechanical equivalent of heat obtained mechanically by Joule in 1849 and electrically by his experiments conducted between 1865 and 1867 really indicated that the B.A. unit of resistance had a value something like 2 per cent. less than the ideal value it was intended to possess.

The British Association Committee had aimed at choosing the absolute unit of resistance and the absolute unit of current, so that when a centimetre, gramme, and second were taken as the fundamental units of length, mass, and time, the power given to any circuit stated in ergs per second should be equal to the product of the resistance of the circuit in absolute units into the square of the current in absolute units. Hence, by sending a current, the absolute value of which Joule himself measured with great accuracy, through a resistance the value of which was determined by direct comparison with the B.A. unit, he was able to calculate the power

in ergs per second given to the circuit. If, however, the B.A. unit was, say, 2 per cent. too small, then Joule would *over*-estimate the total energy given to his calorimeter by 2 per cent., and, consequently, would arrive at a value for the mechanical equivalent of heat 2 per cent. *larger* than the true value.

Thinking, however, that the high value of his result arose from imperfections in his apparatus, Joule effected a number of improvements, and then carried out a fresh series of determinations. But, in spite of all the precautions which he took, his final value of the mechanical equivalent of heat obtained electrically was about 1·4 per cent. higher than the value which he had previously obtained by stirring water.

No explanation of the discrepancy just referred to was forthcoming, and the B.A. unit was employed as the practical unit of resistance in Great Britain during the next ten years; the Siemens unit, however, continued to be used as the standard in Germany and some other countries.

Redeterminations of the ohm were carried out by Lorenz in 1873, by F. Kohlrausch in 1874, by H. F. Weber in 1877, by Rowland in 1878, and by Rayleigh and Schuster in 1881; and although the methods of experimenting employed in these five investigations were radically different, the results all agreed in showing that the resistance of the B.A. standard coil was something like 1 per cent. too small. There could be no doubt then that some mistake must have been made by the British Association Committee either in carrying out the experiment described in page 572, or in reducing the results of the measurements.

In 1881 the International Electric Exhibition was held in Paris, and the modern industry of electrical engineering may almost be said to date its existence from that year. A subsidy of £8,000 was given by the French Government towards the expenses of this Exhibition, and a further subsidy of £4,000 for defraying the cost

of holding an International Congress of Electricians. The Congress was divided into three sections, one of which was entirely occupied with the consideration of the steps to be taken to secure the general adoption of "an international system of electrical units."

Everything depended on the selection of the unit of *resistance*, and, consequently, the attention of Section I. of the Congress was mainly devoted to the discussion of this unit. The German representatives strongly urged that the unit proposed and constructed by Siemens possessed the great merits of simplicity of definition and comparative facility for being reproduced if destroyed, and, therefore, that the Siemens unit was the one that ought to secure universal recognition. On the other hand, the English representatives, while admitting that the difficulties connected with the absolute determination of the ohm had led to the introduction of an error of over 1 per cent. in the concrete standard issued by the British Association, advocated the importance of the system in which the unit of resistance was based on the fundamental units of length, mass, and time, and not on the qualities of some special material like mercury.

Of the other nations represented at the Congress, some supported the Germans, while others sympathised with the system that had been developed by the British Association; and ultimately, after a week's animated debating, the following resolutions were unanimously adopted as the result of a very happy compromise which was arrived at:—

"1. For electrical measurements, the fundamental units, the centimetre for length, the gramme for mass, and the second for time (C.G.S.), are adopted.

"2. The practical units, the *Ohm* and the *Volt*, are to retain their existing definitions—10^9 for the Ohm and 10^8 for the Volt.

"3. The unit of resistance (*Ohm*) is to be represented by a column of mercury one square millimetre in section at the temperature of zero Centigrade.

"4. An International Commission is to be appointed to determine, by fresh experiments, the length of a column of mercury one square millimetre in section at a temperature of zero Centigrade, which for practical purposes is to represent the Ohm.

"5. The current produced by a Volt through an Ohm is to be called an *Ampere.*

"6. The quantity of electricity given by an Ampere in a second is to be called a *Coulomb.*

"7. The capacity defined by the condition that a Coulomb charges it to a Volt is to be called a *Farad.*"

By the adoption of the preceding seven resolutions the Congress agreed—

(*a*) To accept the British Association system of absolute units for international use, but not the concrete standard which the B.A. Committee had issued as representing 10^9 C.G.S. absolute electromagnetic units of resistance.

(*b*) To meet the wishes of the Germans by employing as the practical standard the resistance of a column of mercury 1 square millimetre in section at 0°C.; but, instead of selecting a purely arbitrary length like that of 1 metre, as Siemens had done, to ascertain by a series of fresh experiments the length of such a column which had the resistance of 10^9 C.G.S. units.

(*c*) To pay a graceful compliment to the French nation, in whose country the Congress was held, by employing for the future the names of two French experimenters, Ampère and Coulomb, for the units of current and quantity, to which no names had been previously given * with general consent.

The next "International Conference for the Determination of the Electrical Units" was held in Paris in October, 1882. Professor Mascart described the various methods that were known for determining the length of

* The names of *Weber* for the unit of quantity, and *Weber per Second,* or *Oerstedt,* for the unit of current had been used by some writers.

the column of mercury 1 square millimetre in cross-section, which had a resistance of 10^9 C.G.S. units at 0°C.; and the relative values of these methods, together with the results that had been obtained by their use, were considered.

Professor von Helmholtz expressed the view that, of all the investigations on the value of the ohm of which accounts had been published up to that time, those of Lord Rayleigh appeared to be the only ones that had been carried out with the necessary degree of accuracy.

According to the experiments of Lord Rayleigh, 1 B.A. unit equalled $0{\cdot}9867 \times 10^9$ C.G.S. units, and the required length of the column of mercury, which under the specified conditions had a resistance of 1 ohm, was 106·24 centimetres. Other experimenters, however, obtained a somewhat shorter length; and the Commission of 1882 was ultimately led to the following resolutions:—

FIRST RESOLUTION.

"*The Commission considers that the determinations made up to the present time do not possess the amount of agreement that is necessary to fix the numerical value of the Ohm in terms of the mercury column.*

"*It considers, therefore, that further experiments should be carried out.*

"*Without being able to give an authoritative opinion regarding the different methods that have not yet been put into practice, it is of opinion that the following are suitable for giving very exact result:—*

"1. *Induction of a current in a closed circuit.* (*Kirchhoff.*)

"2. *Induction by the earth.* (*W. Weber.*)

"3. *Damping of the motion of swinging magnets.* (*W. Weber.*)

"4. *British Association apparatus.*

"5. *Lorenz method.*

"*In addition, it is desirable that a new determination should be made of the quantity of heat produced by a*

current of known strength, this experiment being for the purpose either of controlling the value of the ohm or of settling with greater accuracy the mechanical equivalent of heat."

SECOND RESOLUTION.

"*The Conference expresses the wish that the French Government will take the necessary steps to place the same standard, or several standards, of resistance at the service of experimenters who are engaged in absolute measurements, in order to render it easy for comparisons to be made.*

"*The Commission is of opinion that, as soon as the results of the different investigations show an agreement which permits of an approximation to the one-thousandth part being arrived at, it will be desirable to stop at this approximation, and use it to fix the practical standard of resistance.*

"In conclusion, the Commission expressed the wish:

"*That the French Government will see fit to transmit to each of the Governments represented at the Conference its desire that, in view of the importance and urgency of a practical solution, it will take the necessary steps to encourage the researches of its people relating to the determination of the electrical units."*

By 1884 a number of new determinations of the value of the ohm had been carried out, and the results, as far as they were generally known at the holding of the second session of the International Conference at Paris in April, 1884, were as follows:—

I.—MEAN ACTION ON A MAGNETIC NEEDLE OF A CURRENT INDUCED IN A ROTATING FRAME.

Date and Observer.	Value of the Ohm Expressed in Centimetres of Mercury 1 Square Millimetre in Section at 0°C.
1865. Committee of the British Association	104·83
1881. Lord Rayleigh and Schuster ...	105·96
1882. Lord Rayleigh	106·27
1882. H. F. Weber...	106·13

II.—CALORIMETRIC METHOD.

1866. Joule	106·22

III.—Steady P.D. Induced in a Rotating Disc Balanced against the P.D. Produced by a Battery.

Date and Observer.	Value of the Ohm Expressed in Centimetres of Mercury 1 Square Millimetre in Section at 0°C.
1873. Lorenz (preliminary)	107·10
1883. Lord Rayleigh and Mrs. Sidgwick ...	106·22
1884. Lorenz	106·19
1884. Lenz	106·13

IV.—Discharge Induced in a Frame when Turned through an angle of 180° in a Magnetic Field.

1874. F. Kohlrausch	105·91
1884. Mascart, De Nerville, and Benoít ...	106·31
1884. G. Wiedemann	106·19

V.—Discharge Induced in a Coil by Altering the Current in Another Coil.

1878. Rowland	105·76
1882. Glazebrook	106·28
1884. Mascart, De Nerville and Benoît ...	106·31
1884. H. F. Weber...	105·37
1884. Roïti	105·90

VI.—Damping of the Vibration of a Magnet.

1882. Dorn	105·46
1884. Wild	105·68
1884. H. F. Weber...	105·26
Mean of the whole	105·97

In consequence of the difference in the methods adopted in carrying out the investigations as well as in the skill possessed by the various experimenters, it would have been right, scientifically, to give different weights to the results before taking the mean. This, however, was regarded as too delicate a matter to undertake; and although it seemed pretty certain that the true number exceeded 106·2 centimetres, it was thought

better to accept 106, the nearest whole number to the arithmetic mean, in order to avoid all question of national jealousy. The resolutions of the 1884 Conference were, therefore, as follows :—

"1. *The legal ohm is the resistance of a column of mercury of a square millimetre cross-section and* 106 *centimetres in length, at a temperature of melting ice.*

"2. *The Conference expresses the wish that the French Government will transmit this resolution to the various States, and recommend the international adoption of it.*

"3. *The Conference recommends the construction of primary standards in mercury in accordance with the resolution previously adopted, and the concurrent employment of sets of secondary resistances in solid alloys which shall be frequently compared amongst one another, and with the primary standard.*

"4. *The ampere is the current the absolute value of which is* 10^{-1} *in electromagnetic units.*

"5. *The volt is the electromotive force which maintains a current of one ampere in a conductor the resistance of which is one legal ohm.*"

Many people considered at the time that the Conference had come to a decision too hurriedly, and that, had the matter been postponed for a year or two, the value of the ohm in terms of the column of mercury might have been stated with greater accuracy. Experience, however, showed that unless the decision had been postponed for several years, nothing would have been gained from the delay ; for the results obtained up to the end of the next year, 1885, were as follows :—

Method	No. II., 1885.	Fletcher	105·95
„	No. III., 1884.	Rowland, Kimball, and Duncan	106·29
„	No. III., 1885.	Lorenz	105·93
„	No. V., 1884.	Rowland and Kimball	106·31
„	No. V., 1885.	Himstedt	105·98
		Mean	106·09

And the mean of these five new determinations does not differ much from the mean of the results published before the holding of the 1884 Conference.

The next step was the resolution arrived at by the British Association Committee on Electrical Standards at their meeting in Birmingham, in 1886, to recommend to the English Government—

"(1) To adopt for a term of ten years the Legal Ohm of the Paris Congress as a legalised standard sufficiently near to the absolute Ohm for commercial purposes.

"(2) That at the end of the ten years' period the Legal Ohm should be defined to a closer approximation to the absolute Ohm.

"(3) That the resolutions of the Paris Congress with respect to the Ampere, the Volt, the Coulomb, and the Farad be adopted.

"(4) That the Resistance Standards belonging to the Committee of the British Association on Electrical Standards now deposited at the Cavendish Laboratory at Cambridge be accepted as the English Legal Standards conformable to the adopted definition of the Paris Congress."

The English Government, however, decided that it was premature to take any action in the matter, and, therefore, although many resistance boxes graduated in "legal ohms" were constructed in England during the next few years (*see* note, page 600), neither this unit of resistance nor any other of the electrical units just referred to had any *legal* value in Great Britain.

Although the 1884 Conference thought it politic to take the arithmetic mean of all the results given in the table on page 581 in order to arrive at the value of the legal ohm, there was no question that certain of the numbers were much less trustworthy than others. For example, an examination of the calculations that had been made by the British Association Committee in

1865 brought to light certain errors that had not previously been detected.

Further, in 1885 Prof. Mascart pointed out that the method of determining the value of the ohm by measuring the damping of the vibration of a magnet, when swinging in a closed coil, was likely to give erroneous results from the alteration of the permanent magnet by the currents induced in the coil; and he proved that this error so introduced would tend to make the length of the column of mercury corresponding with the ohm appear to be too small.

Later on, when discussing the results obtained by Roïti and Himstedt from their measurements of the effect of a series of currents induced in a secondary coil, on starting and stopping a current in a primary coil, Prof. Mascart drew attention to the fact that, since it was necessary to disconnect the galvanometer from the secondary coil at every make, or else at every break, of the current in the primary coil, there was considerable risk of part of the induced current being lost. And he pointed out that such a diminution in the mean value of the induced secondary current would make the required length of the mercury column appear to be too low.

It is further to be noticed that Lord Rayleigh's 1882 result may be regarded as superseding that found by Schuster and himself in 1881, that the value obtained by Lorenz in 1873 was professedly but a provisional one, and that the value arrived at by H. F. Weber, using method No. V., was manifestly too small.

We shall, therefore, obtain a more accurate mean if we neglect the first and second results of method No. I., the first result of method No. III., the fourth result of method No. V., all the results obtained with method No. VI., and the last result in the second table on page 583. When this is done, and when Rowland's 1878 value of 105·76 centimetres is replaced by 106·16, which was afterwards found to more accurately represent the result of his test, we obtain as the mean of all the remaining values 106·17 centimetres.

In 1890 the account of a very accurate "Determination of the Specific Resistance of Mercury in Absolute Measure" by Lorenz's method was communicated by J. Viriamu Jones to the Royal Society, from which it followed that 106·307 centimetres was the required length of the mercury column which represented the ohm.* This number, in consequence of the great care that had been taken by Prof. Jones in arriving at it, may with safety be used to discriminate between the various lengths previously published, and it is seen that it is *closely* in accord with the result 106·29 obtained by Rowland, Kimball, and Duncan in 1884 by the use of this same method No. III., as well as with the value 106·31, which represents the result obtained in each of three separate investigations also carried out in 1884—viz. by Mascart, De Nerville, and Benoît, using method No. IV., by the same experimenters using method No. V., and by Rowland and Kimball, also using method No. V. It also differs but little from Lord Rayleigh's result, 106·27, obtained by using method No. I., or from that deduced by Glazebrook, 106·28, from the employment of method No. V.

There is, then, a very strong reason for believing that the length of 106·3 centimetres is correct to the first four figures.

In December, 1890, the Board of Trade appointed the representatives of the Board of Trade, General

* The bobbin of the coil used in this investigation was made of brass, and yielded a little when it was being turned. This caused it to acquire a slightly elliptical shape with a difference in the lengths of the axes of about 1 part in 1,300. The value 106·307 given above for the ohm was decided on the assumption that the coil was truly circular, but, in a communication made to the Physical Society in May, 1896, Professor Jones has proved that the correction for the ellipticity is about 7 parts in 100,000. Hence, this determination of the specific resistance of mercury leads to the result that the length of the mercury column 1 square millimetre in cross-section, which has a resistance of 1 ohm at 0°C., is 106·300 centimetres (*see* page 599).

Post Office, Royal Society, British Association, and Institution of Electrical Engineers, whose names are given in a note to page 24, "to be a Committee to consider and report whether any, and if so, what, action should be taken by the Board of Trade under Section 6 of the Weights and Measures Act, 1889, with a view of causing new denominations of Standards for the measurement of electricity for use for trade to be made and duly verified."

The first report of this Committee was issued in July, 1891. It contained sixteen resolutions, of which the following were the most important:—

"1. That it is desirable that new denominations of standards for the measurement of electricity should be made and approved by Her Majesty in Council as Board of Trade standards.

"3. That the standard of electrical resistance should be denominated the ohm, and should have the value 1,000,000,000 in terms of the centimetre and second.

"4. That the resistance offered to an unvarying electric current by a column of mercury of a constant cross-sectional area of one square millimetre, and of a length of 106·3 centimetres, at the temperature of melting ice, may be adopted as one ohm.

"5. That the value of the standard of resistance constructed by a committee of the British Association for the Advancement of Science in the years 1863 and 1864, and known as the British Association unit, may be taken as ·9866 of the ohm.

"6. That a material standard, constructed in solid metal, and verified by comparison with the British Association unit, should be adopted as the standard ohm.

"9. That the standard of electrical current should be denominated the ampere, and should have the value one-tenth (0·1) on terms of the centimetre, gramme, and second.

"10. That an unvarying current which, when passed

through a solution of nitrate of silver in water, in accordance with the Specification attached to this Report, deposits silver at the rate of 0·001118 of a gramme per second, may be taken as a current of one ampere.

"12. That instruments constructed on the principle of the balance, in which, by the proper disposition of the conductors, forces of attraction and repulsion are produced, which depend upon the current passing, and are balanced by known weights, should be adopted as the Board of Trade standards for the measurement of current, whether unvarying or alternating.

"13. That the standard of electrical pressure should be denominated the volt, being the pressure which, if steadily applied to a conductor whose resistance is one ohm, will produce a current of one ampere.

"14. That the electrical pressure at a temperature of 62°F. between the poles, or electrodes, of the voltaic cell known as Clark's cell, may be taken as not differencing from a pressure of 1·433 volt, by more than an amount which will be determined by a sub-committee appointed to investigate the question, who will prepare a specification for the construction and use of the cell.

"16. That instruments constructed on the principle of Sir W. Thomson's Quadrant Electrometer used idiostatically, and, for high-pressure, instruments on the principle of the balance, electrostatic forces being balanced against a known weight, should be adopted as Board of Trade standards for the measurement of pressure, whether unvarying or alternating."

Next followed the Specification (*see* § 6, pages 23–26) which was referred to in Resolution 10, and a Draft Order in Council proposed by the Committee for Her Majesty's signature.

In August, 1892, on the occasion of the meeting of the British Association at Edinburgh, there was a conference of its Committee on Electrical Standards with

Professor von Helmholtz, the director of the Imperial Physico-Technical Institute of Berlin, Dr. Guillaume, of the Bureau International des Poids et Mesures of France, and Professor Carhart, of the University of Michigan, U.S.A.

Professor von Helmholtz pointed out that in order to measure the bore of a narrow glass tube we must fill it with mercury and weigh it (*see* page 570), and therefore that it would be better to specify the weight than the cross-section of the column of mercury 106·3 centimetres in length that at 0°C. represented the ohm. He stated that from experiments carried out in his laboratories this weight was found to be 14·452 grammes, which, therefore, he had already recommended the German Government to adopt. He also mentioned that in the recommendations to his Government he had taken 15°C. as the standard temperature for the specification of the E.M.F. of the Clark's cell, and that at 15°C. the value was 1·434 volt.

The British Association Committee accordingly adopted resolutions in conformity with the preceding, and transmitted these resolutions to the Board of Trade. In consequence of this, the Committee of the Board of Trade, after further deliberation, issued a supplementary report in November, 1892, in which their former Resolution 4 was replaced by—

"4. That the resistance offered to an unvarying electric current by a column of mercury, at the temperature of melting ice, 14·4521 grammes in mass of a constant cross-sectional area, and of a length of 106·3 centimetres, may be adopted as one ohm,"

and their former Resolution 14 by—

"14. That the electrical pressure at a temperature of 15° Centigrade between the poles, or electrodes, of the voltaic cell, known as Clark's cell, prepared in accordance with the Specification attached to this report, may be taken as not differing from a pressure of 1·434 volt by more than one part in one thousand."

Then followed the Specification referred to in Resolu-

tion 14, which will be found in full in § 145, pages 467–469.

This substitution of the Centigrade for the Fahrenheit scale of temperature was only made after some discussion; for this supplementary report was the first document issued by the Board of Trade in which the Centigrade scale was officially recognised in Great Britain.

By 1892, then, both the English and the German Governments were advised to adopt the resistance, at 0°C., of a column of mercury 106·3 centimetres long, of uniform cross-section, and weighing 14·4521 grammes, as the value of the ohm; whereas the French Government, some nine years before, had legalised as the ohm the resistance, at 0°C., of a column of mercury only 106 centimetres in length. Hence, before any material unit of resistance could receive international support, it was necessary to summon another international congress. The United States Government was, therefore, advised to utilise the occasion of the holding of the World's Fair at Chicago in 1893 by inviting the other Governments to co-operate with it in sending representatives to constitute a "Chamber of Delegates" for selecting the units of electrical measure. Five delegates were nominated by America, and the Governments of Great Britain, Germany, and France were each asked to nominate an equal number, while three, two, and in some instances one, were allotted to other countries.

Ten countries, as enumerated on page 173, were actually represented in the Chamber, and, after many sittings, it was agreed to adopt certain units, to each of which the name *international* was to be affixed. The definitions of the *international ampere* and *international volt* were the same as those recommended by the Committee of the Board of Trade in the previous year, and the definition of the *international ohm* only differed from that of the Board of Trade ohm in that, while the latter had been defined as having "the value 1,000,000,000 in

terms of the centimetre and second," coupled with the statement "that the resistance offered . . . by a column of mercury . . . may be adopted as one ohm," the *international ohm* was defined as "based upon the ohm equal to 10^9 units of resistance of the C.G.S. system of electromagnetic units, and is represented by the resistance offered . . . by a column of mercury," &c.

Hence, the resistance of the specified column of mercury, which is the *secondary* definition of the ohm in the Board of Trade system, was taken as the *primary* definition in the international system. As, however, the specification of the mercury column was exactly the same in the two cases, and as the resistance of this column is believed to represent the ideal ohm to a high degree of accuracy, no practical difference in the value of the ohm was introduced by this variation in the form of the definition.

From the preceding pages it will be seen that, while the Paris Congress and the British Association Committee on Electrical Standards had defined and named the units of quantity and capacity as well as those of resistance, current, and E.M.F., the Board of Trade had confined its attention to the three latter. The Chamber of Delegates, on the other hand, embodied in their definitions not only the five units just referred to, but also the unit of work—the joule (*see* page 317)—the unit of power—the watt (*see* page 326)—and a new name, the "*henry*," for the unit of self-induction (*see* page 250), this name being selected partly because some of the earliest work on self-induction had been carried out by Prof. Henry, of America, and partly out of compliment to the nation at whose invitation the Chamber had been summoned.

The definition adopted for this sixth unit was:—
"As the unit of induction, the *henry*, which is the induction in a circuit when the electromotive force induced in this circuit is one international volt, while

the inducing current varies at the rate of one ampere per second."

The recommendations made by the Chamber of Delegates, and adopted by the International Congress in August, 1893, were passed by both Houses of Congress in July, 1894, in the form of an "Act to define and establish the Units of Electrical Measure," which, after receiving the signature of the President of the United States, became law in that country.

In August, 1894, the Committee of the Board of Trade submitted their final report to the President of the Board, and stated that since the International Congress held in Chicago had adopted, almost without change, the definitions proposed by the committee in 1892, "they saw no reason for further delay in the legislation of standards." The committee appended to this report a revised Draft Order in Council, which they had prepared, and Mr. Glazebrook's Notes to the Specification of the Clark cell (*see* pages 470–473). An Order in Council in the suggested form was made by Her Majesty on the 23rd August, 1894, and so became law. The following is the text :—

AT THE COURT AT OSBORNE HOUSE, ISLE OF WIGHT,

The 23rd day of August, 1894.

PRESENT,

THE QUEEN'S MOST EXCELLENT MAJESTY IN COUNCIL.

WHEREAS by "The Weights and Measures Act, 1889," it is among other things enacted that the Board of Trade shall from time to time cause such new denominations of standards for the measurement of electricity as appear to them to be required for use in trade to be made and duly verified.

And whereas it has been made to appear to the Board of Trade that new denominations of standards are required for use in trade based upon the following units of electrical measurement, viz.:—

1. The Ohm, which has the value 10^9 in terms of the centimetre and the second of time and is represented by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice 14·4521 grammes in mass of a constant cross-sectional area and of a length of 106·3 centimetres.

2. The Ampere, which has the value $\frac{1}{10}$ in terms of the centimetre, the gramme and the second of time and which is represented by the unvarying electric current which when passed through a solution of nitrate of silver in water, in accordance with the specification appended hereto and marked A, deposits silver at the rate of 0·001118 of a gramme per second.

The Volt which has the value 10^8 in terms of the centimetre, the gramme and the second of time, being

the electrical pressure that if steadily applied to a conductor whose resistance is one ohm will produce a current of one ampere, and which is represented by ·6974 ($\frac{1000}{1434}$) of the electrical pressure at a temperature of 15°C. between the poles of the voltaic cell known as Clark's cell set up in accordance with the specification appended hereto and marked B.

And whereas they have caused the said new de-denominations of standards to be made and duly verified.

NOW, THEREFORE, Her Majesty, by virtue of the power vested in Her by the said Act, by and with the advice of Her Privy Council, is pleased to approve the several denominations of Standards set forth in the schedule hereto as new denominations of Standards for electrical measurement.

C. L. PEEL.

SCHEDULE.

I.—STANDARD OF ELECTRICAL RESISTANCE.

A standard of electrical resistance denominated one Ohm being the resistance between the copper terminals of the instrument marked "Board of Trade Ohm Standard Verified 1894" to the passage of an unvarying electrical current when the coil of insulated wire forming part of the aforesaid instrument and connected to the aforesaid terminals is in all parts at a temperature of 15·4°C.

II.—STANDARD OF ELECTRICAL CURRENT.

A standard of electrical current denominated one ampere being the current which is passing in and through the coils of wire forming part of the instrument marked "Board of Trade Ampere Standard Verified 1894" when on reversing the current in the fixed coils the change in the forces acting upon the suspended coil in its sighted position is exactly balanced by the force

exerted by gravity in Westminster upon the iridio-platinum weight marked A and forming part of the said instrument.

III.—STANDARD OF ELECTRICAL PRESSURE.

A standard of electrical pressure denominated one Volt being one hundredth part of the pressure which when applied between the terminals forming part of the instrument marked "Board of Trade Volt Standard Verified 1894," causes that rotation of the suspended portion of the instrument which is exactly measured by the coincidence of the sighting wire with the image of the fiducial mark A before and after application of the pressure and with that of the fiducial mark B during the application of the pressure, these images being produced by the suspended mirror and observed by means of the eyepiece.

In the use of above standards the limits of accuracy attainable are as follows:—

For the Ohm, within one-hundredth part of one per cent.

For the Ampere, within one-tenth part of one per cent.

For the Volt, within one-tenth part of one per cent.

The coils and instruments referred to in this schedule are deposited at the Board of Trade Standardising Laboratory, 8, Richmond Terrace, Whitehall, London.

SPECIFICATIONS referred to in the foregoing Order in Council.

Specification A.

In the following specification the term silver voltameter means the arrangement of apparatus by means of which an electric current is passed through a solution of nitrate of silver in water. The silver voltameter measures the total electrical quantity which has passed during the time of the experiment, and, by noting this time, the time average of the current, or if the current has been kept constant, the current itself, can be deduced.

In employing the silver voltameter to measure currents of about 1 ampere the following arrangements should be adopted. The kathode on which the silver is to be deposited should take the form of a platinum bowl not less than 10 centimetres in diameter, and from 4 to 5 centimetres in depth.

The anode should be a plate of pure silver some 30 square centimetres in area and 2 or 3 millimetres in thickness.

This is supported horizontally in the liquid near the top of the solution by a platinum wire passed through holes in the plate at opposite corners. To prevent the disintegrated silver which is formed on the anode from falling on to the kathode, the anode should be wrapped round with pure filter paper, secured at the back with sealing-wax.

The liquid should consist of a neutral solution of pure silver nitrate, containing about 15 parts by weight of the nitrate to 85 parts of water.

The resistance of the voltameter changes somewhat as the current passes. To prevent these changes having too great an effect on the current, some resistance besides that of the voltameter should be inserted in the circuit. The total metallic resistance of the circuit should not be less than 10 ohms.

Method of making a Measurement.

The platinum bowl is washed with nitric acid and distilled water, dried by heat, and then left to cool in a desiccator. When thoroughly dry it is weighed carefully.

It is nearly filled with the solution, and connected to the rest of the circuit by being placed on a clean copper support to which a binding screw is attached. This copper support must be insulated.

The anode is then immersed in the solution so as to be well covered by it and supported in that position; the connections to the rest of the circuit are made.

Contact is made at the key, noting the time of contact. The current is allowed to pass for not less than half an hour, and the time at which contact is broken is observed. Care must be taken that the clock used is keeping correct time during this interval.

The solution is now removed from the bowl, and the deposit is washed with distilled water and left to soak for at least six hours. It is then rinsed successively with distilled water and absolute alcohol and dried in a hot-air bath at a temperature of about 160°C. After cooling in a desiccator it is weighed again. The gain in weight gives the silver deposited.

To find the current in amperes, this weight, expressed in grammes, must be divided by the number of seconds during which the current has been passed, and by 0·001118.

The result will be the time-average of the current, if during the interval the current has varied.

In determining by this method the constant of an instrument the current should be kept as nearly constant as possible, and the readings of the instrument observed at frequent intervals of time. These observations give a curve from which the reading corresponding to the mean current (time-average of the current) can be found. The current, as calculated by the voltameter, corresponds to this reading.

Specification B.

On the Preparation of the Clark Cell.

Definition of the Cell.

The cell consists of zinc, or an amalgam of zinc with mercury, and of mercury in a neutral saturated solution of zinc sulphate and mercurous sulphate in water, prepared with mercurous sulphate in excess.

Preparation of the Materials.

1. *The Mercury.*—To secure purity it should be first treated with acid in the usual manner, and subsequently distilled *in vacuo.*

2. *The Zinc.*—Take a portion of a rod of pure redistilled zinc, solder to one end a piece of copper wire, clean the whole with glass-paper or a steel burnisher, carefully removing any loose pieces of the zinc. Just before making up the cell dip the zinc into dilute sulphuric acid, wash with distilled water, and dry with a clean cloth or filter paper.

3. *The Mercurous Sulphate.*—Take mercurous sulphate, purchased as pure, mix with it a small quantity of pure mercury, and wash the whole thoroughly with cold distilled water by agitation in a bottle; drain off the water, and repeat the process at least twice. After the last washing, drain off as much of the water as possible.

4. *The Zinc Sulphate Solution.*—Prepare a *neutral* saturated solution of pure ("pure re-crystallised") zinc sulphate by mixing in a flask distilled water with nearly twice its weight of crystals of pure zinc sulphate, and adding zinc oxide in the proportion of about 2 per cent. by weight of the zinc sulphate crystals to neutralise any free acid. The crystals should be dissolved with the aid of gentle heat, but the temperature to which the solution is raised should not exceed 30°C. Mercurous sulphate treated as described in 3 should be added in the proportion of about 12 per cent. by weight of the zinc sulphate crystals to neutralise any free zinc oxide remaining, and the solution filtered, while still warm, into a stock bottle. Crystals should form as it cools.

5. *The Mercurous Sulphate and Zinc Sulphate Paste.*—Mix the washed mercurous sulphate with the zinc sulphate solution, adding sufficient crystals of zinc sulphate from the stock bottle to ensure saturation, and a small quantity of pure mercury. Shake these up well together to form a paste of the consistence of cream. Heat the paste, but not above a temperature of 30°C. Keep the paste for an hour at this temperature, agitating it from time to time, then allow it to cool; continue to shake it occasionally while it is cooling. Crystals of zinc sulphate should then be distinctly visible, and should be distributed throughout the mass; if this is not the case add more crystals from the stock bottle, and repeat the whole process.

This method ensures the formation of a saturated solution of zinc and mercurous sulphates in water.

To set up the Cell.

The cell may conveniently be set up in a small test-tube of about 2 centimetres' diameter, and 4 or 5 centimetres deep. Place the mercury in the bottom of this tube, filling it to a depth of, say, ·5 centimetre. Cut a cork about ·5 centimetre thick to fit the tube; at one side of the cork bore a hole through which the zinc rod can pass tightly; at the other side bore another hole for the glass tube which covers the platinum wire; at the edge of the cork cut a nick through which the air can pass when the cork is pushed into the tube. Wash the cork thoroughly with warm water, and leave it to soak in water for some hours before use. Pass the zinc rod about 1 centimetre through the cork.

Contact is made with the mercury by means of a platinum wire about No. 22 gauge. This is protected from contact with the other materials of the cell by being sealed into a glass tube. The ends of the wire project from the ends of the tube; one end forms the terminal, the other end and a portion of the glass tube dip into the mercury.

Clean the glass tube and platinum wire carefully, then heat the exposed end of the platinum red hot, and insert it in the mercury in the test-tube, taking care that the whole of the exposed platinum is covered.

Shake up the paste and introduce it without contact with the upper part of the walls of the test-tube, filling the tube above the mercury to a depth of rather more than 1 centimetre.

Then insert the cork and zinc rod, passing the glass tube through the hole prepared for it. Push the cork gently down until its lower surface is nearly in contact with the liquid. The air will thus be nearly all expelled, and the cell should be left in this condition for at least twenty-four hours before sealing, which should be done as follows:—

Melt some marine glue until it is fluid enough to pour by its own weight, and pour it into the test-tube above the cork, using sufficient to cover completely the zinc and soldering. The glass tube containing the platinum wire should project some way above the top of the marine glue.

The cell may be sealed in a more permanent manner by coating the marine glue, when it is set, with a solution of sodium silicate, and leaving it to harden.

The cell thus set up may be mounted in any desirable manner. It is convenient to arrange the mounting so that the cell may be immersed in a water bath up to the level of, say, the upper surface of the cork. Its temperature can then be determined more accurately than is possible when the cell is in air.

In using the cell sudden variations of temperature should as far as possible be avoided.

The form of the vessel containing the cell may be varied. In the H form, the zinc is replaced by an amalgam of 10 parts by weight of zinc to 90 of mercury. The other materials should be prepared as already described. Contact is made with the amalgam in one leg of the cell, and with the mercury in the other, by means of platinum wires sealed through the glass.

The drafting of the American "Specification of the Practical Application of the Definitions of the Ampere and Volt" was deputed by Congress to the National Academy of Sciences, and in February, 1895, the President of the Academy submitted to the Home Secretary of the United States the report drawn up by the committee appointed by the Academy.

The specification so prepared for the use of the silver voltameter was nearly identical with that recommended by the Committee of the Board of Trade (*see* pages 23 to 26 and 595 to 596), but that for the Clark's cell dealt exclusively with the H form. Otherwise, the American and English specifications were generally in accord.

In the note on page 586 it was explained that, after correcting for the ellipticity of the coil used by Professor Viriamu Jones in his determination of the specific resistance of mercury in absolute measure, it followed

that the length of the mercury column 1 square millimetre in cross-section, which had a resistance of 1 ohm at 0°C., was 106·300 centimetres. On using, however, Lorenz's apparatus to test the resistance of a copy of the Board of Trade standard ohm, and assuming that this really represents the resistance at 0°C. of a column of mercury 1 square millimetre in cross-section and 106·3 centimetres in length, Professor Jones finds that the true ohm must have a value equivalent to that of 106·326 centimetres of mercury, or 106·319 centimetres after allowing for the ellipticity of the coil.

We are, therefore, not yet sure of the *fifth* figure in the preceding number, but it is to be expected that this will shortly be obtained with accuracy by the employment of the very carefully-made Lorenz's apparatus that has been constructed for Professor Callendar of the McGill University, Montreal, and which is now—September, 1896—being tested at the City and Guilds of London Central Technical College by Prof. Viriamu Jones and the author.

Note to page 584.—In accordance with the resolution passed at the meeting of the Electrical Standards Committee of the British Association in 1884, the "legal ohm" coils constructed in England were intended to represent the resistance of a column of mercury 106 centimetres in length. But, as a matter of fact, they were made equal to 1·0112 B.A. unit, for in 1884 it was believed that the specific resistance of mercury at 0°C. was $0{\cdot}9540 \times 10^{-4}$ B.A. unit; and, therefore, that 1·0112 B.A. unit was equal to the resistance at 0°C. of 106 centimetres of mercury 1 square millimetre in cross-section. Subsequent measurements, however, showed that the specific resistance of mercury was more nearly $0{\cdot}9535 \times 10^{-4}$ B.A. unit; hence a "legal ohm" constructed in England really represented the resistance of 106·05 centimetres of mercury, and was, therefore, about 5 parts in 10,000 too large.

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